l'Observatoire de Paris | SYRTE
Systèmes de Référence Temps-Espace

A rotational model of Phoebe

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L.Cottereau et al (2010)

Past Studies dealing with Phoebe'rotation

- 1905 : first ephemerids (Ross) Motion of Phoebe retrograde
- Jacobson (1998) new orbital elements
- Emelyanov (2007) Ephemerides of Phoebe from 1904-2027
- Aleshkina (2010) inertial parameters of Phoebe
- Phoebe is only non-synchronous satellite of Saturn with rather well known physical parameters

Why study the motion of Phoebe

	Phoebe
Eccentricity (J2000.0)	0.1648
Inclination (J2000.0)	$151^\circ.64$
Ascending node (J2000.0)	$54^\circ.317$
Obliquity	$23^\circ.95$
Period of rotation	0.386396 d
Triaxiality: $\frac{A-B}{4C}$	–0.011125
Dyn. flattening: $\frac{2C-A-B}{2C}$	0.06465
geo-equatorial coordinates of the pole of Phoebe	$\alpha_{\mathrm{p}}^{0} = 356^\circ.90$ and $\delta_{\mathrm{p}}^{0} = 77^\circ.80$
geo-equatorial coordinates of the pole of Saturn	$\alpha_{S0} = 40^\circ.589$ and $\delta_{S0} = 83^\circ.537$

