



Testing gravitation using asteroids observations with Gaia

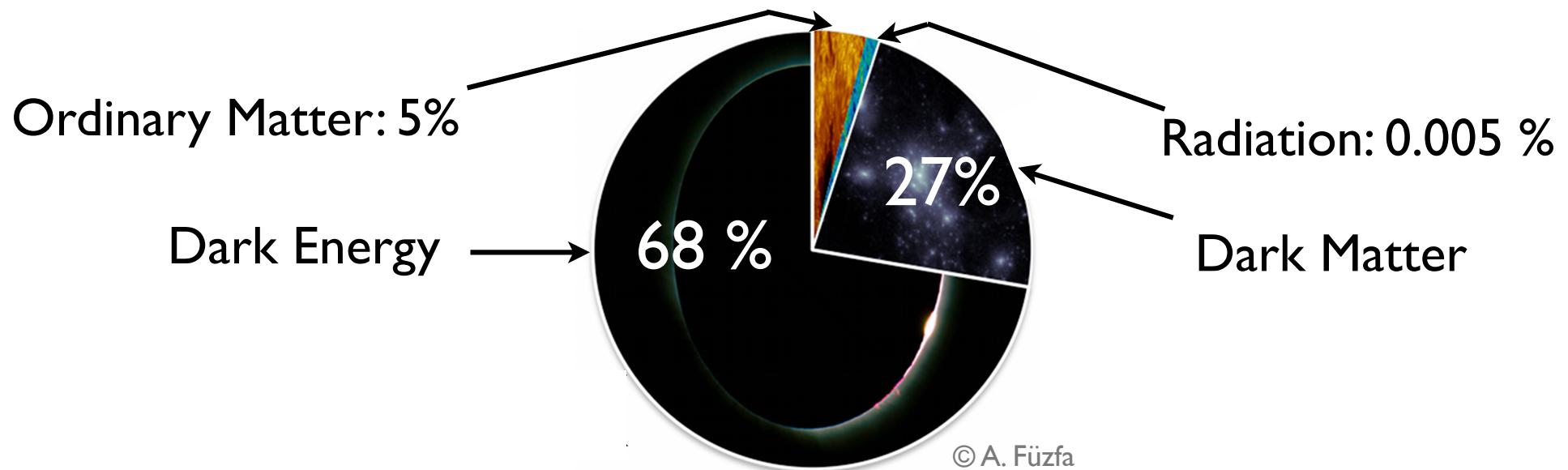
A. Hees - Rhodes University, South Africa

D. Hestroffer and P. David - IMCCE, Paris Obs.

C. Le Poncin-Lafitte - LNE-SYRTE, Paris Obs.

Motivations to test GR

- **Quantum theory of gravity:**
 - GR: classic theory (not a quantum theory)
 - at high energy: quantum effects should appear
 - useful to study black holes and the Planck Era
- **Unification** of all fundamental interactions: unify Standard model of particles with gravitation
- Cosmological and galactic observations required **Dark Matter** and **Dark Energy**: not directly observed so far \Rightarrow hints of a deviation from GR ?



Basic principles of GR

I) Equivalence Principle:

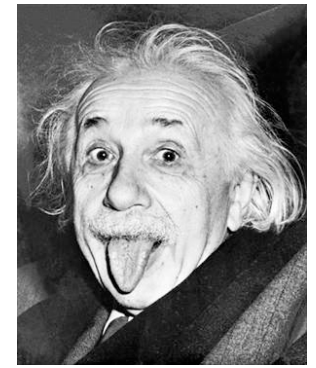
- 3 facets: Universality of free fall, Local Position/Lorentz Invariance
- very well tested (10^{-13} with Eötvash experiments and with Lunar Laser Ranging ; 10^{-4} with grav. redshift ; no variation of constants)¹
- more accurate measurement needed: alternative (string) theories predict violation smaller² → MICROSCOPE accuracy 10^{-15}
- **Gravitation** \Leftrightarrow **space-time curvature** (described by a metric $g_{\mu\nu}$)
- free-falling masses follow **geodesics** of this metric and ideal clocks measure proper time

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

¹ C. Will, LRR, 9, 2006

² T. Damour, CQG, 29-184001, 2012

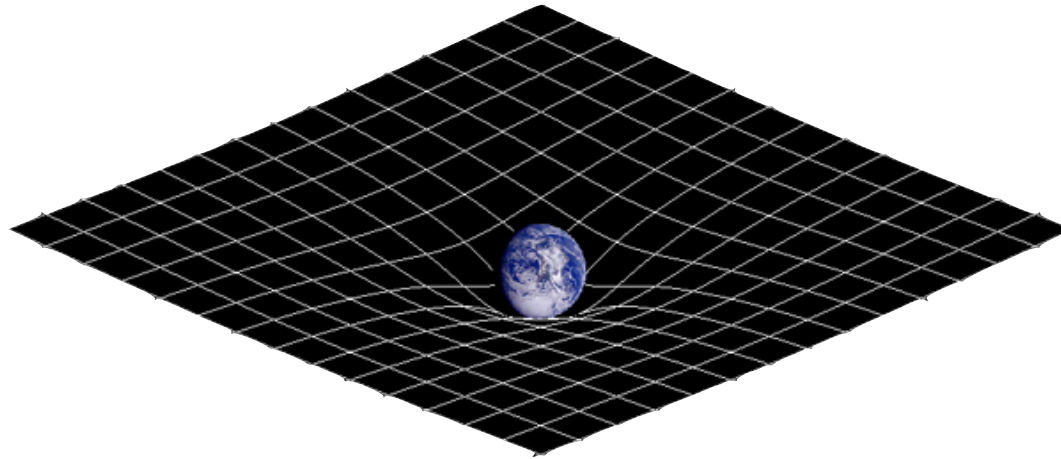
Basic principles of GR



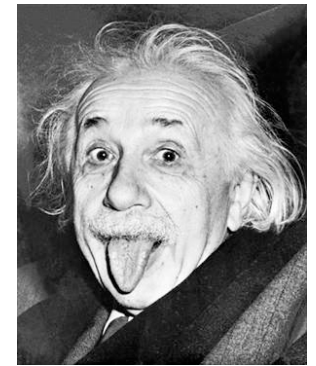
II) Field equations (determination of the metric):

