The Time Transfer Function as a tool for modeling light propagation

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l'Observatoire de Paris
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Systèmes de Référence Temps-Espace

Accuracy of observations improves

- BepiColombo, JUNO, JUICE (3GM and PRIDE), … : main objective: internal structure of planets/satellites. Radioscience @ the level of cm for the range and μm/s for the Doppler
- GAIA: astrometric observations @ the level of 10 μas
- GRAVITY: astrometric observations around our galactic center @ the level of 10 μas
- AGP/GAME: astrometric test of GR at the level of μas (around the Sun)
- THEIA/NEAT: astrometric observations (exoplanets) at the level of ~ 50 nas- 0.1 μas

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Similar improvement in the theoretical modeling needed

Light propagation is crucial in the modelling of Sol. Sys. observations

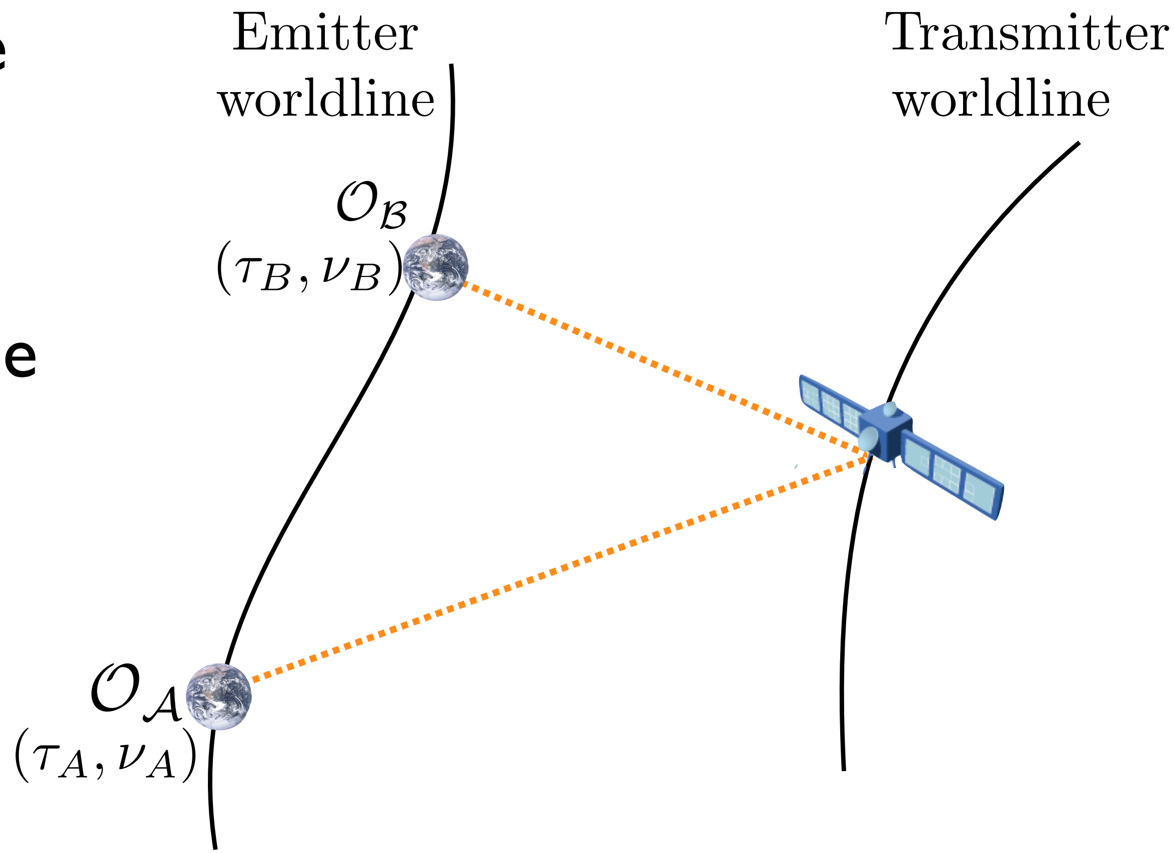
1) Range observable

- Difference in proper time

$$\mathrm{Range} = c(\tau_B - \tau_A)$$

- Depends on the difference in coord. time (amongst other parameters)

$$t_B - t_A$$



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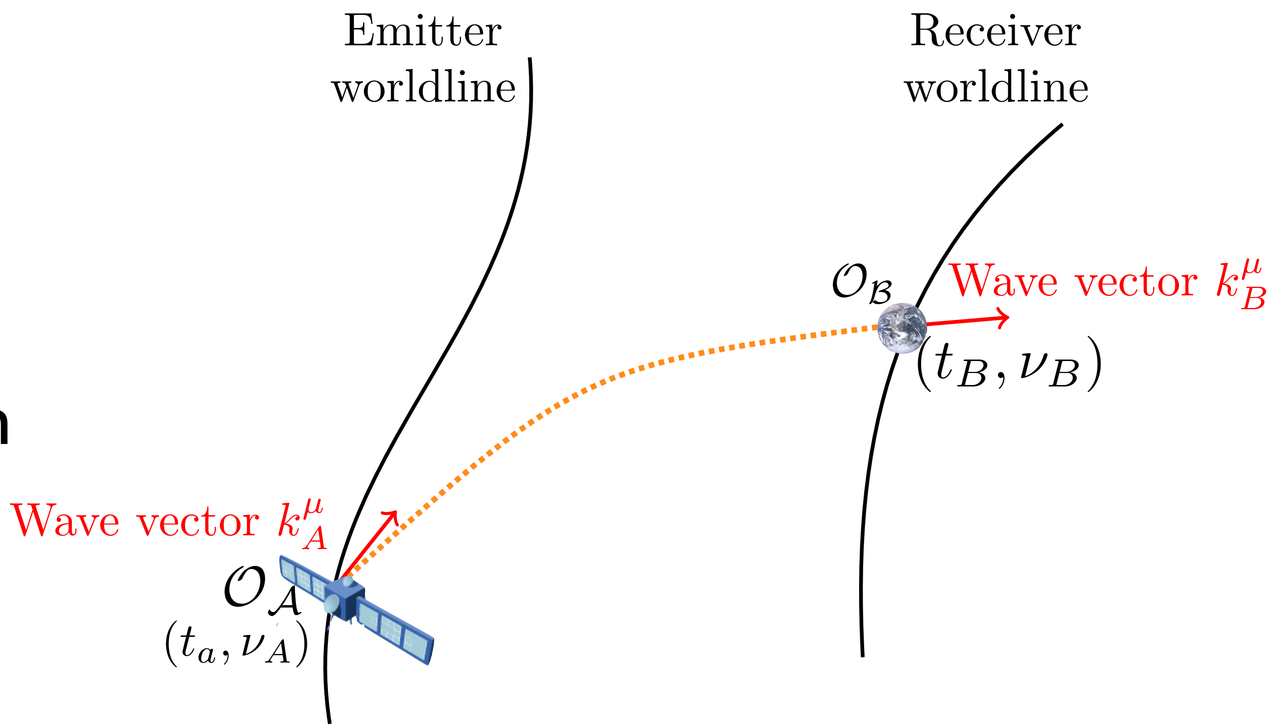
2) Doppler observable

