Gravitational red shift
Atomic clocks versus atomic gravimeters

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Gravitational red shift

One of the most important predictions of Einstein's theory of general relativity (GR)

Two clocks located at two different gravitational potentials do not oscillate at the same frequency

Purpose of this lecture

Review the measurements of the red shift and the tests of GR using precise nuclear or atomic clocks

Discuss the arguments presented in a recent publication according to which atomic interferometers using ultracold atoms could be also considered as clocks oscillating at the Compton frequency $\nu = mc^2/h$ (where m is the rest mass of the atom) and could measure the red shift with a higher precision

Explain why we don't agree with this interpretation

Previous tests of the gravitational red shift

Pound and Rebka experiment PRL **4**, 337 (1960)

The gamma ray emitted by one solid sample of iron (^{57}Fe) is absorbed by another identical sample located 22.5 m below

Very narrow line width (Mössbauer effect) allowing one to measure the red shift between the emitter and the receiver

This red shift is measured by moving the emitter in order to introduce a Doppler shift compensating the red shift

Uncertainty on the order of 10^{-2}

Vessot experiment Gen. Rel. and Grav. **10**, 181 (1979)

Hydrogen maser launched on rocket at an altitude of 10,000 km with its frequency compared with another maser one earth

Uncertainty on the order of 10^{-4}

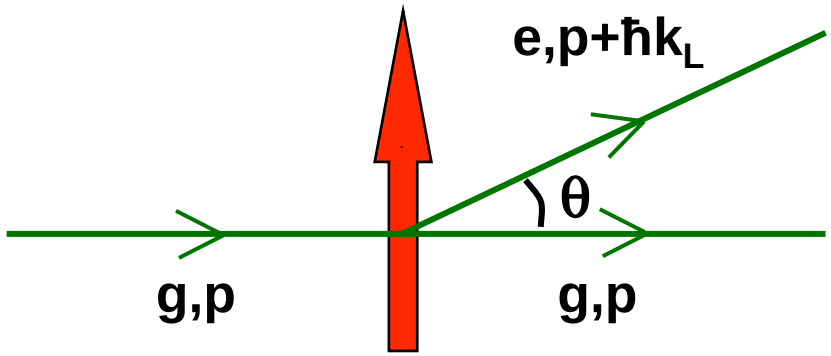
ACES clock in space

Expected uncertainty on the order of $2x10^{-6}$

C. Salomon *et al* , C. R. Acad. Sci. Paris, t.2, Série IV, p. 1313-1330 (2001)

Beam splitter for atomic de Broglie waves

A two-level atom in g crosses at right angle a resonant laser beam. The interaction time corresponds to a $\pi/2$ pulse. When the atom exits the laser beam, it is in linear superposition of g, **p** and e, **p** $+\hbar\mathbf{k}_L$ where p is the initial momentum along the laser



