Effects of asteroids on the orbital motions of terrestrial planets

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Abstract

The present planetary ephemerides, as INPOP08 (Fienga et al.,2009) [1] and DE405 (Standish et al.,1998) are subject to a lack of accuracy, because of the perturbations arising from a large number of asteroids. Those perturbations could reach a few kilometers in several decades in the case of Mars for instance. So it seems appropriate to study in details the individual specific effects of these asteroids.

As an illustration this work deals with the evaluation of the individual effects of largest asteroids of the solar system on the orbits of the terrestrial planets Mercury, Venus, Earth and Mars.

Our methodology consists of several steps:

- ⋆ A numerical integration of the orbits of the planets with and without the disturbing asteroid from which we want to know the effects.
- ⋆ A determination of the signal representing the effects, by simple substraction.
- ⋆ The analysis of the signal by the method of
  - – FFT (Fast Fourier Transform) to determine the most significant sinusoidal oscillations.
  - – Adjustment of the signal by the set of sinusoids determined in the previous step.

This type of study is interesting in many fields, such as planetary ephemerides, as well as spatial navigation, to understand better the effects of each asteroid taken individually on the terrestrial planets. Note that this type of study is a continuation of previous studies (Williams, 1984 [3]; Mouret et al.,2009 [2])

Introduction

The motion of a given planet around the sun can be considered at first approximation as a Keplerian motion perturbed by the other planets and the small bodies of the solar system. Each of these perturbations must be treated either analytically or numerically, and can be measured as a change of the planet's osculating orbital elements $(a, e, i, \Omega, \varpi = \Omega + w$ and $L = \varpi + M)$ determined from the perturbing function $\Re$, according to Lagranges formula

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial \Re}{\partial L}$$

$$\frac{de}{dt} = -\frac{\sqrt{1-e^2}}{na^2 e}\left(1-\sqrt{1-e^2}\right)\frac{\partial \Re}{\partial L} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial \Re}{\partial \varpi}$$

$$\frac{di}{dt} = -\frac{1}{na^2\sqrt{1-e^2}\sin i}\left[\frac{\partial \Re}{\partial \Omega} + (1-\cos i)\left(\frac{\partial \Re}{\partial \varpi} + \frac{\partial \Re}{\partial L}\right)\right]$$

$$\frac{d\Omega}{dt} = \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial \Re}{\partial i}$$

$$\frac{d\varpi}{dt} = \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial \Re}{\partial e} + \frac{1-\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial \Re}{\partial i}$$

$$\frac{dL}{dt} = n - \frac{2}{na}\frac{\partial \Re}{\partial a} + \sqrt{1-e^2}\left(\frac{1-\sqrt{1-e^2}}{na^2 e}\right)\frac{\partial \Re}{\partial e} + \frac{1-\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial \Re}{\partial i}$$

To evaluate the effects of a given asteroid on the terrestrial planets we use the numerical integration (Runge-Kutta of the 12th order), in the frame of the 9-body problem (the Sun and the eight planets without asteroids), then of the 10-body problem (the Sun and the eight planets together with the given asteroid). Then we determine the differential variations of orbital parameters of the planet by simple subtraction of the two signals obtained. After we perform the frequency analysis of the data, using fast Fourier transform (FFT) to determine the leading frequencies. At last we carry out a nonlinear regression in which the differential data are modeled by least-square method following an equation of type :

$$F(t) = \sum_{i=1}^{N} A_i \sin(f_i t) + B_i \cos(f_i t) + C_i t \sin(f_i t) + D_i t \cos(f_i t)$$

We have also calculated the individual influences of each asteroid on the distance from the EMB (Earth-Moon barycenter) to the given planet and the orientation vector of this planet as seen from the EMB ,which are very important parameters in space navigation and astrometry.