- Ratio of proper frequency $D = \frac{\nu_B}{\nu_A} = \left(\frac{d\tau}{dt}\right)_A \left(\frac{d\tau}{dt}\right)_B^{-1} \frac{k_0^B}{k_0^A} \frac{1+\beta_B^i \hat{k}_i^B}{1+\beta_A^i \hat{k}_i^A}$

with $\beta^i = v^i/c$ and

$$\hat{k}_i = \frac{k_i}{k_0}$$

- Wave vector at emission and reception needed



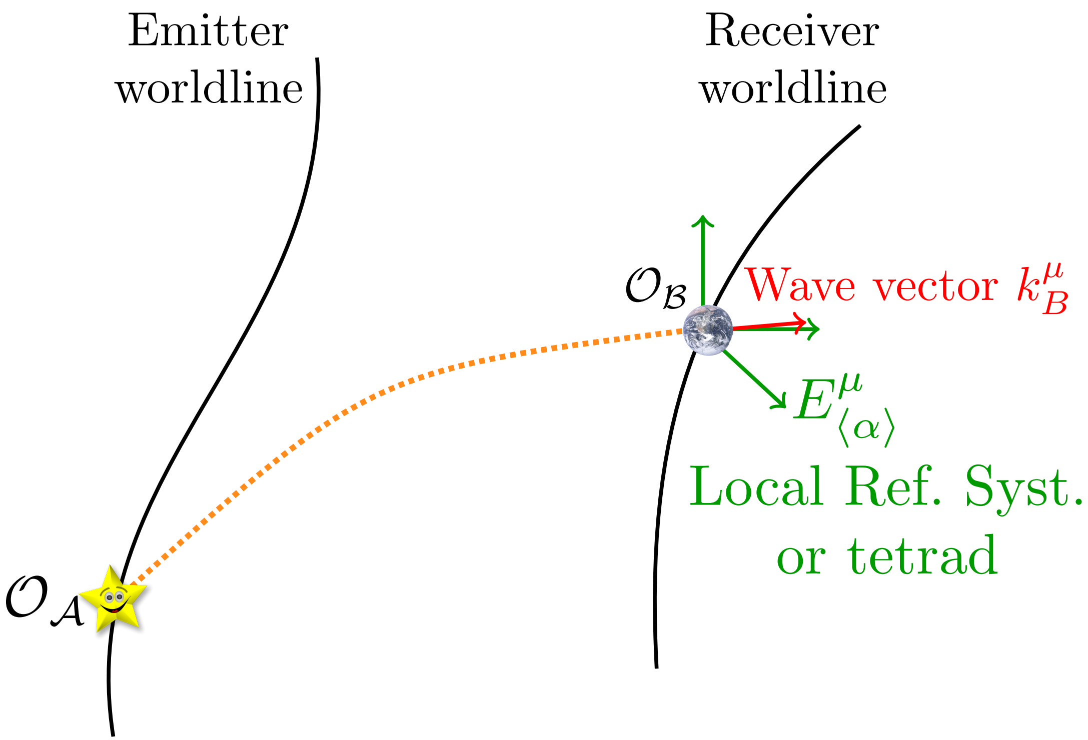
Light propagation is crucial in the modelling of Sol. Sys. observations

3) Astrometric observable & VLBI

- Direction of observation of the light ray in a local reference system (or tetrad)

$$n^{\langle i \rangle} = -\frac{E^0_{\langle i \rangle} + E^j_{\langle i \rangle}\hat{k}^B_j}{E^0_{\langle 0 \rangle} + E^j_{\langle 0 \rangle}\hat{k}^B_j}$$

- Wave vector at reception needed

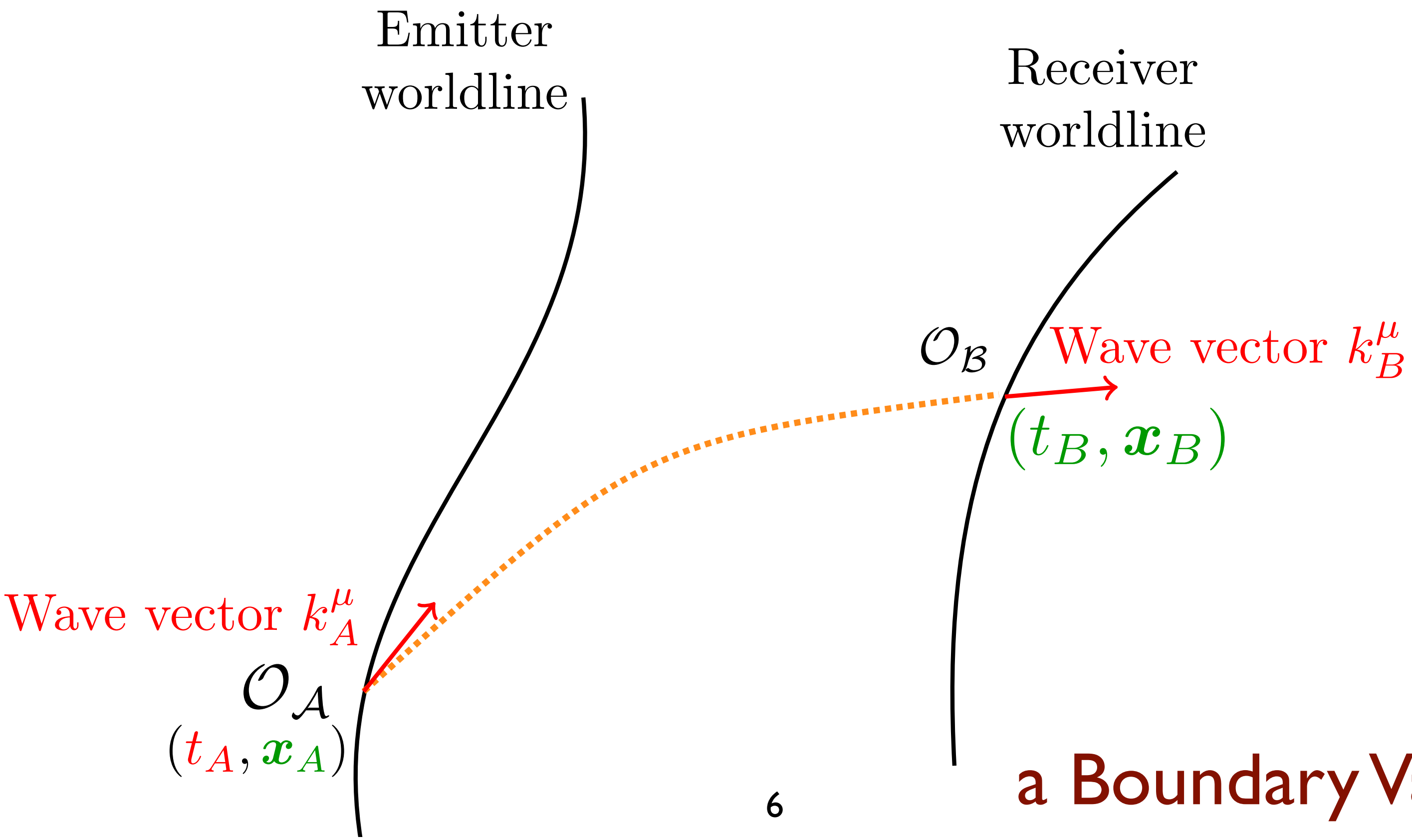


How to determine the light propagation ?