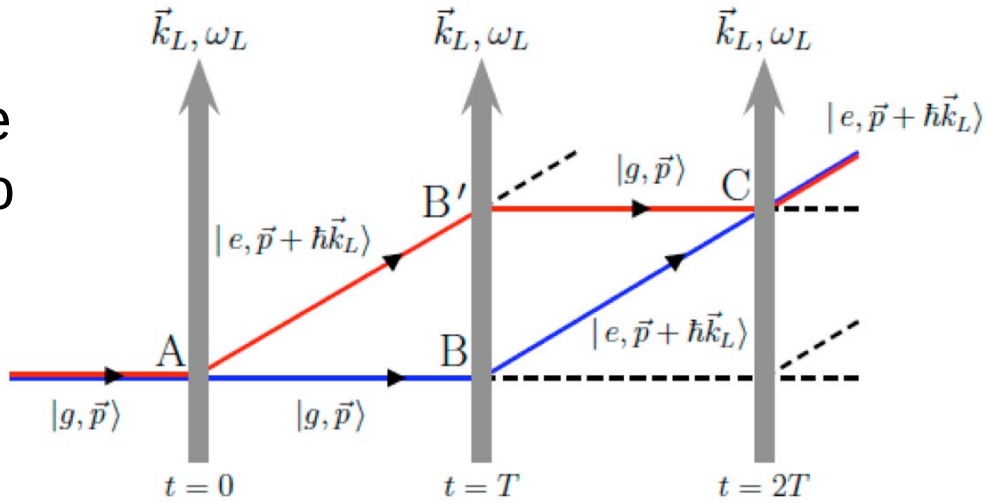
$$\tan\theta = \frac{\hbar k_L}{p}$$

C. Bordé, Phys. Lett. **A 140**, 10 (1989)

Atomic interferometer

The interaction time with the second laser corresponds to a π pulse

Analogy with a Mach-Zehnder interferometer



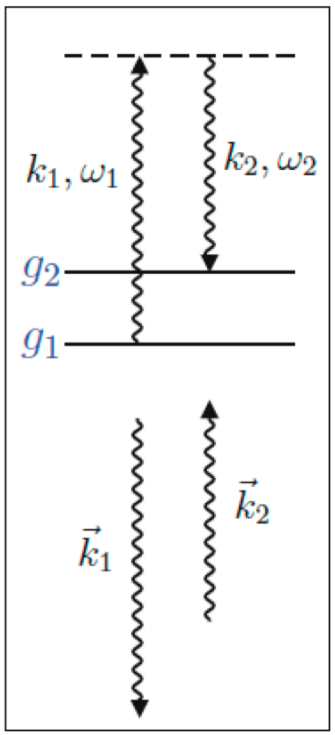
Interferometer of Kasevich and Chu

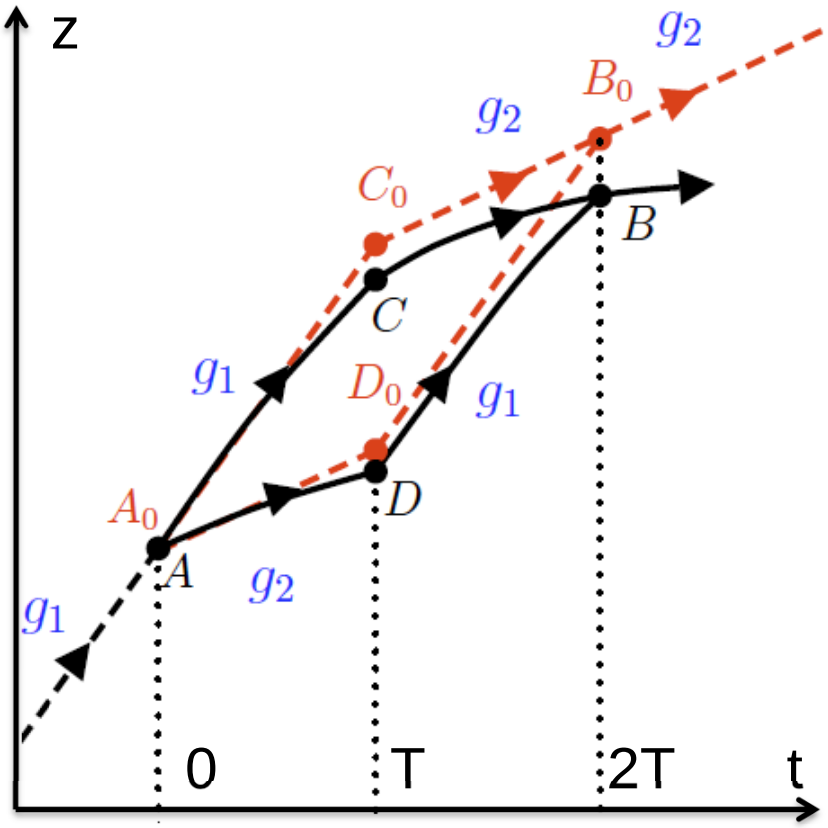
P.R.L. 67, 181 (1991)

g and e are replaced by 2 hyperfine states g_1 and g_2 coupled by a Raman transition stimulated by 2 counter propagating lasers

Momentum transfer: $\hbar\left(\vec{k}_1 - \vec{k}_2\right) \simeq 2\hbar\vec{k}_1$

The free fall of the 2 wave packets in the gravitational field g induces a phase shift, proportional to g, between the 2 wave packets





$A_0C_0B_0D_0A_0$
(g = 0)
Straight lines

ACBDA
(g ≠ 0)
Parabolas

Free fall
$DD_0=CC_0=gT^2/2$
$BB_0=2gT^2$

$$D_0C_0 = \hbar kT / m$$

$$k = \left|\vec{k}_1 - \vec{k}_2\right|$$

Feynman's approach for calculating the phase shift

$K\left(z_b t_b, z_a t_a\right) =$ Probability amplitude for the particle to arrive in z_b at time t_b if it starts from z_a at time t_a. Feynman has shown that:

$K\left(z_b t_b, z_a t_a\right) = \mathcal{N} \sum_\Gamma \exp\left(i S_\Gamma / \hbar\right)$ $\mathcal{N}$: Normalization coefficient