- Einstein Equations:
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

space-time curvature (metric) \Leftrightarrow matter-energy content



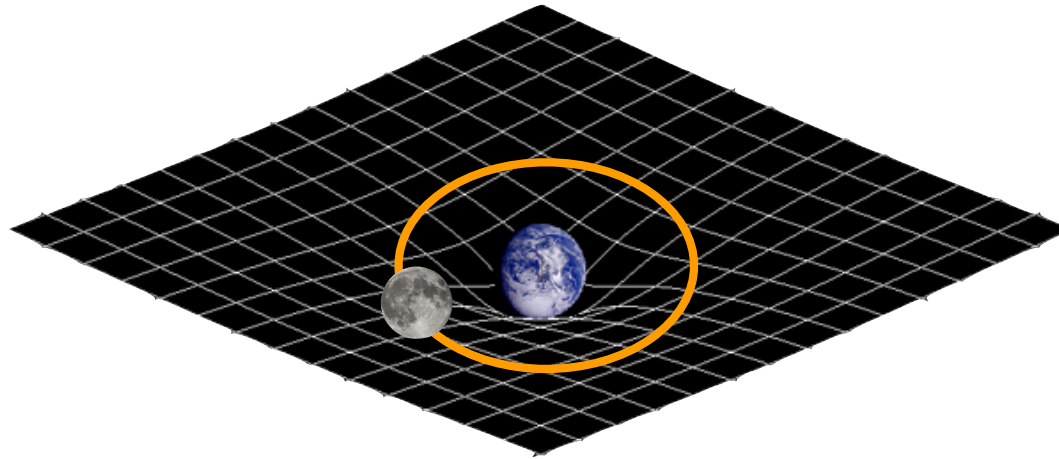
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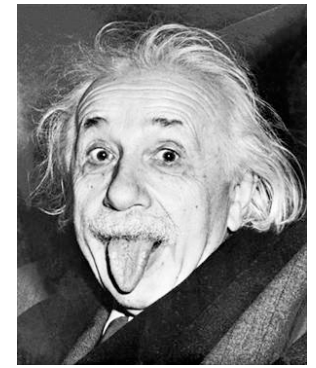
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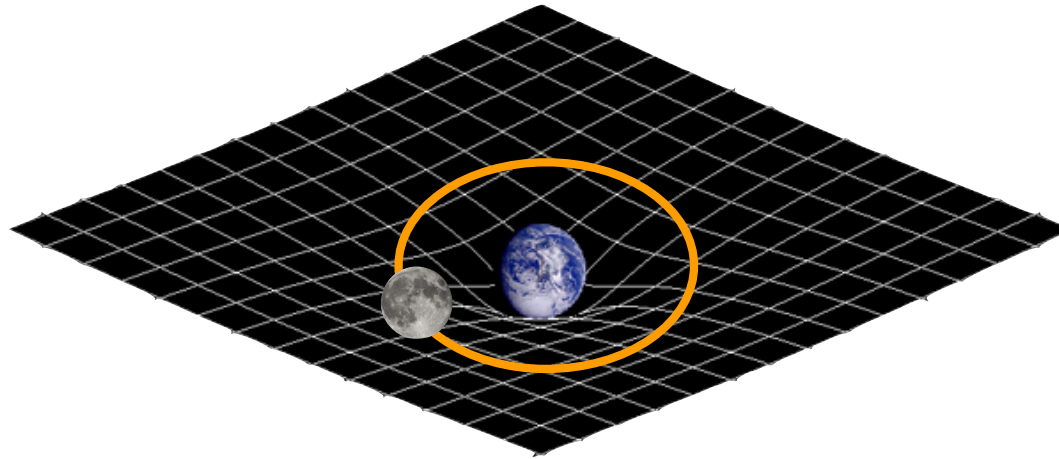
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- important effects for space-mission:
 - dynamics \neq from Newton (ex.: advance of the perihelion)
 - proper time (measured by ideal clocks) \neq coordinate time
 - coordinate time delay for light propagation (Range/Doppler)
 - light deflection (VLBI, astrometry)

Tests of the gravitational dynamics

- How to test the form of the metric/the Einstein field equations ? Two frameworks widely used so far:

¹ C. Will, LRR, 9, 2006
“Theory and Experiment in Grav. Physics”, C. Will, 1993

² E.G. Adelberger, Progress in Part. and Nucl. Phys., 62/102, 2009
“The Search for Non-Newtonian gravity”, E. Fischbach, C. Talmadge, 1998 5

Tests of the gravitational dynamics

- How to test the form of the metric/the Einstein field equations ? Two frameworks widely used so far:

I) Parametrized Post-Newtonian Formalism¹

- powerful phenomenology making an interface between theoretical development and experiments
- metric parametrized by 10 dimensionless coefficients
- γ and β whose values are 1 in GR

$$ds^2 = (1 + 2\phi_N + 2\beta\phi_N^2 + \dots)dt^2 - (1 - 2\gamma\phi_N + \dots)d\vec{x}^2$$

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II) Fifth force formalism²

- modification of Newton potential of the form of a Yukawa potential

$$\phi(r) = \frac{GM}{c^2 r} \left(1 + \alpha e^{-r/\lambda} \right)$$

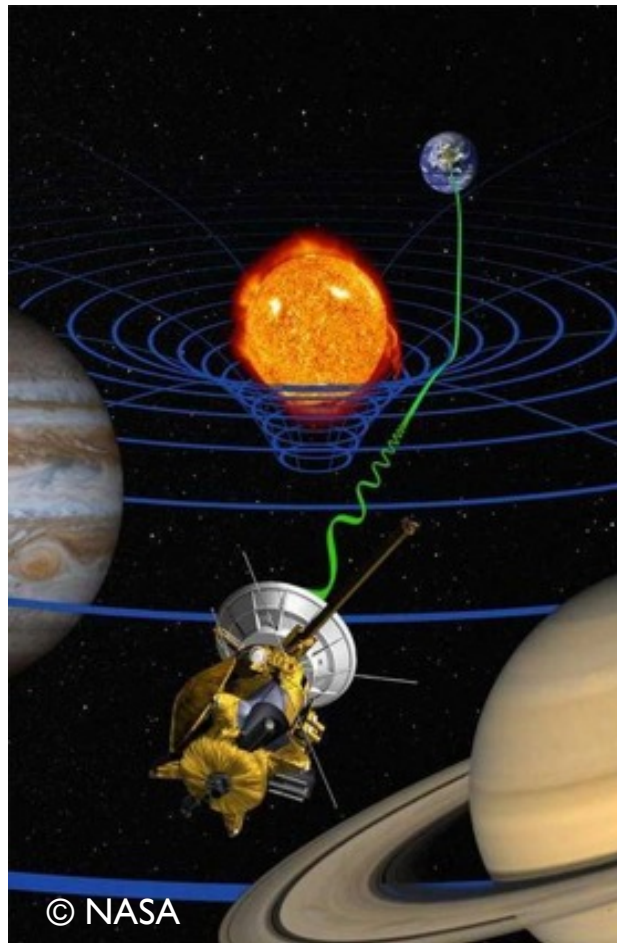
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Constraints on PPN parameters