- At the geometric optics approximation: photons follow null geodesics

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0 \qquad\qquad k^\mu k_\mu = 0$$

with $k^\mu = \frac{dx^\mu}{d\lambda}$ the tangent vector



a Boundary Value Problem

Methods to solve the null geodesic eqs.

- Full numerical integration of the null geodesic eqs. with a shooting method

see A. San Miguel, Gen. Rel. Grav. 39, 2025, 2007

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- Exact analytical solution for some metrics: Schwarzschild and Kerr (solution with Jacobian/Weierstrass elliptic functions)

see for example: de Jans, Mem. de l'Ac. Roy. de Bel., 1922
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- Analytical solutions for low gravitational field:
  - 1 pM Schwarzschild metric — see E. Shapiro, PRL 13, 26, 789, 1964
  - moving monopoles at 1pM order — see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999; S. Klioner, A & A, 404, 783, 2003
  - static extended bodies with multipolar expansion at 1pM — see S. Kopeikin, J. of Math. Phys., 38, 2587
  - 2 pM Schwarzschild metric — see G. Richter, R. Matzner, PRD 28, 3007, 1983; S. Klioner, S. Zschocke, CQG 27, 075015, 2010

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- Use of the eikonal equation:
  - perturbative solution for spherically symmetric space-time

see for example N. Ashby, B. Bertotti, CQG 27, 145013, 2010

… and the Time Transfer Functions

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

- The (reception) Time Transfer Function - TTF - is defined by

$$t_B - t_A = \mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B)$$

- The TTF is solution of an eikonal equation well adapted to a perturbative expansion

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- The TTF is solution of an eikonal equation well adapted to a perturbative expansion
- The derivatives of the TTF are of crucial interest since

$$\hat{k}_i^A = c\frac{\partial \mathcal{T}_r}{\partial x_A^i} \qquad \hat{k}_i^B = -c\frac{\partial \mathcal{T}_r}{\partial x_B^i}\left[1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1} \qquad \frac{k_0^B}{k_0^A} = 1 - \frac{\partial \mathcal{T}_r}{\partial t_B}$$

Range, Doppler, astrometric observables can be written in terms of the TTF and its derivatives

Post-Minkowskian expansion of the TTF

see P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

- A pM expansion of the TTF: $\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \dfrac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}_r^{(n)}$

- Computation with an iterative procedure involving integrations over a straight line between the emitter and the spatial position of the receiver !

- Example at 1 pM: $\mathcal{T}_r^{(1)} = \dfrac{R_{AB}}{2c} \displaystyle\int_0^1 \left[g_{(1)}^{00} - 2N_{AB}^i g_{(1)}^{0i} + N_{AB}^i N_{AB}^j g_{(1)}^{ij} \right]_{z^\alpha(\lambda)} d\lambda$

  with $z^\alpha(\lambda)$ the straight Mink. null path between em. and rec.

- Main advantages:

  - analytical computations relatively easy

  - very well adapted to numerical evaluation

Analytical results in Schwarzschild space-time

see B. Linet and P. Teyssandier, CQG 30, 175008, 2014
P. Teyssandier, 2014, arXiv: 1407.4361

- A "simplified" iterative method has been developed for static spherically symmetric geometry

$$ds^2 = \left(1 - 2\frac{m}{r} + 2\beta\frac{m^2}{r^2} - \frac{3}{2}\beta_3\frac{m^3}{r^3} + \dots\right)dt^2 - \left(1 + 2\gamma\frac{m}{r} + \frac{3}{2}\epsilon\frac{m^2}{r^2} + \frac{1}{2}\gamma_3\frac{m^3}{r^3} + \dots\right)d\boldsymbol{x}^2$$

- In GR: $\gamma = \beta = \epsilon = \beta_3 = \gamma_3 = 1$

- A pM expansion of the TTF: $\mathcal{T} = \dfrac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}^{(n)}$

and the corresponding derivatives have been computed up to the 3rd pM order

Analytical results in Schwarzschild space-time

- A pM expansion of the TTF: $\mathcal{T} = \dfrac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}^{(n)}$

$$\mathcal{T}^{(1)} = \frac{(1+\gamma)m}{c} \ln \frac{r_A + r_B + |\boldsymbol{x}_B - \boldsymbol{x}_A|}{r_A + r_B - |\boldsymbol{x}_B - \boldsymbol{x}_A|}$$

see E. Shapiro, PRL 13, 26, 789, 1964

$$\mathcal{T}^{(2)} = \frac{m^2}{r_A r_B} \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c} \left[\kappa \frac{\arccos \boldsymbol{n}_A . \boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} - \frac{(1+\gamma)^2}{1 + \boldsymbol{n}_A . \boldsymbol{n}_B} \right]$$

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
S. Klioner, S. Zschocke, CQG 27, 075015, 2010

$$\mathcal{T}^{(3)} = \frac{m^3}{r_A r_B} \left(\frac{1}{r_A} + \frac{1}{r_B} \right) \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c(1 + \boldsymbol{n}_A . \boldsymbol{n}_B)} \left[\kappa_3 - (1+\gamma)\kappa \frac{\arccos \boldsymbol{n}_A . \boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} + \frac{(1+\gamma)^3}{1 + \boldsymbol{n}_A . \boldsymbol{n}_B} \right]$$

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with $\kappa = 2 + 2\gamma - \beta + \dfrac{3}{4}\epsilon$

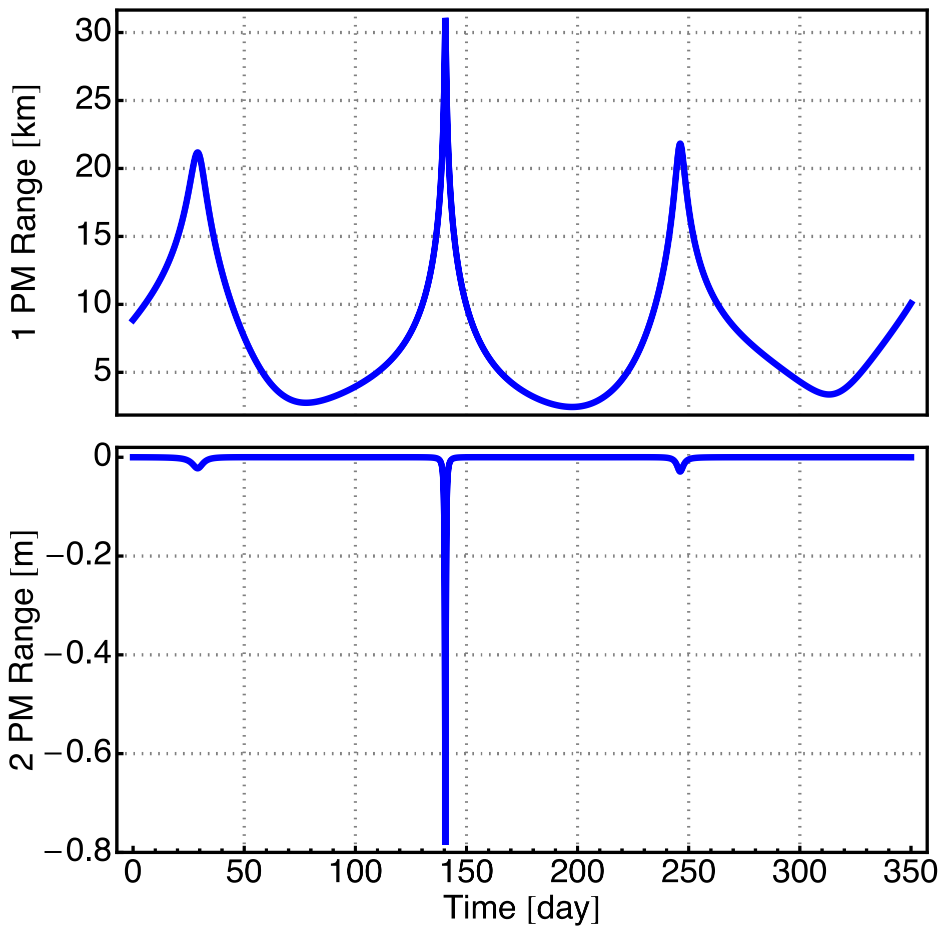
$\kappa_3 = 2\kappa - 2\beta(1+\gamma) + \dfrac{1}{4}(3\beta_3 + \gamma_3)$