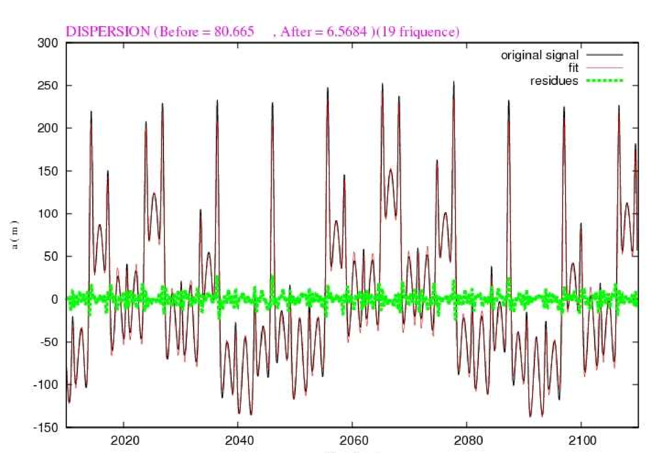
We present below an example of our results which consist in tables showing the coefficients of Fourier and Poisson components for the orbital elements of each terrestrial planet with respect due to each asteroid, and the corresponding curves (the initial signal, the adjustment determined by our analysis and the residuals). In each case: (planet, astroid, orbital element) we find that our fit is satisfactory, since the post-fit residuals are significantly lower than the original signal.

Gravitational influence of Ceres on the semimajor axis of Mars

BIAS : -0.781486 $\times 10^{+5}$
LINEAR : 0.834730 $\times 10^{+2}$
T^2 : -0.221018 $\times 10^{-1}$

PERIOD (Day)	PERIOD (Year)	SIN $\times 10^4$	COS $\times 10^4$	$\times 10^4$	T SIN $\times 10^0$	T COS $\times 10^0$	$\times 10^0$	amplitud $\times 10^2$	Asteroid
37126.67	101.65	1.5936	- 0.1788	1.6036	- 7.7484	0.7782	1.1005	0.1664	1_ Ceres
17079.57	46.76	0.0805	0.0941	0.1238	- 0.3470	- 0.4554	0.6441	1.1071	1_ Ceres
4849.40	13.28	- 0.0024	0.0022	0.0032	0.0144	- 0.0085	0.0120	0.3714	1_ Ceres
3752.00	10.27	0.0038	0.0056	0.0068	0.0076	- 0.0154	0.0217	0.6514	1_ Ceres
3050.05	8.35	- 0.0018	0.0000	0.0018	0.0087	- 0.0077	0.0109	0.1710	1_ Ceres
1876.35	5.14	0.0015	0.0029	0.0033	0.0040	- 0.0178	0.0251	0.2576	1_ Ceres
1250.96	3.42	0.0009	- 0.0003	0.0009	- 0.0028	- 0.0013	0.0018	0.0871	1_ Ceres
1161.21	3.18	- 0.0003	- 0.0025	0.0025	- 0.0017	0.0058	0.0081	0.1678	1_ Ceres
887.00	2.43	0.0001	- 0.0022	0.0022	- 0.0041	- 0.0064	0.0090	0.3611	1_ Ceres
717.51	1.96	0.0003	0.0034	0.0034	- 0.0033	- 0.0214	0.0302	0.1386	1_ Ceres
580.74	1.59	- 0.0011	- 0.0004	0.0012	- 0.0137	0.0163	0.0230	0.5175	1_ Ceres
502.92	1.38	- 0.0011	- 0.0001	0.0011	0.0027	0.0089	0.0126	0.1875	1_ Ceres
443.48	1.21	- 0.0003	0.0011	0.0011	0.0042	- 0.0031	0.0044	0.0787	1_ Ceres
387.15	1.06	- 0.0009	0.0018	0.0020	0.0136	- 0.0116	0.0164	0.2128	1_ Ceres
350.96	0.96	0.0019	0.0001	0.0019	- 0.0058	- 0.0037	0.0052	0.0988	1_ Ceres
320.96	0.88	- 0.0001	0.0004	0.0004	0.0009	- 0.0039	0.0055	0.0563	1_ Ceres
314.74	0.86	- 0.0011	0.0010	0.0015	0.0045	- 0.0028	0.0039	0.0519	1_ Ceres
290.36	0.79	0.0011	- 0.0005	0.0012	- 0.0091	0.0021	0.0030	0.0857	1_ Ceres
269.60	0.74	0.0005	0.0002	0.0006	- 0.0027	0.0016	0.0022	0.0486	1_ Ceres
DISPERSION	Before= 80.665229				After= 6.568432				
AMPLITUDE	Before= 390.753521				After= 49.948845				