- Measurement of the Shapiro time delay with Cassini¹

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

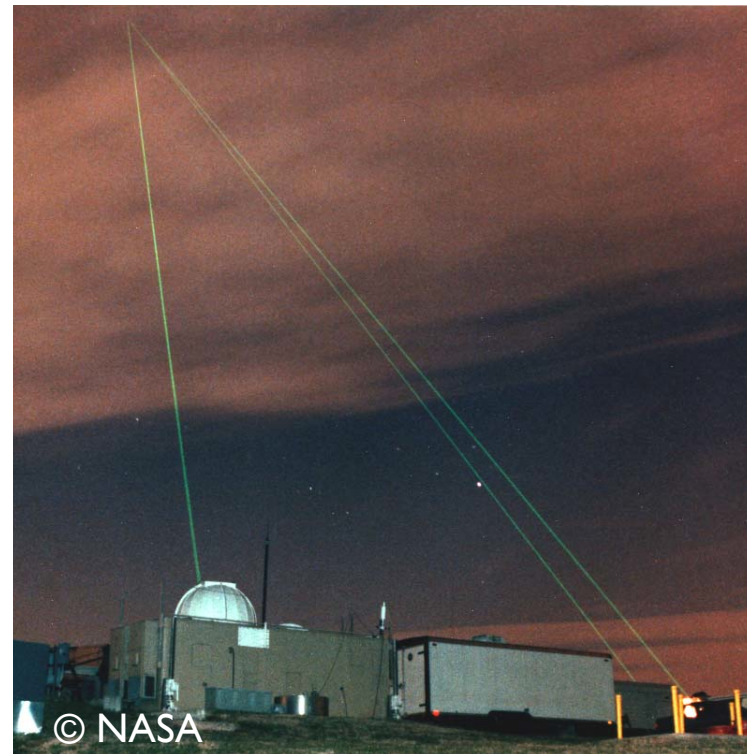


- Planetary ephemerides INPOP²

$$\beta - 1 = (0.2 \pm 2.5) \times 10^{-5}$$

- Dynamic of the orbit of the Moon with LLR³

$$\beta - 1 = (2.1 \pm 1.1) \times 10^{-4}$$



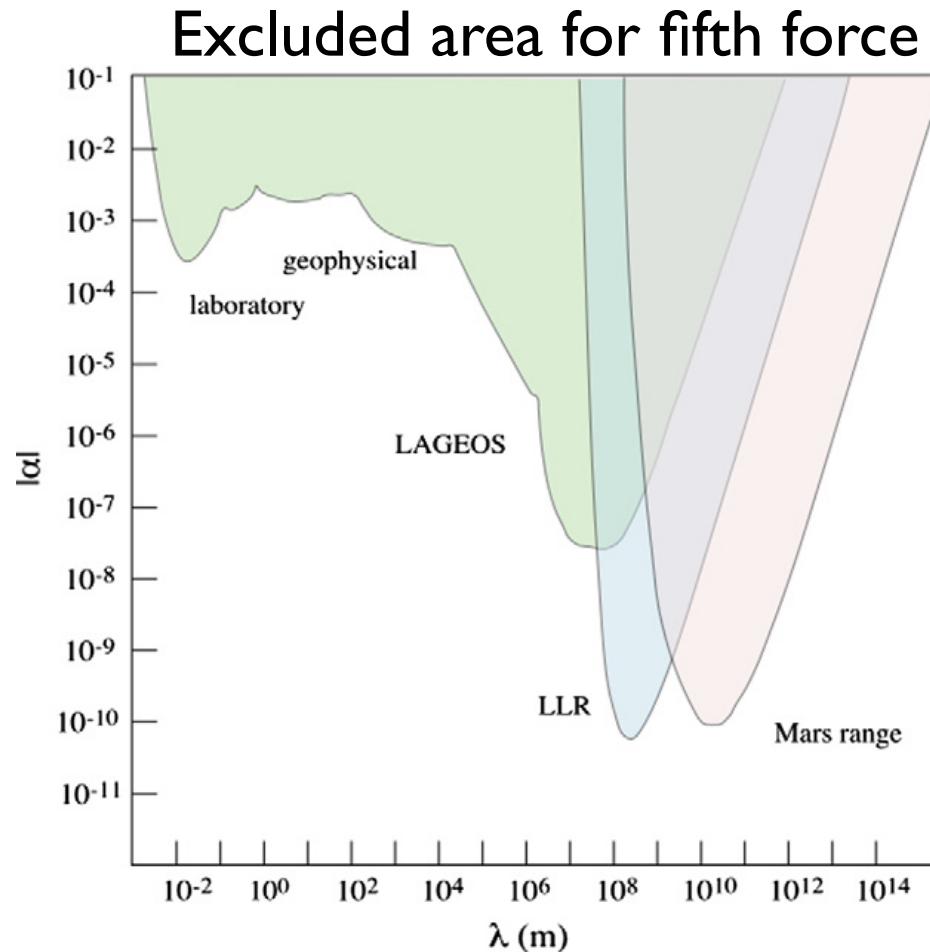
¹ B. Bertotti, L. Iess, P. Tortora, Nature, 425/374, 2003

² A. Verma et al, A & A, 561, A115, 2014

³ J. Williams, S. Turyshev, D. Boggs, IJMP D, 18/1129, 2009

Fifth force formalism

- Search for a deviation of the Newton potential of the form of a Yukawa potential¹ $\phi(r) = \frac{GM}{c^2 r} \left(1 + \alpha e^{-r/\lambda} \right)$



- Very good constraints in this formalism **except at** small and large distances

from A.Konopliv et al,
Icarus, 211/401, 2011

¹ E.G. Adelberger, Progress in Part. and Nucl. Phys., 62/102, 2009
"The Search for Non-Newtonian gravity", E. Fischbach, C. Talmadge, 1998

Is it enough ?

- Still strong motivations to improve the current tests:
 - tensor-scalar theories “naturally” **converging towards GR**¹
 - **screening theories**: modification of GR “hidden” in certain region of space-time: chameleons², symmetron³, Vainshtein mechanism⁴
 - tensor-scalar theories with a **decoupling** of the scalar field⁵

We have strong motivations to pursue this kind of tests!

¹ T. Damour, K. Nordtvedt, PRD 48/3436 and PRL 70/2217, 1993

² J. Khoury, A. Weltman, PRD 69/044026 and PRL 93/171104, 2004

³ K. Hinterbichler, et al, PRD84/103521 and PRL104/231301, 2010

⁴ A. Vainshtein, Phys. Let. B, 39/393, 1972

⁵ T. Damour, A. Polyakov, Nucl. Phys. B, 1994

O. Minazzoli, A. Hees, PRD 88/1504, 2013

Is it necessary to go beyond ?