and $\boldsymbol{n}_{A/B} = \dfrac{\boldsymbol{x}_{A/B}}{r_{A/B}}$

Is it necessary to go to the 3rd order?

- In a conjunction geometry, at each order n, there are enhanced terms proportional to $(1+\gamma)^n$

- Ex. with Earth-BepiColombo range (accuracy ~ 10 cm)
$\Rightarrow$ 2pM term needed



- Ex. with SAGAS: link between spacecraft in the outer Solar System to measure γ at 10^{-8}
$\Rightarrow$ accuracy at the mm level

  $\Rightarrow$ 3pM term needed

$r_c/R_\odot$	$\mathcal{T}^{(2)}_{\rm enh}$	$\mathcal{T}^{(2)}_{\kappa}$	$\mathcal{T}^{(3)}_{\rm enh}$
1	-5 m	37 cm	10 cm
2	-1.3 m	18 cm	0.6 mm
5	-21 cm	7 mm	15 μm

see P. Teyssandier, 2014, arXiv: 1407.4361
A. Hees, S. Bertone, C. Le Poncin-Lafitte, PRD 89, 064045, 2014

Is it necessary to go to the 3rd order?

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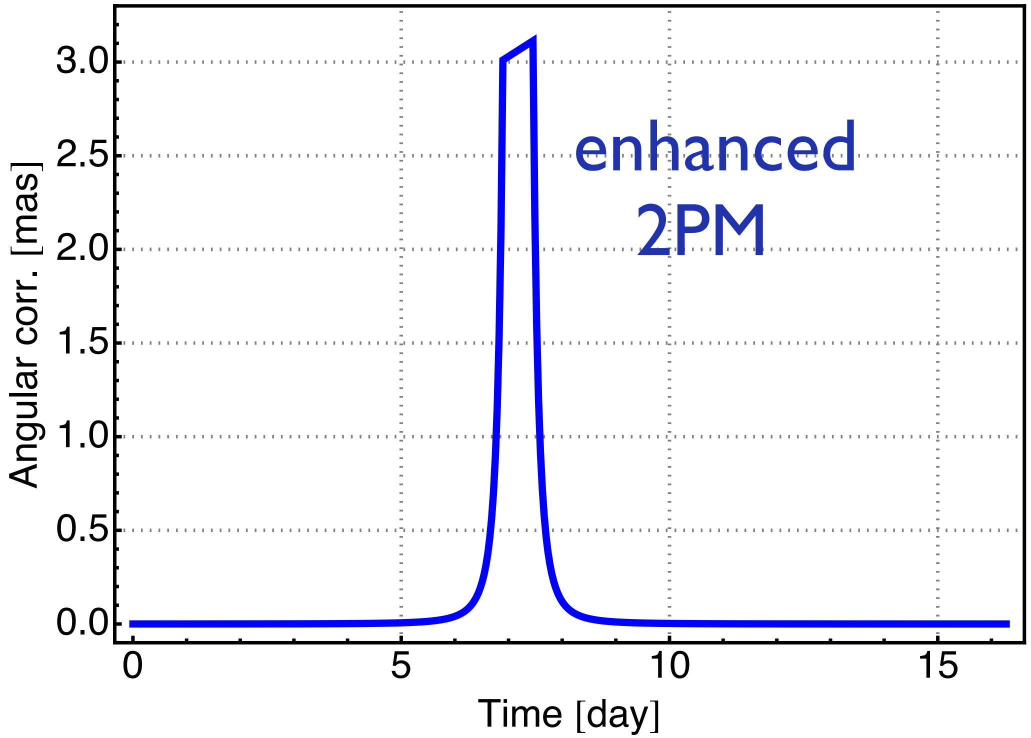
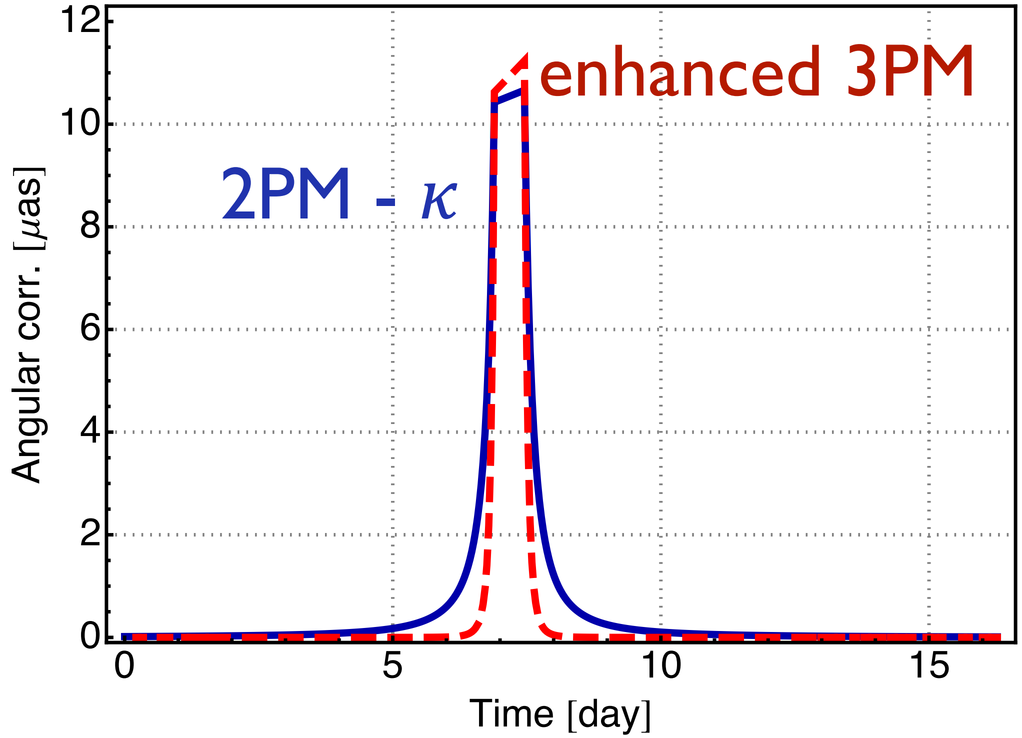
- Ex. with light deflection for Sun grazing rays: AGP space mission (old GAME). Expected accuracy: μas
$\Rightarrow$ 3pM term needed