Fourier and Poisson coefficients and the amplitudes for each frequency



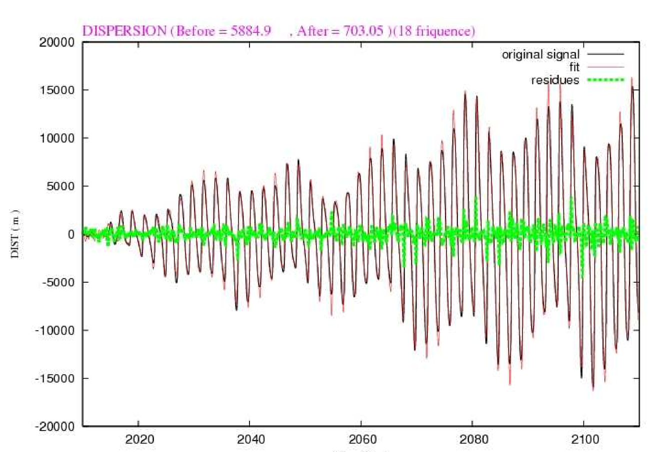
initial signal, fitting curve and post-fit residuals

Gravitational influence of Ceres on the EMB-Mars distance

BIAS : -0.205742 $\times 10^{+6}$
LINEAR : 0.200262 $\times 10^{+3}$
T^2 : -0.487244 $\times 10^{-1}$

PERIOD (Day)	PERIOD (Year)	SIN $\times 10^6$	COS $\times 10^6$	$\times 10^6$	T SIN $\times 10^3$	T COS $\times 10^3$	$\times 10^3$	amplitud $\times 10^4$	Asteroid
6088.53	16.67	0.0052	0.0176	0.0183	- 0.0030	- 0.0087	0.0123	0.2488	1_ Ceres
5217.28	14.28	0.0311	0.0023	0.0312	- 0.0152	- 0.0015	0.0021	0.1358	1_ Ceres
913.09	2.50	- 0.0222	- 0.0156	0.0271	0.0110	0.0073	0.0104	0.0583	1_ Ceres
890.77	2.44	0.0085	0.0164	0.0184	- 0.0039	- 0.0080	0.0113	0.4097	1_ Ceres
811.68	2.22	0.0465	0.0136	0.0485	- 0.0226	- 0.0068	0.0096	0.4211	1_ Ceres
779.93	2.14	0.0361	0.2852	0.2875	- 0.0202	- 0.1417	0.2004	1.2629	1_ Ceres
763.25	2.09	- 0.0950	0.2300	0.2489	0.0466	- 0.1116	0.1578	0.2339	1_ Ceres
745.31	2.04	- 0.0527	- 0.0381	0.0650	0.0250	0.0191	0.0270	0.3677	1_ Ceres
689.13	1.89	0.0315	0.0018	0.0315	- 0.0155	- 0.0012	0.0016	0.2572	1_ Ceres
676.34	1.85	0.0277	0.0399	0.0485	- 0.0133	- 0.0196	0.0277	0.1318	1_ Ceres
419.86	1.15	- 0.0102	- 0.0203	0.0227	0.0051	0.0100	0.0141	0.0636	1_ Ceres
396.79	1.09	0.1282	0.1267	0.1803	- 0.0635	- 0.0606	0.0857	0.1331	1_ Ceres
392.32	1.07	- 0.9134	- 1.3365	1.6188	0.4636	0.6462	0.9139	0.6544	1_ Ceres
389.98	1.07	- 2.5966	4.7104	5.3787	1.2210	- 2.3582	3.3349	1.4097	1_ Ceres
388.76	1.06	- 1.8107	2.0888	2.7643	0.9508	- 0.9632	1.3622	8.3153	1_ Ceres
260.88	0.71	- 0.0035	- 0.0132	0.0136	0.0020	0.0064	0.0091	0.1590	1_ Ceres
259.02	0.71	- 0.0140	0.0103	0.0173	0.0070	- 0.0048	0.0068	0.0843	1_ Ceres
195.31	0.53	- 0.0105	- 0.0156	0.0189	0.0053	0.0077	0.0108	0.0814	1_ Ceres
DISPERSION	Before= 5884.903369				After= 703.050511				
AMPLITUDE	Before= 31323.238410				After= 8331.534175				