$\sum_\Gamma$: Sum over all possible paths connecting $z_a t_a$ to $z_b t_b$

S_Γ : Action along the path Γ : $S_\Gamma = \int_{t_a}^{t_b} L\left[z(t), \dot{z}(t)\right] dt$ L : Lagrangian

Formulation equivalent to the one based on the Schrödinger equation and leading to the principle of least action in the limit $S \gg \hbar$

If L is a quadratic function of z and $\dot{z}$, one can show that the sum over Γ reduces to a single term corresponding to the classical path $z_a t_a \rightarrow z_b t_b$ for which the action, S_{cl} is extremal

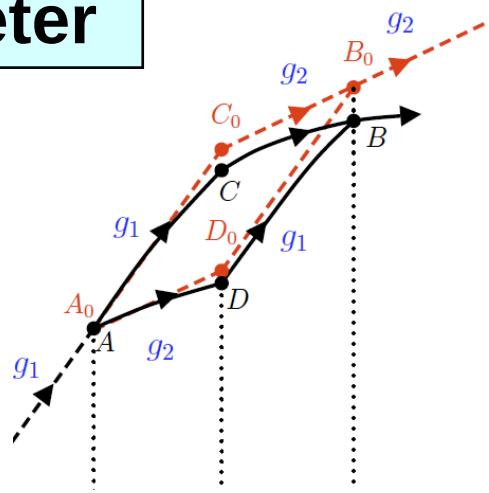
$$K\left(z_b t_b, z_a t_a\right) = F(t_b, t_a) \exp\left\{i S_{\text{cl}}\left(z_b t_b, z_a t_a\right) / \hbar\right\}$$

$F(t_b, t_a)$: independent of z_a and z_b

For an atom in a gravitational field, $L = m\dot{z}^2 / 2 - mgz$ is a quadratic function of z and $\dot{z}$. This approach can be applied

For a review of Feynman's approach applied to interferometry, see:
P. Storey and C.C-T, J.Phys.II France, **4**, 1999 (1994)

Application to the KC interferometer



Propagation along the perturbed trajectories

$$\delta\phi_{\text{prop}} = \frac{1}{\hbar}\left[S_{\text{cl}}(AC) + S_{\text{cl}}(CB) - S_{\text{cl}}(AD) - S_{\text{cl}}(DB)\right]$$

The classical action along a path joining z_a,t_a to z_b,t_b can be exactly calculated

$$S_{\text{cl}}(z_a t_a, z_b t_b) = \frac{m}{2}\frac{(z_b - z_a)^2}{t_b - t_a} - \frac{mg}{2}(z_b + z_a)(t_b - t_a) - \frac{mg^2}{24}(t_b - t_a)^3$$

Using this equation, one finds $\delta\phi_{\text{prop}} = 0$ Exact result

Phase shift due to the interaction with the lasers

Because of the free fall, the laser phases are imprinted on the atomic wave function, not in C_0, D_0, B_0, but in C, B, D

This phase shift is expected to scale as the free fall in units of the laser wavelength, i.e. as gT^2/λ, on the order of kgT^2

Result of the calculation $\delta\phi_{\text{laser}} = kgT^2$

The lasers act as rulers which measure the free fall of atoms

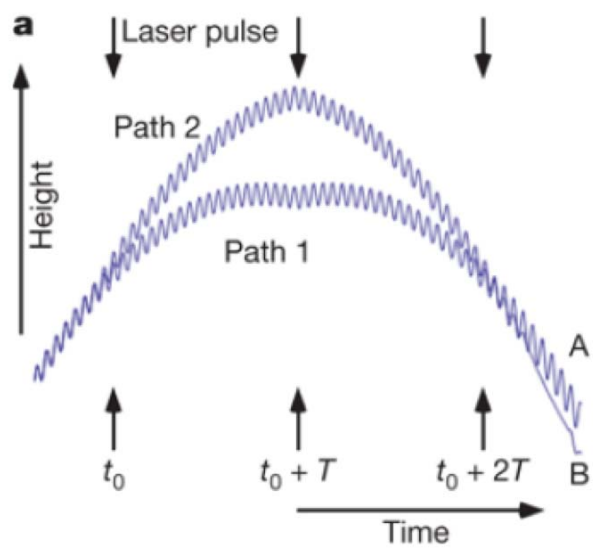
A recent proposed re-interpretation of this experiment

