Relativistic astrometry and Time Transfer Functions

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Abstract

The Gaia mission, expected for launch in 2013, will observe 10^9 stars with an angular precision up to 1 microarcsecond (μas). At this accuracy, General Relativity (GR) plays a role in the propagation of a light ray : one has to take into account the light deflection by all principal Solar System bodies and the aberration due to the satellite motion. Almost all of the studies devoted to relativistic astrometry are based on the integration of the null geodesic differential equations. However, the gravitational deflection of light rays can be calculated by a different method, based on the determination of the Time Transfer Functions (TTF) between two arbitrary points-events. We give a brief review of the preliminary results obtained with this method.

General principle

Let's take a light signal emitted at a point event $x_A = (t_A, \vec{x}_A)$ and received at a point event $x_B = (t_B, \vec{x}_B)$, considering the *IAU 2000* [2] barycentric reference system (BCRS) and a deflector body at $x_P = (t_P, \vec{x}_P)$.
We have to calculate k^{x_B}, the tangent vector to the null geodesic at x_B.

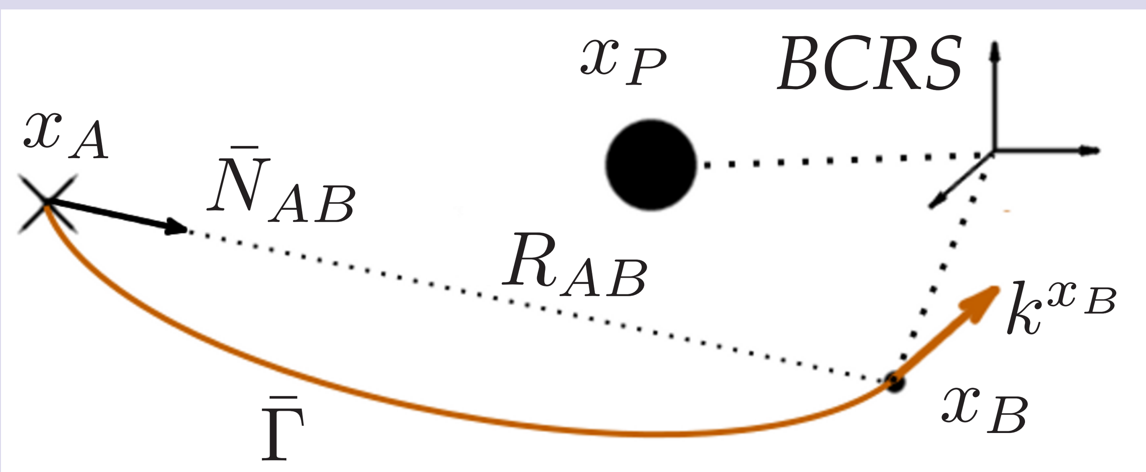

Fig : Light propagation between x_A and x_B

Knowing the signal time delay $\mathcal{T}_r$ between x_A and x_B, this vector can be computed using :

$$\hat{k}_i^{x_B} = \frac{k_i^{x_B}}{k_0^{x_B}} = -c\frac{\partial \mathcal{T}_r}{\partial x_B^i}\left[1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1},$$

where $\mathcal{T}_r$ can be calculated using the TTF formalism [1].

Calculating k_μ using $\mathcal{T}_r$

Using a standard post-Newtonian approximation of GR and the *IAU 2000* [2] development for the gravitational potentials W and W^i, we can write an equation for the tangent vector $\hat{k}_i^{x_B}$:

$$\hat{k}_i^{x_B} = -N_{AB}^i - \frac{1}{c^2}(\gamma+1)\int_0^1 \left[W N_{AB}^i + (1-\lambda)R_{AB}\partial_{x^i}W\right] d\lambda$$
$$-\frac{1}{c^3}\int_0^1 \left[-4W^i + (1-\lambda)R_{AB}N_{AB}^i\left(-4\partial_{x^i}W^i + (\gamma+1)\partial_t W\right)\right] d\lambda$$
$$+\mathcal{O}(c^{-4}),$$

where the integral is calculated along the Minkowskian line of sight.

A method for relativistic astrometry

Since this equation is rather difficult to integrate for a non-static space-time, we split the problem in three parts.