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P. Teyssandier, B. Linet, proceedings of JSR 2013, arXiv:1312.3510

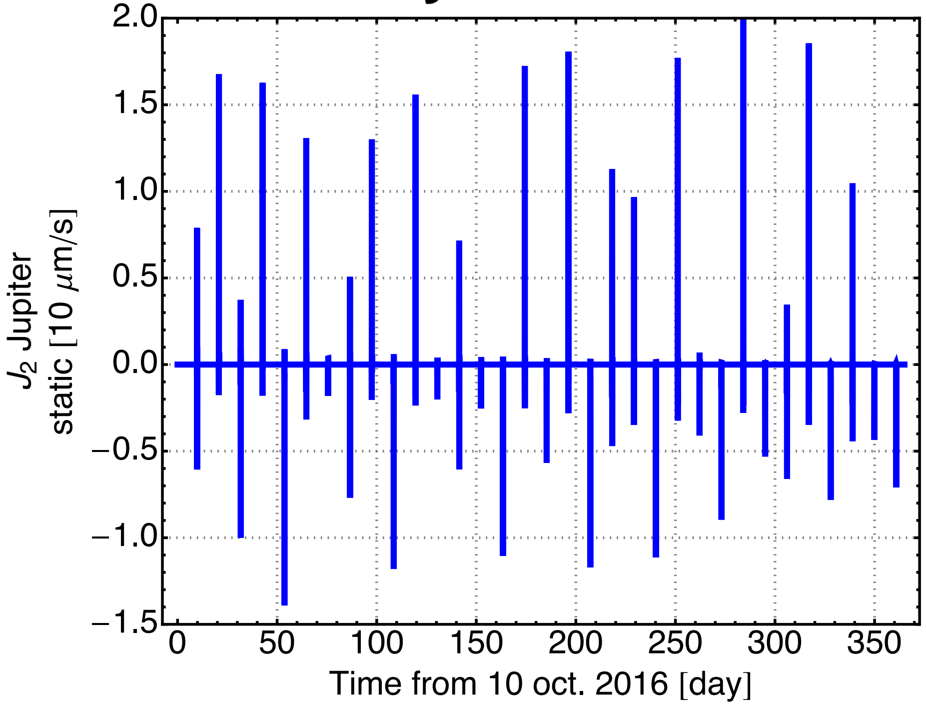
Analytical result around axisymmetric bodies

- Influence of all the multipole moments J_n from the grav. potential

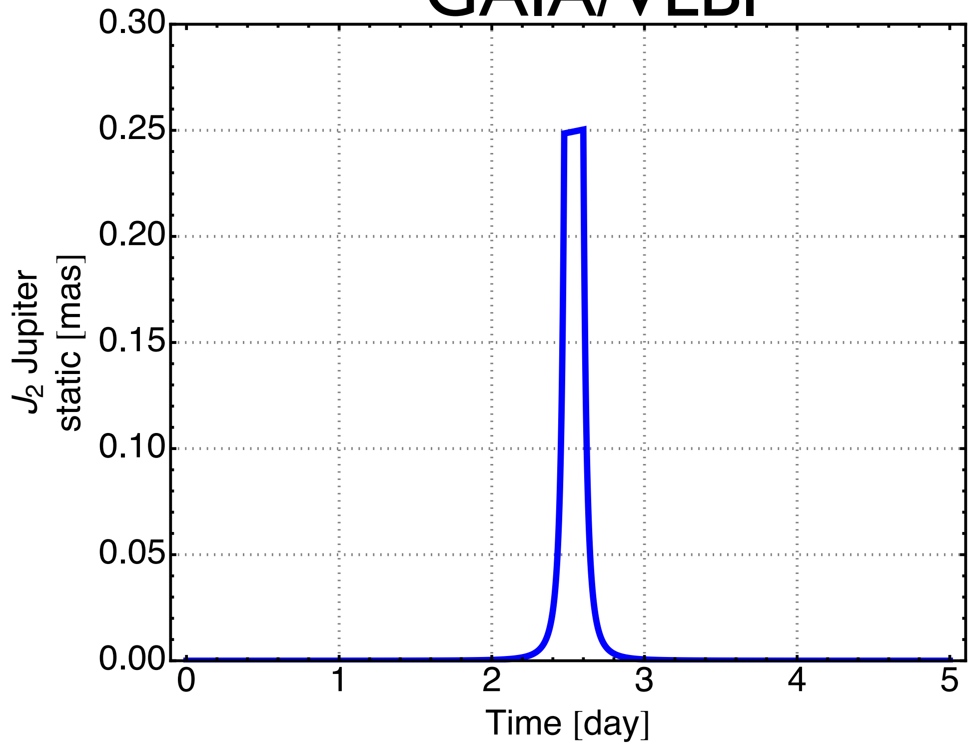
see C. Le Poncin-Lafitte, P. Teyssandier, PRD 77, 044029, 2008 for a computation with the TTF
or S. Kopeikin, J. of Math. Physics 38, 2587, 1997 for another approach

- Influence of Jupiter J_2 on the JUNO Doppler (1 μm/s accuracy) and for GAIA (10 μas acc.)

JUNO



GAIA/VLBI



- terms important for the data analysis for these missions

see A. Hees, et al, PRD 08, 084020, 2014

What happens if the body is moving ?

- At first pM order, the TTF for uniformly moving bodies can easily be derived from the TTF generated by a static body

$$\Delta(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \gamma(1 - \boldsymbol{N}_{AB}.\boldsymbol{\beta})\tilde{\Delta}(\boldsymbol{R}_A + \gamma\boldsymbol{\beta}R_{AB}, \boldsymbol{R}_B)$$

TTF in the moving case

static TTF

with $\boldsymbol{\beta} = \boldsymbol{v}/c, \quad \gamma = (1 - \beta^2)^{-1/2}$

and $\boldsymbol{R}_X$ depends on $\boldsymbol{x}_X, \boldsymbol{\beta}$

- All the analytical results computed for a static source can be extended in the case of a uniformly moving source

see A. Hees, et al, PRD 08, 084020, 2014

Time Transfer around a moving body

- moving monopole:
  - using the previous result:

$$\Delta_M(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = 2\frac{GM_p}{c^2}\gamma_p\left(1 - \boldsymbol{N}_{AB}\cdot\boldsymbol{\beta}_p\right)\ln\frac{|\boldsymbol{R}_{pA} + \gamma_p\boldsymbol{\beta}_p R_{AB}| + R_{pB} + \gamma_p R_{AB}(1 - \boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}{|\boldsymbol{R}_{pA} + \gamma_p\boldsymbol{\beta}_p R_{AB}| + R_{pB} - \gamma_p R_{AB}(1 - \boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}$$

see see A. Hees, et al, PRD 08, 084020, 2014
S. Bertone et al, CQG 31, 015021, 2014 for a pN expansion

- also determined by other methods

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see M. Soffel, W.-B. Han, arXiv:1409.3743