Post Einsteinian Grav.

- phenomenology
- non local field equation:
quantization ?

$$G_{\mu\nu}[k] = \chi_{\mu\nu}^{\alpha\beta}[k]T_{\alpha\beta}[k]$$

- metric: parametrized by
2 arbitrary functions

M.T. Jaekel, S. Reynaud, CQG, 2005

SME

- phenomenology
- violation of Lorentz
symmetry coming from a
fundamental level
- action parametrized by a
tensor $\bar{S}^{\mu\nu}$

Q. Bailey, A. Kostelecky, PRD, 2006

Fab Four

- General 2nd order tensor-
scalar theory
- developed in cosmology:
Dark Energy
- weak-field metric:
parametrized by **4
parameters**

J.P. Bruneton et al, Adv. in Astr., 2012

MOND

- phenomenology
- developed for galactic observations: Dark
Matter (galactic rotation curves)
- main effect in the Solar System: **External
Field Effect**

$$U = \frac{GM}{r} + \frac{Q_2}{2}x^i x^j \left(e_i e_j - \frac{1}{3}\delta_{ij} \right)$$

L. Blanchet, J. Novak, MNRAS, 2011

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~~PPN formalism : γ, β, \dots~~

~~5th force formalism: α, λ~~

Is it necessary to go beyond ?

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Currently: lack of constraints from Solar System for these theories !

Interesting to consider them and to constrain them using Solar System observations

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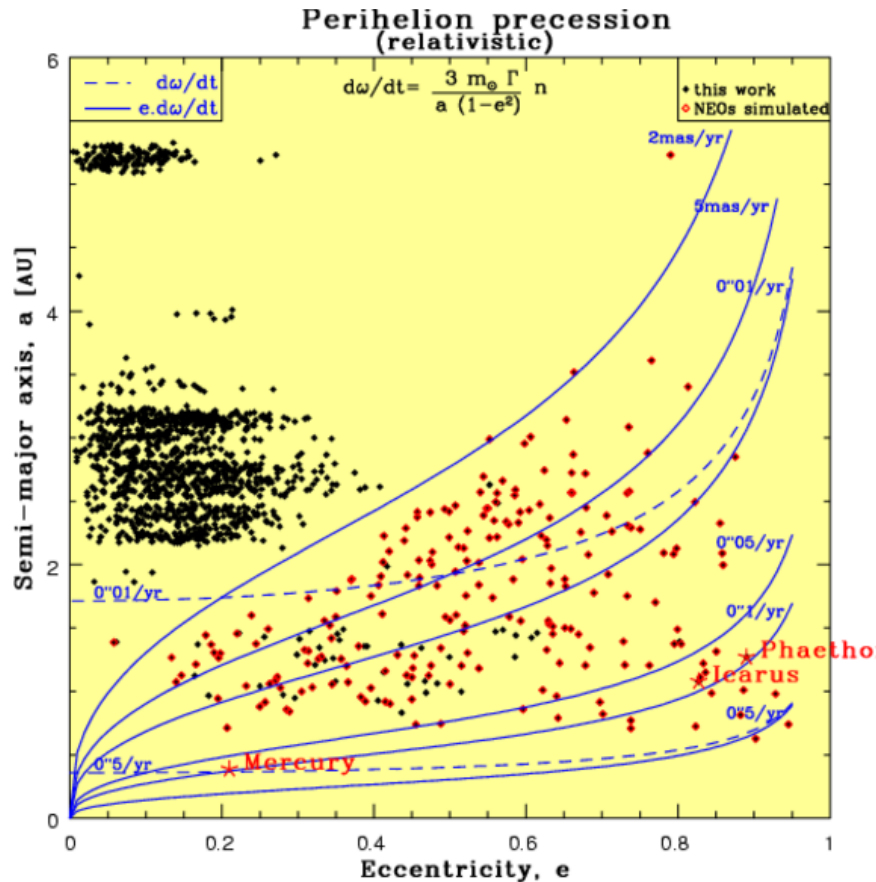
GAIA



- Launched in December 19 2013
- Successor of Hipparcos, it will bring some huge improvements:
 - observation of ~ 1 billion stars, 3D mapping of our galaxy
 - parallax to $25 \mu\text{as}$ and proper motion to $15 \mu\text{as/yr}$
 - colours from low resolution spectro-photometry
 - radial velocities from spectrometer
 - astrometric and photometric measurements for a large number of SSOs, mainly asteroids: high precision on a CCD basis

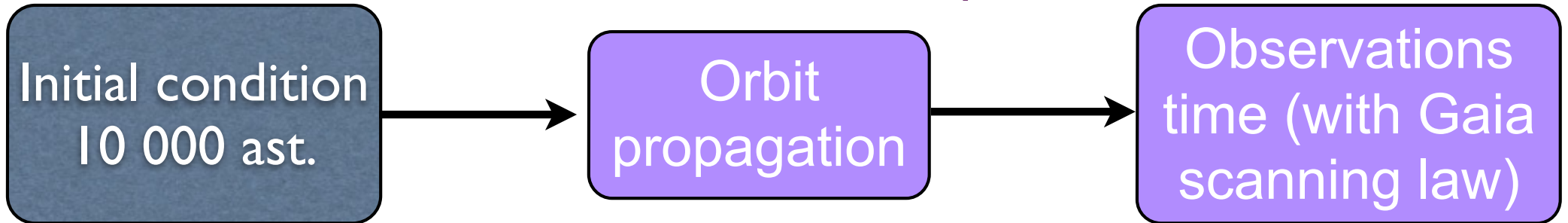
Asteroids: Gaia

- Use GAIA **asteroid observations** to test GR: advantage of a large samples of different orbital parameters (300 000 objects)
 - decorrelation of parameters
 - complementary to planetary ephemerides (different bodies, different type of observations, different method to analyze the data)
- accuracy ~ 0.2-0.5 mas