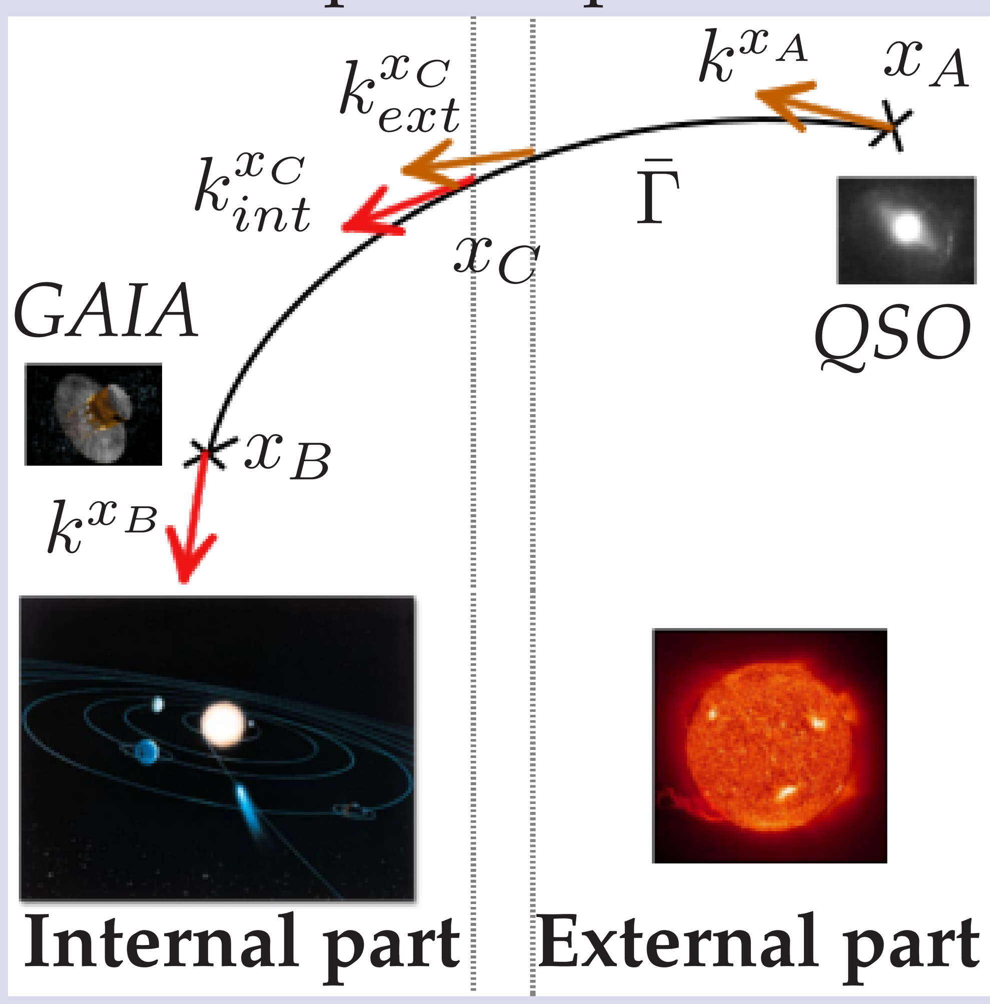

Fig : The 3-parts method

External part : from $x_A \to \infty$ (source at infinity) to x_C, pointlike mass at Solar System Barycenter ;

Internal part : from x_C to x_B, Solar System deflecting bodies moving with a chosen motion law ;

Matching zone : at x_C, defined by setting the angular distance between $k_{int}^{x_C}$ and $k_{ext}^{x_C}$ at less than 1 μas .

The observable

The real observable, the director cosines, is obtained when one substracts the aberration due to the satellite. So, one has to introduce its motion v^i and attitude by means of a tetrad of 4 quadrivectors $E_{\hat{a}}^\mu$ ($\hat{a} = 0, 3$) and project $k_i^{x_B}$ on it :

$$\cos\Phi_{\hat{a}}^{(B)} = -\frac{E_{\hat{a}}^0 + \hat{k}_i^{x_B} E_{\hat{a}}^i}{E_{\hat{0}}^0\left(1 + \hat{k}_i^{x_B}\frac{v^i}{c}\right)}.$$

Conclusions

We present a relativistic astrometric modelling based on the TTF formalism. It's main advantage is to avoid the null-geodesic calculation. Currently, we are working on its development and the comparison to other existing methods.

References

[1] Le Poncin Lafitte, Linet, Teyssandier. *Classical and Quantum Gravity*, 09/2004

[2] Soffel et al. . *The Astronomical Journal*, 12/2003.