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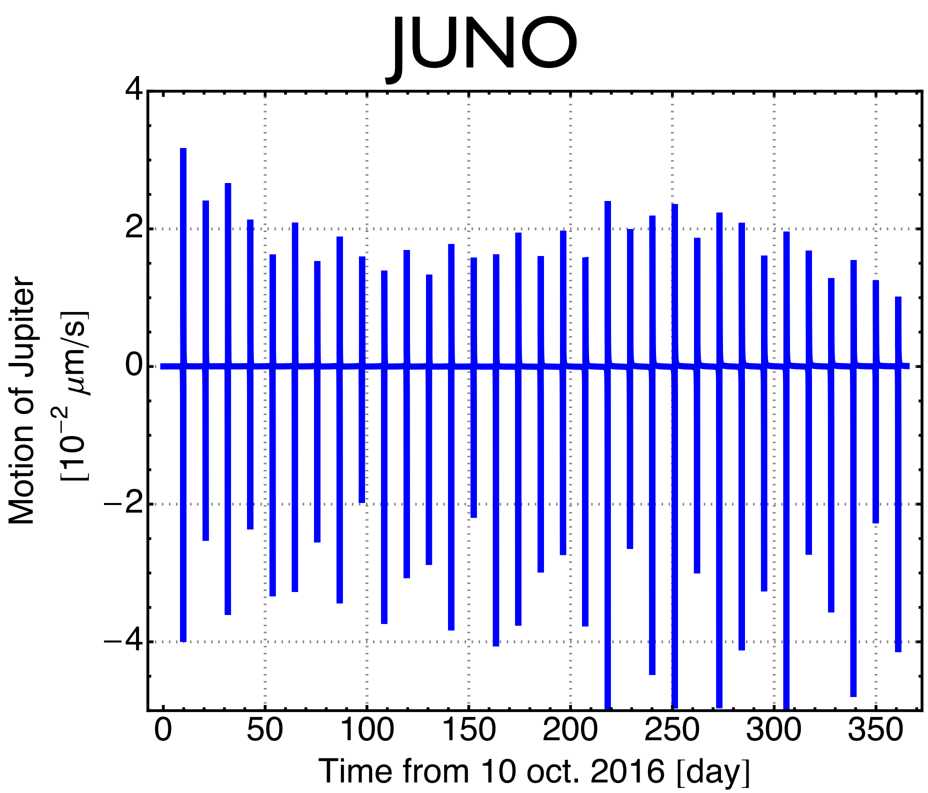
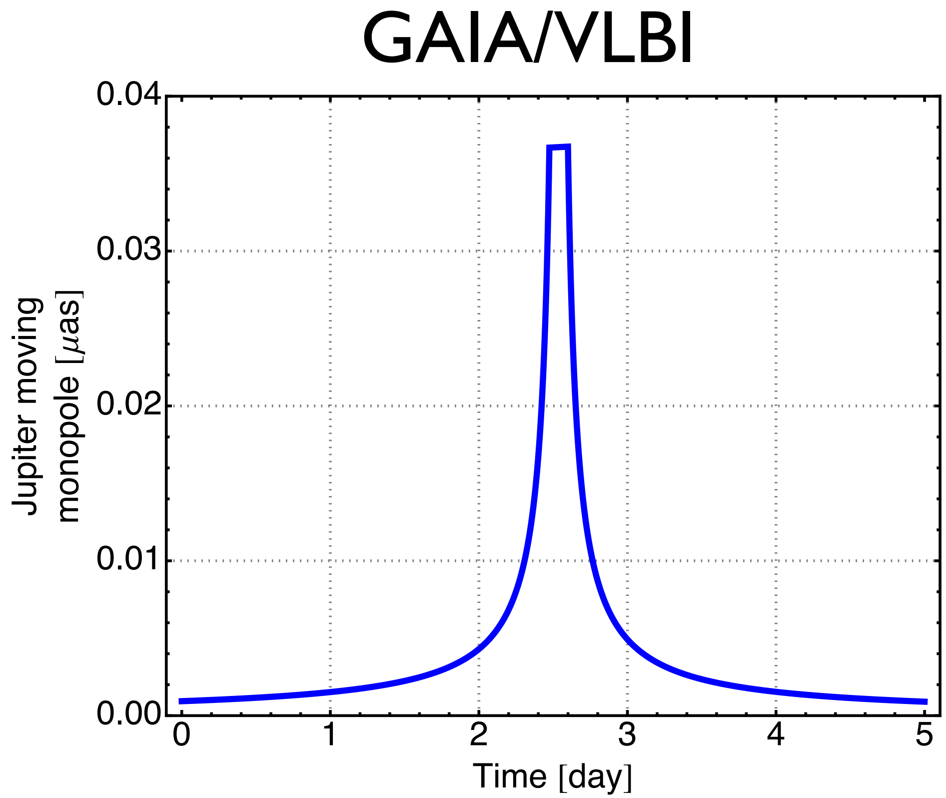
- arbitrarily moving point masses: numerical expression

see A. Hees, et al, PRD 08, 084020, 2014

Ex.: motion of Jupiter

- Influence of Jupiter velocity on the JUNO Doppler (1 μm/s accuracy) and for GAIA (10 μas acc.)





- depend highly on the orbit geometry: conjunction and $\beta.N_{AB}$
- In particular: should be reassessed for JUICE orbit

see A. Hees, et al, PRD 08, 084020, 2014

Numerical evaluation of the TTF

- Iterative procedure involving integrals over a straight line: appropriate for numerical evaluation

see A. Hees, et al, PRD 89, 064045, 2014

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- Iterative procedure involving integrals over a straight line: appropriate for numerical evaluation
- At 1pM order: a simple integral to evaluate

$$\mathcal{T}^{(1)} = \int_0^1 m\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

$$\frac{\partial \mathcal{T}^{(1)}}{\partial x^i_{A/B}} = \int_0^1 m_{A/B}\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

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- At 2pM order: a double integral to evaluate

$$\mathcal{T}^{(2)} = \int_0^1 \int_0^1 n\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

$$\frac{\partial \mathcal{T}^{(2)}}{\partial x^i_{A/B}} = \int_0^1 \int_0^1 n_{A/B}\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(2)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta,\gamma\delta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

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- At 1pM order: a simple integral to evaluate

$$\mathcal{T}^{(1)} = \int_0^1 m\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

$$\frac{\partial \mathcal{T}^{(1)}}{\partial x^i_{A/B}} = \int_0^1 m_{A/B}\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

- At 2pM order: a double integral to evaluate

$$\mathcal{T}^{(2)} = \int_0^1 \int_0^1 n\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

$$\frac{\partial \mathcal{T}^{(2)}}{\partial x^i_{A/B}} = \int_0^1 \int_0^1 n_{A/B}\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(2)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta,\gamma\delta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