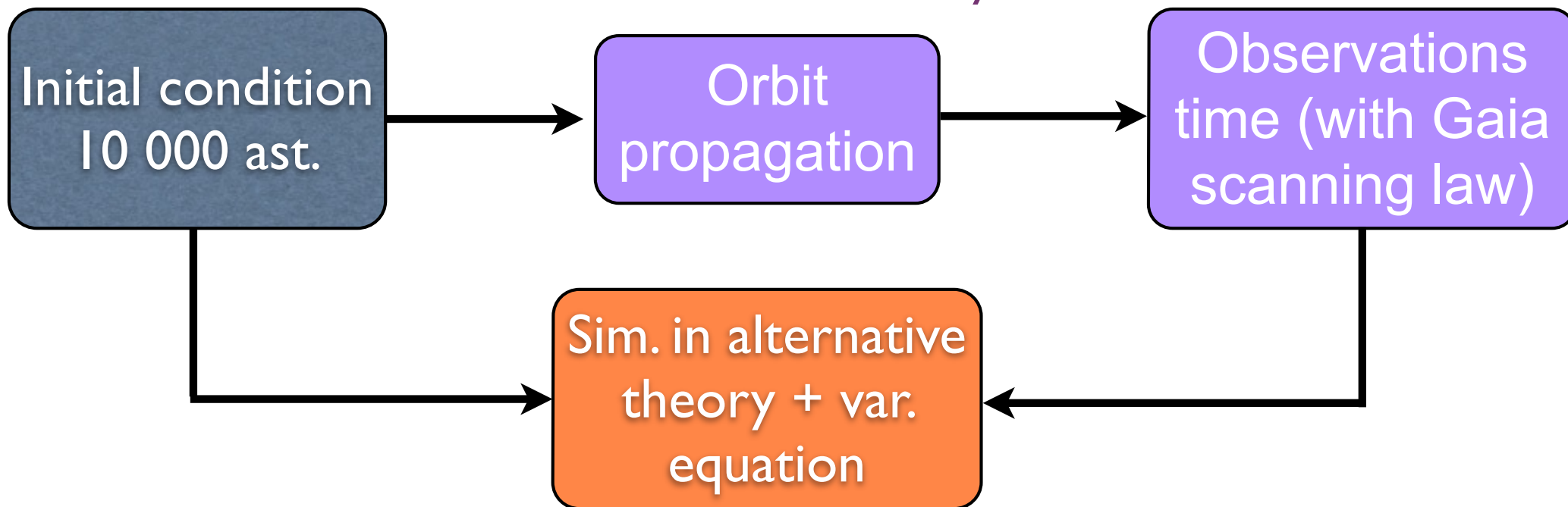
Simulations of Gaia observations

done by Gaia WP DU460



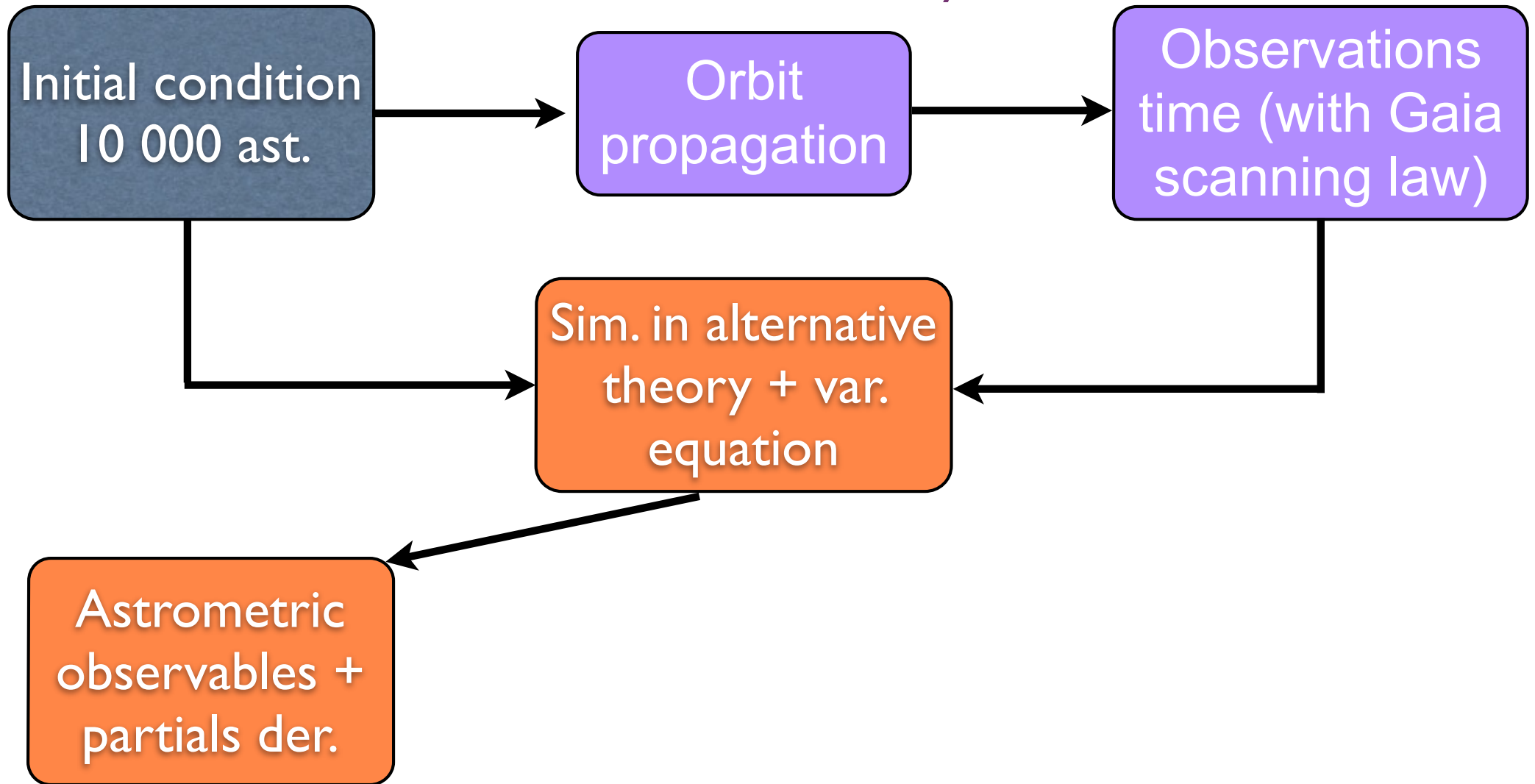
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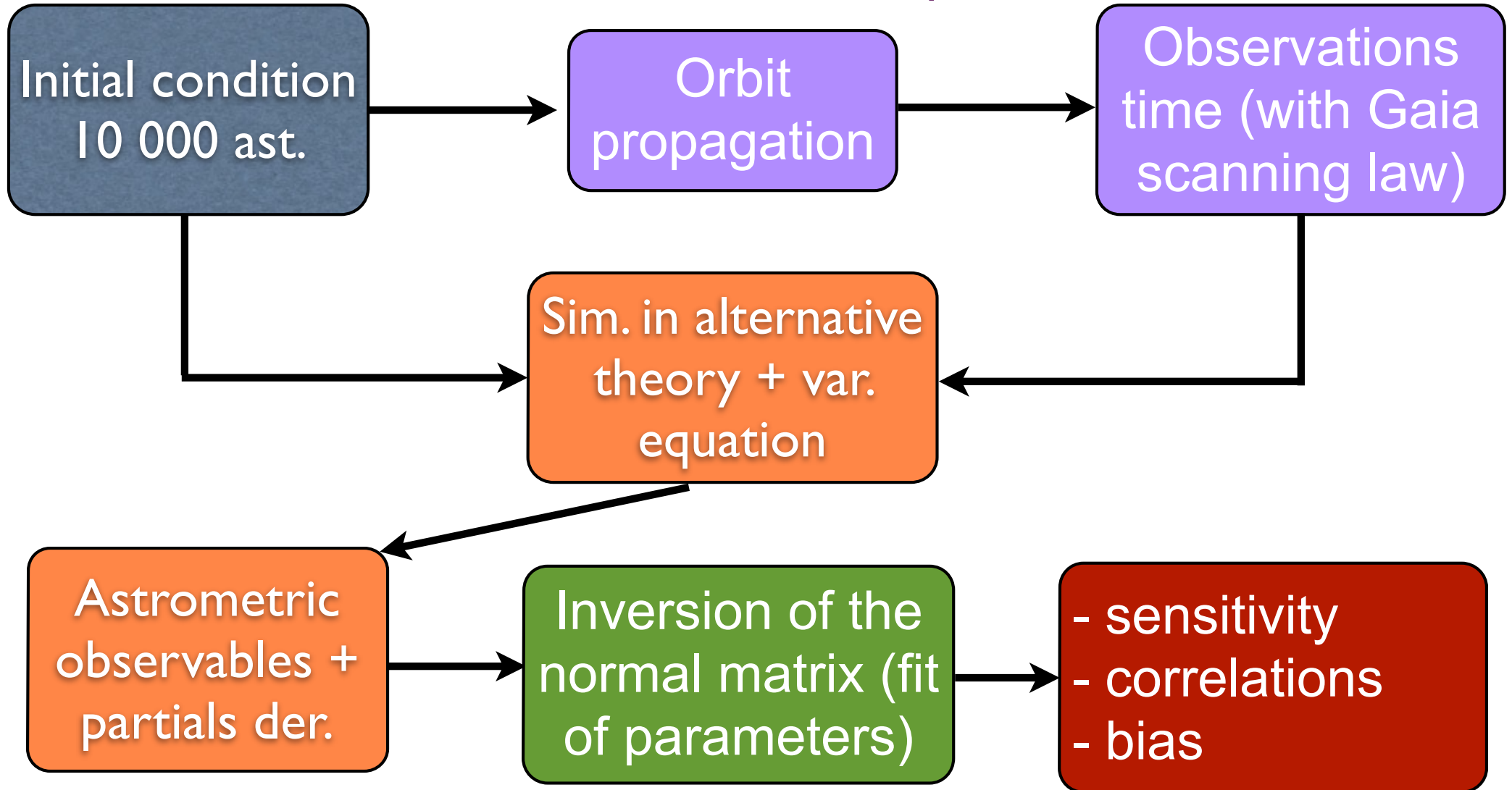
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Simulations of Gaia observations