H. Müller, A. Peters and S. Chu, Nature, **463**, 926 (2010)

The atom is considered as a clock 'ticking" at the Compton frequency

$$\omega_C / 2\pi = mc^2 / h \simeq 3.2 \times 10^{25} \text{ Hz}$$

The "atom-clock" propagates along the 2 arms of the interferometer at different heights and experiences different gravitational red shifts along the 2 paths



In spite of the small difference of heights between the 2 paths, the huge value of ω_C provides the best test of Einstein's red shift

<u>We do not agree with this interpretation</u>

P. Wolf, L. Blanchet, C. Bordé, S. Reynaud, C. Salomon and C. Cohen-Tannoudji, Nature, **467**, E1 (2010)

More detailed paper of the same authors, arXiv:1012.1194v1 [gr-qc]
Published in Class. Quantum Grav. **28**, 145017 (2011)

Our arguments

- The <u>exact</u> quantum calculation of the phase shift due to the propagation of the 2 matter waves along the 2 arms gives zero. The contributions of the term $-mgz$ of L (red-shift) and $mv^2/2$ (special relativistic shifts) cancel out. The contribution of the term $mv^2/2$ cannot be determined and subtracted because measuring the trajectories of the atom in the interferometer is impossible.
- The phase shift comes from the change, due to the free fall, of the phases imprinted by the lasers. The interferometer is thus a gravimeter measuring g and not the red shift. The value obtained for g is compared with the one measured with a falling corner cube
- The interest of this experiment is to test that quantum objects, like atoms, fall with the same acceleration as classical objects, like corner cubes. It tests the universality of free fall.
- If g is changed into $g'=g(1+\beta)$ to describe possible anomalies of the red shift, and if the <u>same</u> Lagrangian, which is still quadratic, is used in all calculations, the previous conclusions remain valid. The signal is not sensitive to the red shift. It measures the free fall in g'

Comparison with real clocks

- The red shift measurement uses 2 clocks A and B located at different heights and locked on the frequency ω of an atomic transition. The 2 measured frequencies ω_A and ω_B are exchanged and compared.
- The 2 clocks are in containers (experimental set ups, rockets,..) that are classical and whose trajectories can be measured by radio or laser ranging. The atomic transition of A and B used as a frequency standard is described quantum mechanically but the motion of A and B in space can be described classically because we are not using an interference between two possible paths followed by the <u>same</u> atom
- The motion of the 2 clocks can thus be precisely measured and the contribution of the special relativistic term can be evaluated and subtracted from the total frequency shift to get the red shift
- In the atomic interferometer, we have a single atom whose wave function can follow 2 different paths, requiring a quantum description of atomic motion. The trajectory of the atom cannot be measured. Nowhere a frequency measurement is performed.

Tests of alternative theories

Most alternative theories use a modified Lagrangian *L* with parameters β_i describing corrections to –*mgz* due to non universal couplings between gravity and other fields (for example, electromagnetic and nuclear energies may have different couplings)

For the analysis of the KC interferometer, this new Lagrangien *L* remains quadratic and the previous analysis remains valid. The phase shift $\delta\Phi_{prop}$ due to the propagation in the interferomer is still equal to 0. The total phase shift comes only from the lasers and gives a measure of the free fall of the atom described by the new Lagrangian

The KC interferometer tests UFF ("Universality of Free Fall"), whereas atomic clocks test UCR ("Universality of Clock Rates"). Both tests are related because it is impossible to violate UFF without violating UCR (Schiff's conjecture). They are however complementary because they are not sensitive to the same linear combinations of the β_i (atomic clock transitions are electromagnetic, but the Pound-Rebka experiment uses a nuclear transition.)

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The model of MPC

The model chosen by MPC, which allows them to obtain $\delta\Phi_{prop} \neq 0$, uses in fact 2 different Lagrangians: the first one, L_1, for calculating the classical trajectories; the second one, L_2, whose integrals along the trajectories calculated with L_1, gives them the phase shift $\delta\Phi_{prop}$.

Such a procedure raises serious problems. The propagator obtained by integrating L_2 along the trajectory calculated withL_1, is no longer the Feynman propagator leading to the Schrödinger equation and to the principle of least action. This procedure thus leads to a violation of quantum mechanics! How can one then calculate a quantum phase shift in an atomic interferometer?

The MPC model would thus require a more careful study to justify such an important modification of the Feynman formulation of quantum mechanics. In the absence of such a justification, we conclude that their re-interpretation of the KC experiment as a test of UCR at the Compton frequency is not correct.