Fourier and Poisson coefficients and the amplitudes for each frequency



initial signal, fitting curve and post-fit residuals

asteroide	Δa [m]	$\Delta e \times a_{Mars}$	Δi ["] $\times 10^{-6}$	$\Delta\Omega$ ["] $\times 10^{-6}$	$\Delta\varpi$ ["] $\times 10^{-6}$	$\Delta\lambda$ ["] $\times 10^{-6}$
1 Ceres	385.99973	1018.25341	837.97915	37190.92552	16391.84563	19419.20264
2 Pallas	193.24460	413.14399	441.88830	12173.16362	3700.64524	18125.33458
3 Juno	29.79749	47.96241	35.77520	869.55154	430.40624	3198.86497
4 Vesta	404.55975	1636.34551	586.32212	8301.55218	14585.61346	64574.80064
6 Hebe	61.28165	319.84333	74.70642	1275.36428	2971.28411	4854.97680
7 Iris	63.86719	135.36684	10.91732	972.29642	1218.83655	1869.09102
8 Flora	22.17044	65.30294	27.75361	131.25152	362.96686	2024.49401
9 Metis	66.42932	218.60586	11.70010	876.90488	2098.95546	2956.91830
10 Hygiea	40.41443	244.13825	38.31212	1254.45965	2798.38321	2998.78852
11 Parthenope	7.79214	51.54678	7.89198	42.35986	505.50615	464.19872
13 Egeria	11.49283	23.51995	6.00169	1626.31059	361.99825	619.36587
14 Irene	12.16370	21.63927	13.84427	483.07604	271.56248	1050.89770
15 Eunomia	22.84708	71.25421	64.06555	1315.85795	470.31980	2029.37351
16 Psyche	6.99493	21.21170	9.41352	229.74675	161.67922	645.65713
17 Thetis	2.17981	7.78699	2.12143	5.41765	68.21111	219.80180
18 Melpomene	6.50754	13.38326	14.99738	180.46621	105.09344	119.62906
19 Fortuna	25.64017	117.05157	1.45028	241.74259	1004.49811	6000.85764
20 Massalia	43.15692	87.96440	0.52064	92.24314	785.42295	2598.67141
21 Lutetia	4.85095	36.77710	0.95140	13.72545	290.94054	488.24872
22 Kalliope	2.24835	8.23317	3.41134	335.01872	66.82759	193.40492
24 Themis	8.29007	35.55968	0.45115	40.77149	263.71602	2145.45218
28 Bellona	13.00662	27.20348	20.89924	241.43927	481.31076	666.97955
29 Amphitrite	14.75650	49.64686	21.15240	273.76387	582.36206	1189.20823
31 Euphrosyne	5.12241	22.00394	5.49823	240.62689	302.94482	359.52655
45 Eugenia	7.74860	23.94010	19.99905	233.45638	290.71744	267.90582
46 Hestia	17.96786	63.84851	10.11392	600.04503	585.88586	1087.43324
47 Aglaja	0.56693	1.52783	0.98627	13.06890	14.46588	3.34039
48 Doris	2.88513	13.51154	4.41259	192.22558	143.69507	293.43395
49 Pales	1.36624	6.58591	0.74581	33.42004	44.87418	46.59249
52 Europa	6.43481	26.91968	13.23229	69.31637	342.50298	1296.25556
65 Cybele	2.39575	7.49899	3.32215	76.06273	71.36757	325.08383
87 Sylvia	1.35168	3.38215	3.47084	213.27108	43.60731	26.55712
88 Thisbe	4.08374	12.87875	6.41856	289.82627	119.82741	613.18320
90 Antiope	0.42065	2.01203	0.04924	0.74104	18.39564	53.38957
107 Camilla	1.45676	3.18390	5.05085	147.73203	63.08296	17.97039
111 Ate	144.29700	397.32342	171.68672	4215.37702	5169.29002	31583.25731
121 Hermione	0.65030	1.99023	0.94898	52.20717	20.46184	138.98339
130 Elektra	2.82965	10.37911	10.78122	110.47445	115.07394	132.25069
165 Loreley	6.50281	27.76875	28.27952	465.04322	345.34894	231.09589
189 Phthia	0.06564	0.31375	0.02352	2.13209	3.13691	8.40850
243 Ida	0.02475	0.03559	0.00574	0.31220	0.46578	2.44193
253 Mathilde	0.11630	0.19786	0.12028	4.48961	2.69194	8.82521
283 Emma	0.35806	1.11882	1.04708	16.94125	9.49715	9.82103