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- local parameters (IC)
- global parameters (grav. theory, J_2 , ...)

Parameters considered

- local parameters: 6 initial conditions / asteroids (60 000 par.)
- global parameters:
 - Solar Quadrupole moment J_2 .
 - Post-Newtonian Parameter β
 - Sun Lense-Thirring effect: depends on the Sun spin S
 - Violation of the Strong Equivalence Principle (Nordtvedt effect): η
 - Fifth Force formalism: (λ, α)
 - Time variation of G : constant \dot{G}/G
 - Periodic variation of G
 - Standard Model Extension formalism: $\bar{s}^{\mu\nu}$
- 10 000 asteroids with astrometric accuracy of 0.2 mas

PPN formalism and Sun J_2

- highly correlated parameters: one secular effect on orbital dynamics (advance of the perihelion)

$$\left\langle \frac{d\omega}{dt} \right\rangle = (2 + 2\gamma - \beta)n \frac{GM}{c^2 a (1 - e^2)} + \frac{3}{2}n \frac{J_2 R^2}{a^2 (1 - e^2)^2}$$

- various asteroids orbital parameters help to decorrelate
- sensitivity:

	J_2	β
GAIA	$\sigma_{J_2} \sim 10^{-7}$	$\sigma_\beta \sim 7 \times 10^{-4}$
INPOP	$(2.24 \pm 0.15) \times 10^{-7}$	$(-0.25 \pm 6.7) \times 10^{-5}$

INPOP results from A. Fienga et al, arXiv:1409.4932, 2014

- correlation ~ 0.56
- Not as good as planetary ephemerides but: independent analysis, not suffering from the same systematics \Rightarrow interesting complementary check

Violations of the SEP

- All alternative theories of gravitation violate the Strong Equivalence Principle (SEP)
- The Universality of Free Fall is violated for self-gravitating bodies
- Nordtvedt effect parametrized by η

$$m_p = m_i + \eta \frac{E_{\text{grav}}}{c^2}$$

$$m_i \vec{a} = m_p \vec{\nabla} U$$

see K. Nordtvedt, Phys. Rev., 169, 1014, 1968

- sensitivity: 9×10^{-4} . The only actual constraint comes from LLR

$$\eta = (4.4 \pm 4.5) \times 10^{-4}$$

see J. Williams et al, IJMPD, 18, 1129, 2009

- NO correlation with J_2 or β PPN

The SEP can help to decorelate J_2 and β

- In the PPN framework

$$\eta = 4\beta - \gamma - 3$$

see C. Will, "Theory and Experiment in Gravitational Physics", 1993

- Instead of estimating 3 independent parameters: J_2 , β , η , we can use the above relation to estimate only 2 parameters: J_2 and β

	J_2	β	ρ
GAIA (no SEP)	$\sigma_{J_2} \sim 10^{-7}$	$\sigma_{\beta} \sim 7 \times 10^{-4}$	0.55
GAIA (with SEP)	$\sigma_{J_2} \sim 9 \times 10^{-8}$	$\sigma_{\beta} \sim 2 \times 10^{-4}$	0.18
INPOP	$(2.24 \pm 0.15) \times 10^{-7}$	$(-0.25 \pm 6.7) \times 10^{-5}$	-

- using the relation between η and β allows to improve the sensitivity but **reduce significantly the correlation !**
- Similar result expected for planetary ephemerides (η never been considered so far)

Lense-Thirring effect

- Relativistic frame dragging effect produced by the rotation of a body (Sun or Earth)
- Detected with the orbit of LAGEOS spacecraft @ the level of 10% (controversy between L. Iorio and I. Ciufolini) see Ciufolini et al, Nature 431, 958, 2004
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- Lense-Thirring impossible to be estimated in planetary ephemerides: completely correlated with J_2 see W. Folkner et al, IPN Prog. Rep. 42, 196, 2014
- Asteroids can decorrelate but Gaia does not have enough accuracy!
- But... **not including the LT in the modeling leads to bias:**
 - 10^{-8} on the J_2 (i.e. 10% of its value)
 - 5×10^{-5} on the β PPN