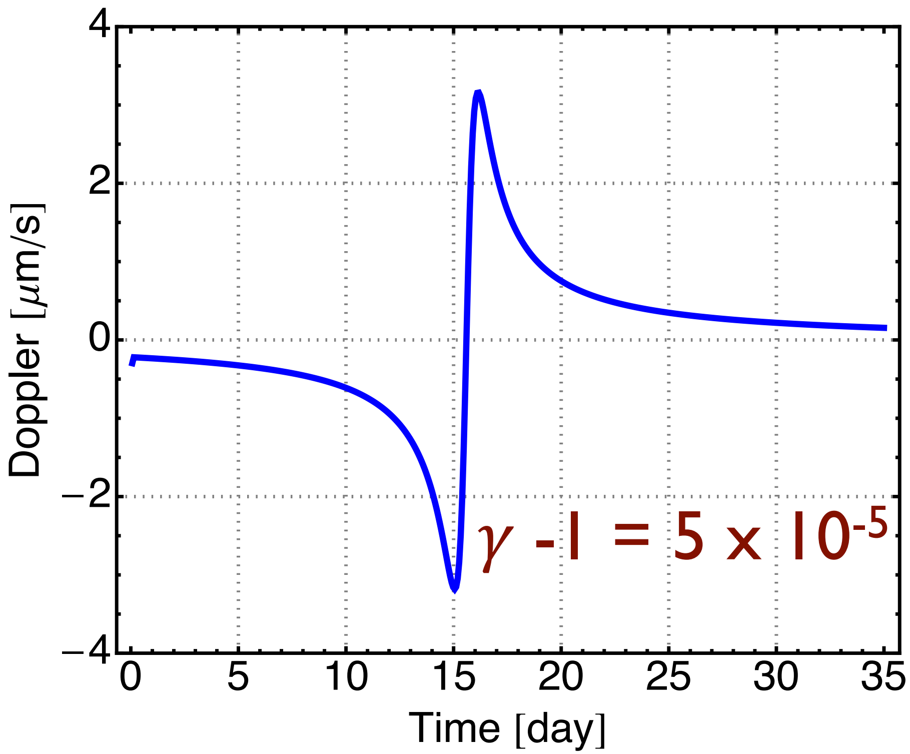
- Numerically efficient ; useful when no analytical solution can be found

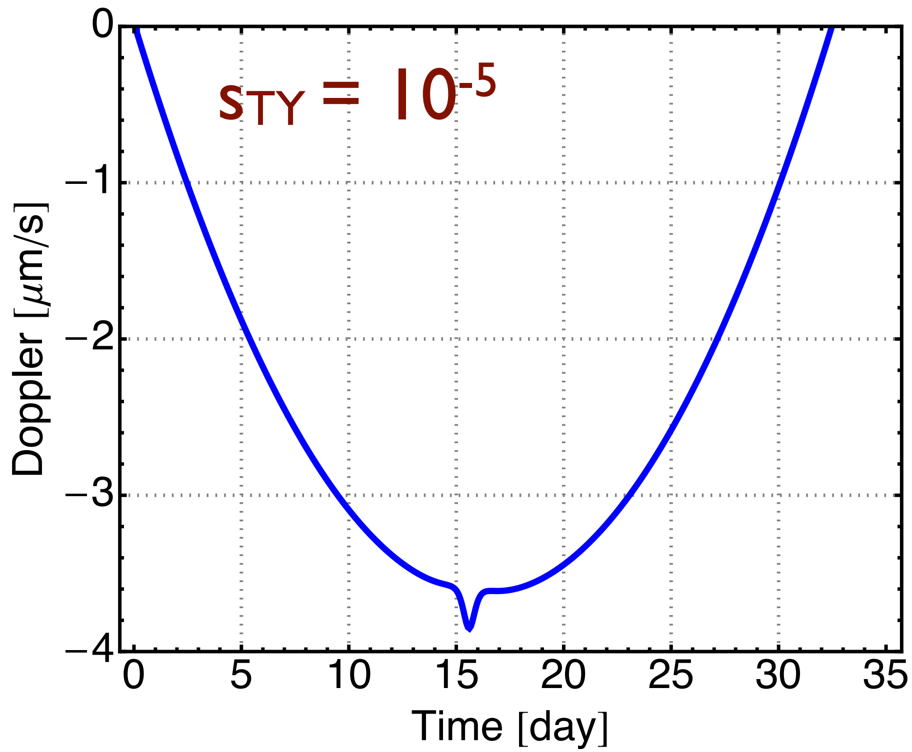
see A. Hees, et al, PRD 89, 064045, 2014

Numerical evaluation of the TTF

- Numerical evaluation appropriate to evaluate effects due to alternative theories of gravitation
- Example: Doppler for 30 days of Cassini tracking between Jupiter and Saturn (“γ experiment”)
- Effect of the γ PPN and of Standard Model Extension s_{TY} on Cassini Doppler

for SME, see Q. Bailey and A. Kostelecky, PRD 74, 045001, 2006





see A. Hees, et al, CQG 29, 235027, 2012

Simulations of observations directly from metric

- New tool that performs Range/Doppler/Astrometric simulations from a specific space-time metric (orbit integration, clock model, light propagation, tetrad propagation,...) and fits of the orbital initial conditions in GR

- Identification of incompressible signals due to the alternative theory: order of magnitude and signature that can eventually be observed in residuals of real data analysis

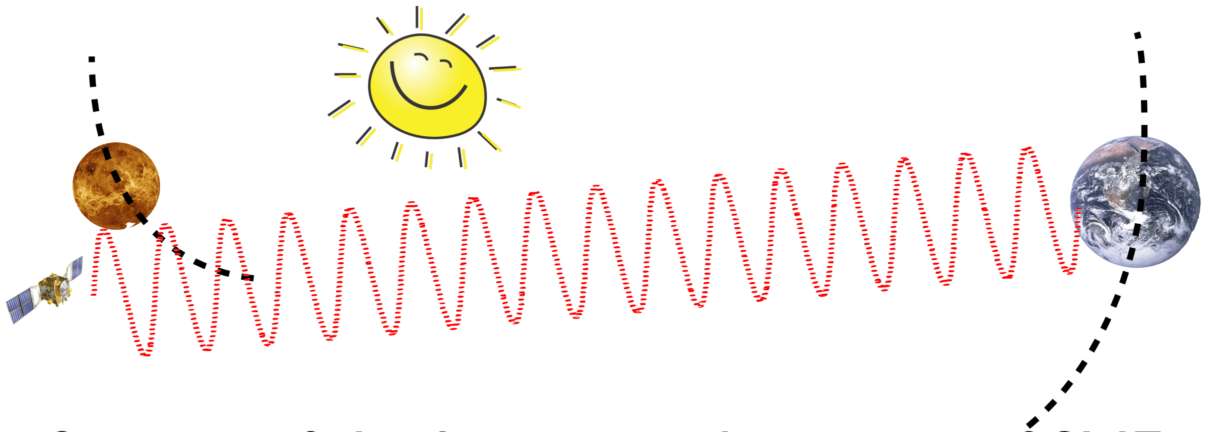
- Very flexible approach: easy to change the gravitation theory (the only thing to change: the expression of the metric)

- What are the effects of alternative theories of gravity on space observations ?