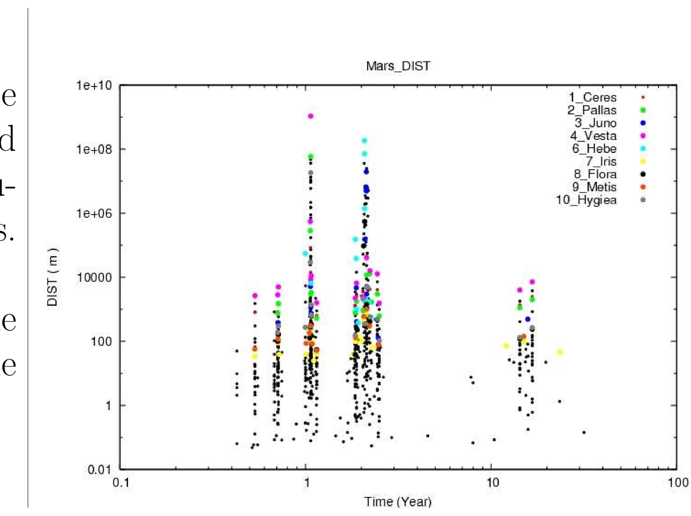
The above table lists :
- – the asteroids whose we study the effects
- – the individual effect in the orbital parameters of Mars, for the periods of 100 years.

Our analyses		Mouret	
Period (years)	amplitude (UA)	Period (years)	amplitude (UA)
1_Ceres			
44.33	3.848×10^{-10}	45.77	4.05×10^{-10}
10.28	3.980×10^{-10}	10.25	3.97×10^{-10}
1.59	3.299×10^{-10}	1.59	3.31×10^{-10}
2.43	2.421×10^{-10}	2.43	2.43×10^{-10}
5.14	1.668×10^{-10}	5.12	1.68×10^{-10}
1.06	1.343×10^{-10}	1.06	1.33×10^{-10}
1.38	1.233×10^{-10}	1.38	1.24×10^{-10}
8.34	1.030×10^{-10}	8.37	1.05×10^{-10}
3.18	9.922×10^{-11}	3.18	9.96×10^{-11}
1.96	7.684×10^{-11}	1.96	7.60×10^{-11}
0.96	6.434×10^{-11}	0.96	6.48×10^{-11}
0.79	5.478×10^{-11}	0.79	5.31×10^{-11}
13.38	4.618×10^{-11}	13.20	4.99×10^{-11}
3.43	4.486×10^{-11}	3.42	4.46×10^{-11}
1.21	4.472×10^{-11}	1.21	4.43×10^{-11}
0.74	3.271×10^{-11}	0.74	3.31×10^{-11}
0.86	3.165×10^{-11}	0.86	3.13×10^{-11}
0.88	2.685×10^{-11}	0.88	2.63×10^{-11}
1.65	2.114×10^{-11}	1.65	2.45×10^{-11}
0.68	2.328×10^{-11}	0.68	2.25×10^{-11}
1.88	2.032×10^{-11}	1.88	2.00×10^{-11}

Comparison between our study and the analytical study of Mouret [2] for the perturbations of Ceres on the orbital semi-major axis of Mars, for the periods of 1000 years

The next Figure present the Amplitude with respect to each frequency found from our analyses of the individual influence of asteroids on the distance EMB-Mars.

There are very clear lines, related to the synodic period of the Mars (as seen from the EMB)



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[2] S. Mouret, J. L. Simon, F. Mignard, and D. Hestroffer. The list of asteroids perturbing the Mars orbit to be seen during future space missions. *Astronomy & Astrophysics*, 508:479–489, December 2009.

[3] J. G. Williams. Determining asteroid masses from perturbations on Mars. *Icarus*, 57:1–13, January 1984.