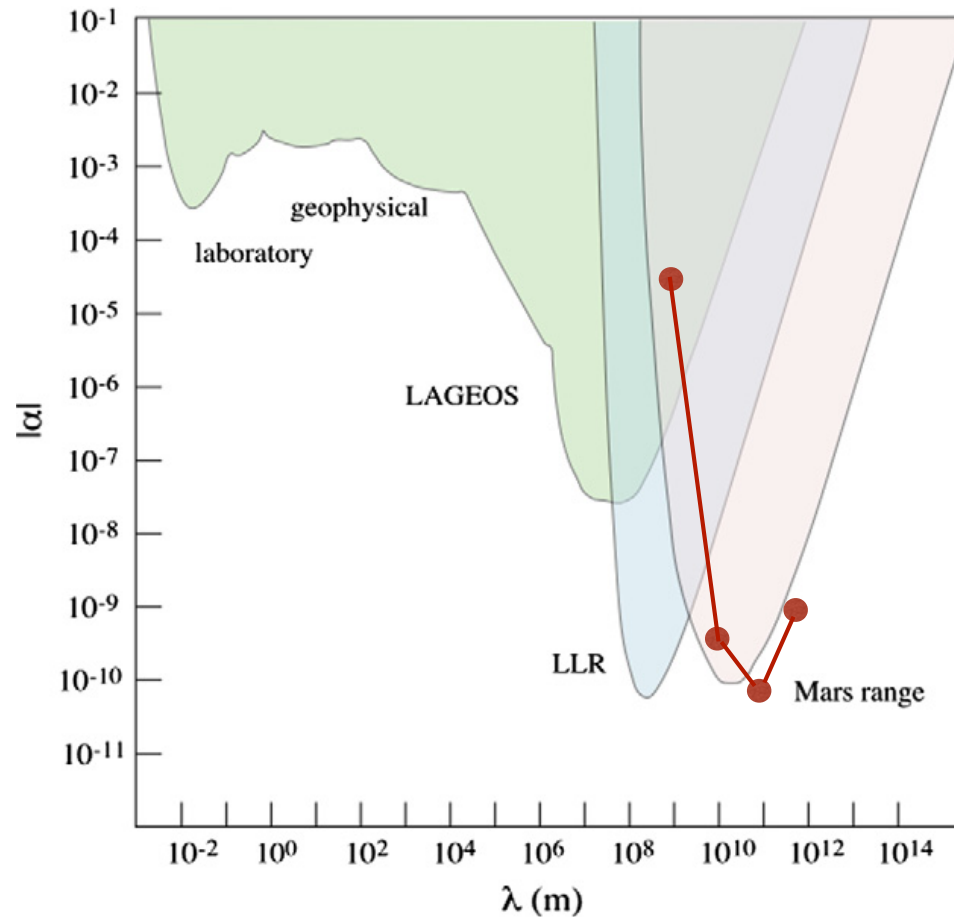
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Similar conclusions seem to hold for planet. ephem. !

Fifth force

- Use GAIA **asteroid observations** to constrain the 5th force parameters

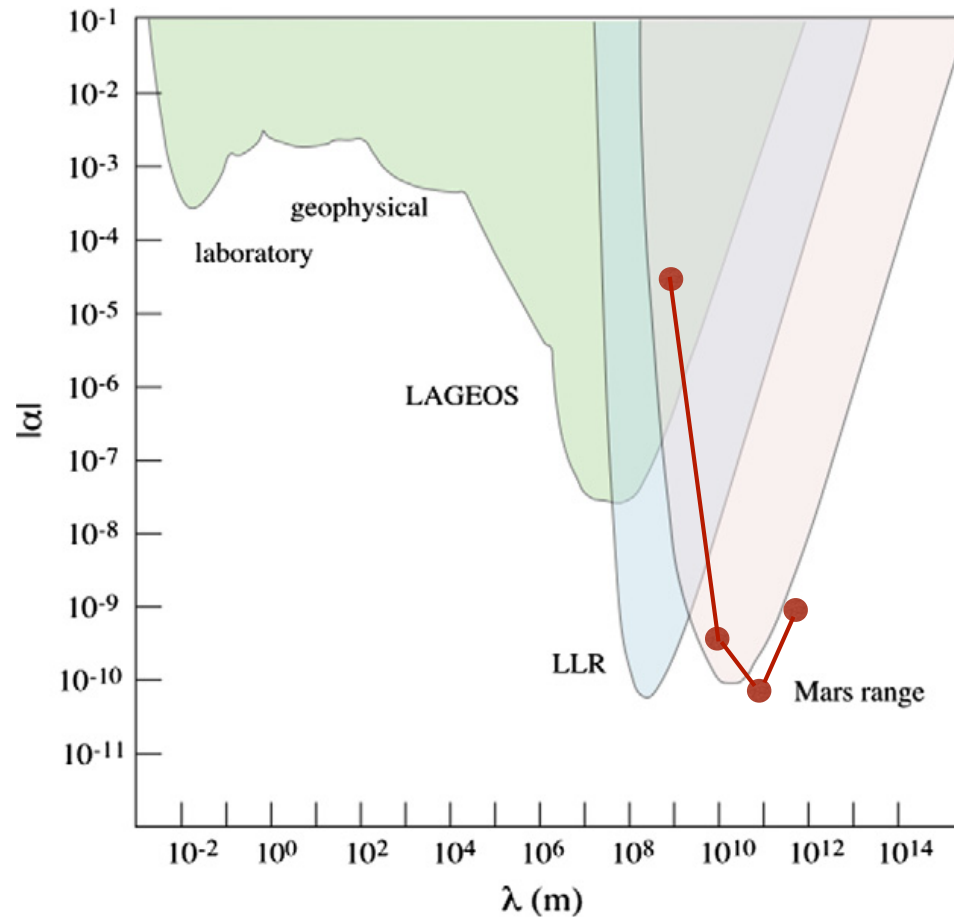


- correlation with Sun mass to be assessed

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Standard Model Extension (SME)

- Recent phenomenology developed to consider hypothetical **violations of the Lorentz invariance** in all sector of physics — violations coming from a more fundamental theory (string theory, loop quantum gravity, non-commutative theory, ...)
- Pure gravity sector¹ depends on 8 parameters $\bar{s}^{\mu\nu}$: Lagrangian based approach (vs PPN based on the metric). The metric **does not enter PPN formalism**
- Quite few analysis in SME framework: LLR and atom interferometry²

