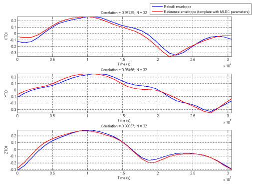
LISA data analysis:
A method for galactic binaries parameters estimation

N. Douillet, J-Y. Vinet
Université de Nice Sophia-Antipolis, ARTEMIS, CNRS, OCA
A.Petiteau, G. Auger
Université Paris VII, Denis Diderot, APC, CNRS

I Frequency and envelope extraction

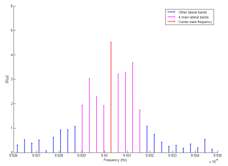
The amplitude of the signal exploited by LISA (optical phase information) is modulated by LISA orbital motion in relation to the source GW signal amplitude.



The shape of the TDI-signal envelope then provides us information (via the pattern beam functions F_+ and F_x) on the sky Location (coordinates ecliptic latitude θ and longitude φ) of the source.

$$E(t) = \left[(h_+ F_+)^2 + (h_\times F_\times)^2 \right]^{1/2}$$

To extract the envelope, we first determine the carrier wave frequency. We then cancel it with an opposite frequency wave.



MLDC 1.1.1a 'blind'

We finally rebuild the envelope by performing a Fourier sum (filtered at low frequencies in the spectrum)

$$s(t)e^{-2i\pi\upsilon_0 t} \Leftrightarrow S(\upsilon) \leftarrow S(\upsilon) * \delta(\upsilon + \upsilon_0) = S(\upsilon + \upsilon_0)$$

$$E(n) = \sum_{k=0}^{7} Y(k) e^{2j\pi(n-1)\frac{(k-1)}{N}}$$

II Creation of an adaptative mesh / sampling to explore the parameter space

We now have to explore the correlation, $C = \int_0^T E(t) \cdot E_0(t) dt$
or the least mean squares (lms) function, $\sigma = \int_0^T \left[E(t) - E_0(t) \right]^2 dt$

on the parameter space $(\lambda_i)_{1 \le i \le M} = (\theta, \varphi, \psi, \iota)$
between the extracted envelope: $E_0 = E(\lambda_0) = E(\theta_0, \varphi_0, \psi_0, \iota_0)$
and a template: $E = E(\lambda) = E(\theta, \varphi, \psi, \iota)$
to estimate the parameters.

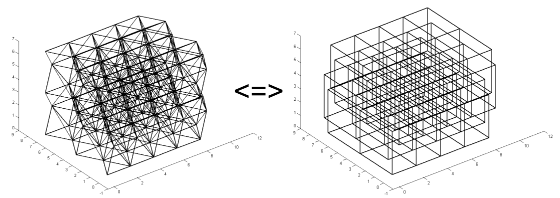
The standard "basic" method when using exhaustive template based search, is to build a regular N-dimension square grid to sample the parameter space before exploring it:

$$F_k = ([\![1;N]\!] \cdot \Delta\theta) \times ([\![1;N]\!] \cdot \Delta\varphi) \times ([\![1;N]\!] \cdot \Delta\psi) \times ([\![1;N]\!] \cdot \Delta\iota)$$

The main drawback of such a method is that its shape doesn't fit the topology of the space: indeed, the distance function in an n-dimension squared grid is Euclidian whereas the least mean squares function (lms) is not. If it happens minima and maxima are too close one to each other, this kind of grid doesn't suit well.

An idea to overcome this problem is to improve the sample-points layout in the parameter space by increasing its density in the concave area and decreasing it in the convex area (aiming for the absolute minimum, and given that our criterion is the lms)

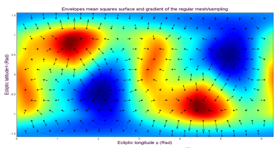
IV Convergence and optimization step



Instead of a squared grid we prefer to use a triangular grid to sample the parameter space, in order to have a more isotropic and more dense mesh.

We have developped an algorithm to build quite easily such a N-dimensional triangular mesh by an iterative process: indeed it is equivalent of several squared meshes shifted from one to the other.

To do this we start from a regular grid of sample-points and make them « migrate » following their gradient opposite direction:



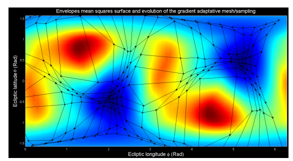
$$[\lambda_i(t + \Delta t)]_{1 \le i \le M} = [\lambda_i(t)]_{1 \le i \le M} - \mu \nabla \sigma_\lambda$$

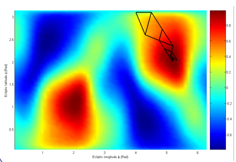
With: $\nabla \sigma_\lambda = 2\int_0^T \left[E(t) - E_0(t) \right] dt \cdot \left[\partial_\theta E(t)\ \partial_\varphi E(t)\ \partial_\psi E(t)\ \partial_\iota(t) \right]^T$

And μ, a coefficient function of the local gradient and the sampling step.

This technique allows to save computational cost by avoiding useless samples.

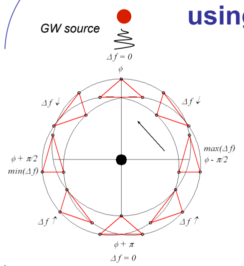
Then we can only keep one point per concave area, the one which has the lowest lms.

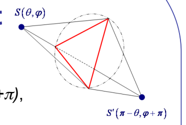




The last step of our algorithm is the convergence step once it has reached the targeted area. This step is direct since convex/concave optimization is a solved problem. Almost all the algorithms used here are quasi-Newton like: Gauss-Newton, Levenberg Marquardt, BFGS.

III Remove LISA main symmetry using Doppler effect



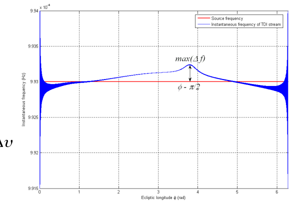


To distinguish between a (θ, φ) source and its opposite in the sky, $(-\theta, \varphi+\pi)$, it is necessary to remove LISA main ambiguity. The only signal envelope is unable to provide us this information.

We have to use the whole TDI signal. LISA orbital motion induce a tiny Doppler shift of the instantaneous frequency.

To estimate this shift is equivalent to compute the sign of the propagation way of the wave.

Detected Doppler shift is very tiny:



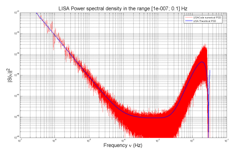
$$f_{max} / f_{min} = \upsilon\left(1 + \frac{2\pi R}{cT}\sin\theta\right) = \upsilon\left(1 \pm \frac{v}{c}\right) = \upsilon \pm \Delta\upsilon$$

$$\left(\frac{\Delta\upsilon}{\upsilon}\right)_{max} = \frac{2\pi R}{cT} \sim 10^{-4} \quad ; \quad \Phi = \varphi \pm \frac{\pi}{2}$$

VI Results on the parameters estimation

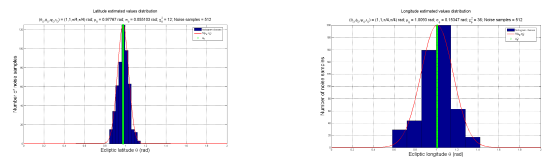
For our simulations, we use LISACode, our french LISA simulator developped by APC lab.

We use here a single source combined with 512 independant noises files, with A SNR ~ 10.



LISA noises power spectral density

We are yet able with our method to estimate 6 parameters over 7, with a satisfying precision: α, the origin phase estimation remains too disturbed by the noise, and very much dependant on the frequency precision.



Latitude and longitude estimated values distributions