[1] A. Hees, B. Lamine et al, CQG, 29/235027, 2012

Simulations of Messenger in SME

- SME: consider violations of the Lorentz symmetry
- metric parametrizing a violation of Lorentz symmetry in the gravitational sector depends[1] on $\bar{s}_{\mu\nu}$: does not enter PPN of fifth force formalisms
- Simulations of two years of Earth-Messenger-Earth Range and Doppler link



- Numerical identification of the linear combinations of SME parameters whose observations depend on

$$\bar{s}_A = \bar{s}_{XX} - 0.72\bar{s}_{YY} - 0.28\bar{s}_{ZZ}$$

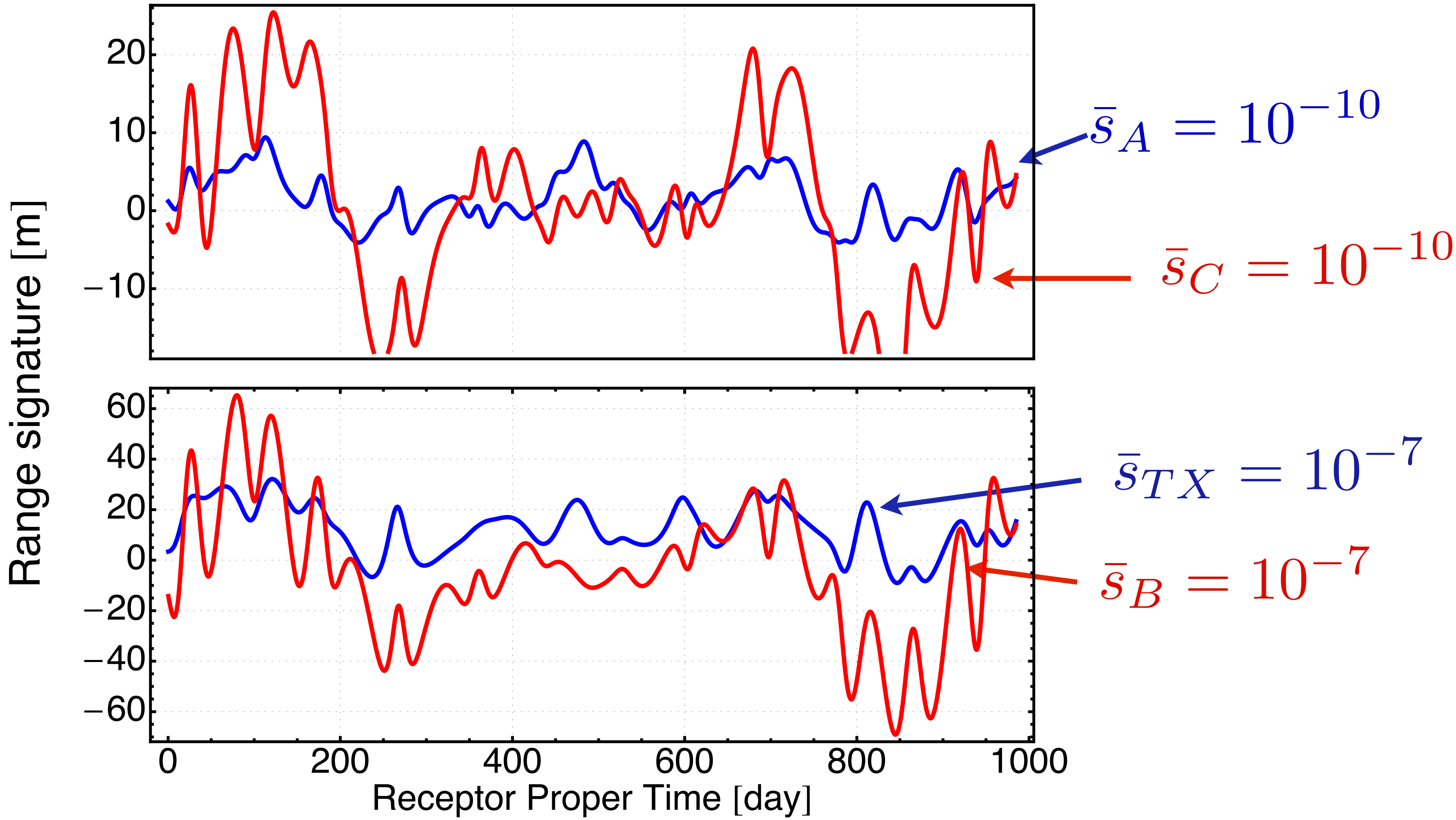
$$\bar{s}_{TX}$$

$$\bar{s}_B = \bar{s}_{TY} + 0.53\bar{s}_{TZ}$$

$$\bar{s}_C = \bar{s}_{XY} + 2.954\bar{s}_{XZ} - 0.26\bar{s}_{YZ}$$

Incompressible signature of SME on Messenger

- Signatures that would be observed in residuals of data analysis if SME is the real theory of gravitation but if data are analyzed in GR



- Can we identify such signatures in residuals of real data analysis ? Can we constrain these parameters ?

Summary of results for SME

- Linear combinations involved in the situations considered determined[1]
- Expected sensitivities[1]:

Messenger

Par.	Uncertainties
$\bar{s}_A$	1.1×10^{-10}
$\bar{s}_{TX}$	3.1×10^{-8}
$\bar{s}_B$	1.4×10^{-8}
$\bar{s}_C$	3.2×10^{-11}

Cassini (Saturn)

Par.	Uncertainties
$\bar{s}_F$	8.6×10^{-11}
$\bar{s}_{TX}$	1.2×10^{-8}
$\bar{s}_G$	1.5×10^{-8}
$\bar{s}_H$	2.3×10^{-11}

- To be compared with previous results obtained with LLR+interferometry[2]

Coeff.	
$\bar{s}^{TX}$	$(0.5 \pm 6.2) \times 10^{-7}$
$\bar{s}^{TY}$	$(0.1 \pm 1.3) \times 10^{-6}$
$\bar{s}^{TZ}$	$(-0.4 \pm 3.8) \times 10^{-6}$
$\bar{s}^{XX} - \bar{s}^{YY}$	$(-1.2 \pm 1.6) \times 10^{-9}$
$\bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}$	$(1.8 \pm 38) \times 10^{-9}$
$\bar{s}^{XY}$	$(-0.6 \pm 1.5) \times 10^{-9}$
$\bar{s}^{XZ}$	$(-2.7 \pm 1.4) \times 10^{-9}$
$\bar{s}^{YZ}$	$(0.6 \pm 1.4) \times 10^{-9}$

Results promising
⇒ give strong motivations to do the real analysis

[1] A. Hees, et al, proceedings of CPT13

[2] J. Battat, J. Chandler, C. Stubbs, PRL, 99/241103, 2007
K. Chung, et al, PRD, 80/016002, 2009

Conclusion

- The TTF is a very nice tool to compute the time transfer, the Doppler and astrometric (VLBI) observations
- Analytical results found (so far):
  - time transfer in Schwarzschild space-time at 1, 2, 3 pM order

    see B. Linet and P. Teyssandier, CQG 30, 175008, 2014

  - time transfer around static axisymmetric body

    see C. Le Poncin-Lafitte, P. Teyssandier, PRD 77, 044029, 2008

  - time transfer around a slowly moving monopole

    see S. Bertone et al, CQG 31, 015021, 2014

  - time transfer around uniformly moving axisymmetric body

    see A. Hees, et al, PRD 08, 084020, 2014
- Very efficient from a numerical point of view

  see A. Hees, et al, PRD 89, 064045, 2014
- Useful to assess order of magnitude of different GR effects but also effects from alternative theories of gravitation

  see A. Hees, et al, CQG 29, 235027, 2012