Parameter	Predicted sensitivity	This work
$\bar{s}^{11} - \bar{s}^{22}$	10^{-10}	$(1.3 \pm 0.9) \times 10^{-10}$
\bar{s}^{12}	10^{-11}	$(6.9 \pm 4.5) \times 10^{-11}$
\bar{s}^{02}	10^{-7}	$(-5.2 \pm 4.8) \times 10^{-7}$
\bar{s}^{01}	10^{-7}	$(-0.8 \pm 1.1) \times 10^{-6}$
$\bar{s}_{\oplus c}$	10^{-7}	$(0.2 \pm 3.9) \times 10^{-7}$
$\bar{s}_{\oplus s}$	10^{-7}	$(-1.3 \pm 4.1) \times 10^{-7}$

¹ Q. Bailey, V.A. Kostelecky, PRD, 74/045001, 2006

² J. Battat, J. Chandler, C. Stubbs, PRL, 99/241103, 2007
K. Chung, et al, PRD, 80/016002, 2009

SME and asteroids

- Main advantage: decorrelation of the SME parameters
- Sensitivity on the 8 independent parameters

SME Parameter	sensitivity (σ)
$\bar{s}^{XX} - \bar{s}^{YY}$	9×10^{-12}
$\bar{s}^{XX} + \bar{s}^{YY} - \bar{s}^{ZZ}$	2×10^{-11}
\bar{s}^{XY}	4×10^{-12}
\bar{s}^{XZ}	2×10^{-12}
\bar{s}^{YZ}	4×10^{-12}
\bar{s}^{TX}	1×10^{-8}
\bar{s}^{TY}	2×10^{-8}
\bar{s}^{TZ}	4×10^{-8}

**1 order of magnitude
improvement wrt current
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1 order of magnitude improvement wrt current constraints

- Correlations between parameters

	$\bar{s}^{XX} - \bar{s}^{YY}$	$\bar{s}^{XX} + \bar{s}^{YY} - \bar{s}^{ZZ}$	\bar{s}^{XY}	\bar{s}^{XZ}	\bar{s}^{YZ}	\bar{s}^{TX}	\bar{s}^{TY}	\bar{s}^{TZ}
$\bar{s}^{XX} - \bar{s}^{YY}$	1							
$\bar{s}^{XX} + \bar{s}^{YY} - \bar{s}^{ZZ}$	0.28	1						
\bar{s}^{XY}	-0.06	-0.01	1					
\bar{s}^{XZ}	-0.17	-0.06	0.46	1				
\bar{s}^{YZ}	-0.16	0.71	0.01	0.01	1			
\bar{s}^{TX}	10^{-3}	-0.01	-0.01	10^{-3}	-0.01	1		
\bar{s}^{TY}	0.03	0.09	0.01	-0.01	0.02	-0.16	1	
\bar{s}^{TZ}	-0.02	-0.1	-0.01	0.01	-0.02	0.13	-0.67	1

reasonable correlations

SME and asteroids

- First possibility to decorrelate all parameters
- Analysis done including the Sun J_2 : similar results ; J_2 decorrelates as well
- Improvement by ~ 1 order of magnitude wrt current constraints
- Need to extend the study to include “gravity-matter SME coupling” (more parameters that include violation of the equivalence principle)

Very promising results expected

Time variation of G

- A lot of alternative theories of gravitation induce a time variation of G (tensor-scalar theory for example)
- Constraining a linear variation in G is standard: \dot{G}/G
- Sensitivity for GAIA: 10^{-12} per year

- Current constraint: $\dot{G}/G = (0.5 \pm 1.6) \times 10^{-13} \text{yr}^{-1}$

INPOP results from A. Fienga et al, arXiv:1409.4932, 2014

$$\dot{G}/G = (0.1 \pm 1.6) \times 10^{-13} \text{yr}^{-1}$$

DE results from A. Konopliv et al, Icarus 211, 401, 2011

Periodic variation of G

- Very recent temporal analysis of G measurements seem to indicate a periodic variation

$$G(t) = \bar{G} + A \sin\left(2\pi \frac{t - t_0}{T}\right)$$

- first estimation by Anderson et al

J. Anderson, et al, Eur. Phys. Letters 110, 10002, 2015

- more careful analysis by Schlamminger et al

S. Schlamminger et al, arXiv:1505.01774, 2015

Fit function	T (years)	$A \times 10^{15}$ ($\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$)	$\bar{G} \times 10^{11}$ ($\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$)	Maximum
from Fig. 1 in [1]	5.93	16.1	6.673 88	09/13/01
sine, fixed T	5.93	10.7	6.673 59	03/14/01
sine, T free	0.77	11.2	6.673 58	02/21/00
sine, T free	6.17	11.0	6.673 54	02/13/01
straight line	n.a.	n.a.	6.674 13	n.a.

- Gaia sensitivity around $\sim 10^{-20}$ for the amplitude, no correlation with Sun J_2 .
- Planetary ephemerides can be used to constrain severely this effect

Conclusion

- Testing GR in the solar system is very challenging but very important:
 - search for small deviations (smaller than present PPN accuracy)
 - search for deviations in extended frameworks (SME is one of them)
- Asteroids observations with GAIA offer **nice opportunities to probe orbital dynamics**
 - large number of orbital parameters: nice to deal with correlations
 - different and independent constraints from planetary ephemerides
- Sensitivity assessed for different alternative gravity framework: PPN parameters, fifth force, SME, variation of G , ...
- In the longer term, combining **GAIA observations with UCLA radar data** may improve the results: complementary observations — currently under investigation

see J.L. Margot and J. Giorgini, proceedings of IAU symp. 261, 2010

BACK UP SLIDES

Basic principles of GR

I) Equivalence Principle: the future...

- theoretical motivations to improve these: string theory, Kaluza-Klein, theories with variable fundamental constants (“principle of absence of absolute structure”), “anthropic principle”, ...

for a review, see T. Damour, CQG, 29-184001, 2012

- Universality of Free Fall:

- **Microscope**: launch in April 2016; test at 10^{-15} see G. Metris’s talk
- Galileo Galilei: Italian proposal; test at 10^{-17}
- STE-QUEST: ESA proposal; quantum test at the level of 10^{-15}

- Local Position Invariance - gravitational redshift:

- **ACES**: launch in 2016; test at 10^{-6}
- Galileo 5 and 6 GNSS satellites: failed launched eccentric orbit appropriate for redshift tests (comparison of onboard clocks to clocks on Earth). Full sensitivity study (stochastic noise and systematics):
sensitivity at 2×10^{-5} with one year of data

see Delva P., Hees A., et al, proceedings of Moriond 2015 and coming publication