Plan

1°) **Method**

- Parametrization of the rigid rotation of Phoebe
- Equation of the motion

2°) **Phoebe’s ephemerides**

- Semi-major axis
- The mean elements of the orbital parameters

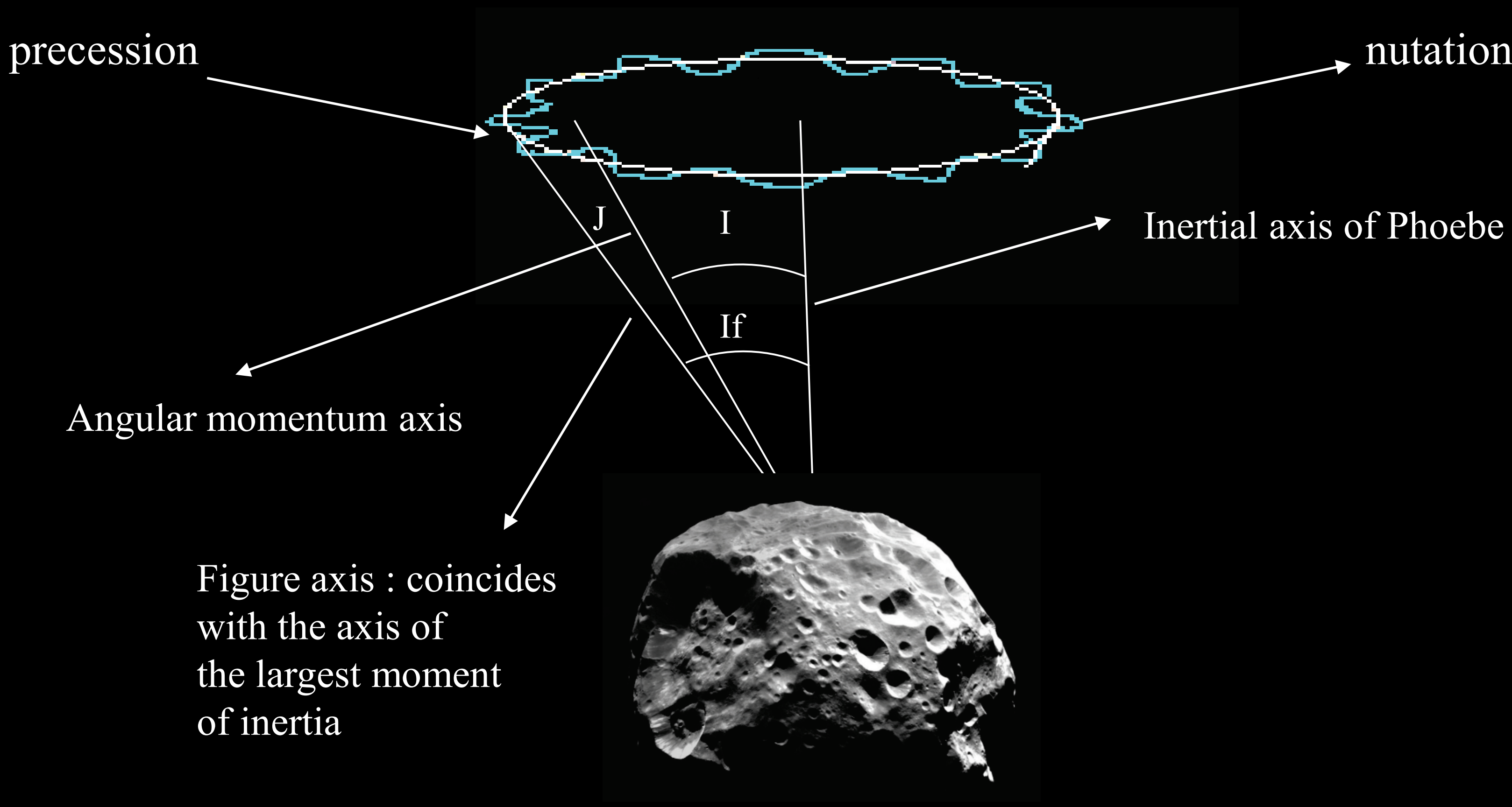
3°) **Results**

- Obliquity of Phoebe
- Numerical results
- Effect of the triaxiality of Phoebe
- Comparison with Venus

4°) **Construction of an analytical model**

- Analytical developments
- Analytical results and problems

Precession-nutation motion



Motion of the angular momentun axis is
Described by the angle h, I (Andoyer, 1923
where I is the obliquity and h the precession-nutation
angle in longitude

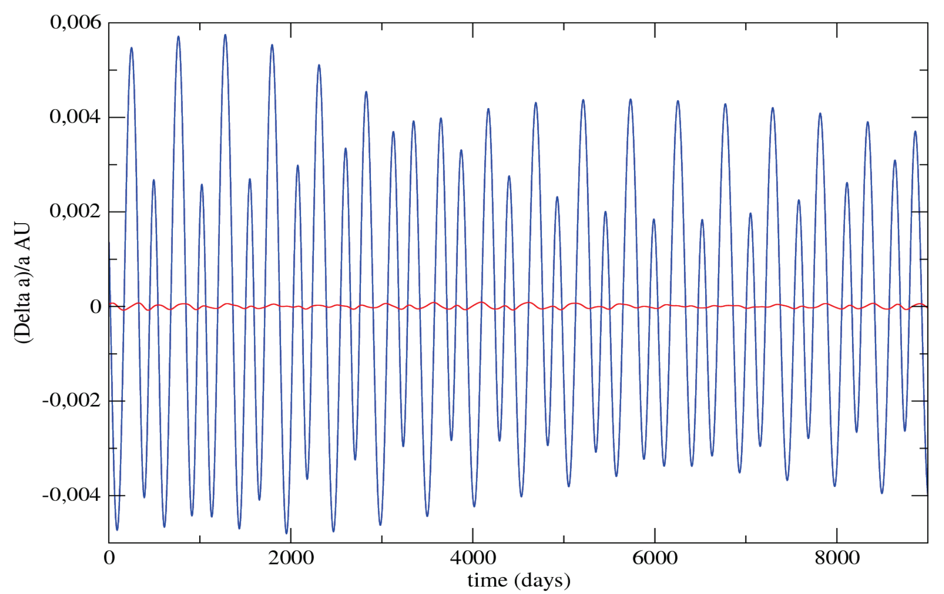
Inertial plane : the orbit of Phoebe at J2000.0.
The reference point is the intersection
between the orbital plane and the mean
equator of Phoebe at J2000.0

Motion of Venus due to external disturbing body

$$K'' = F_o + E + E' + U_1$$

$$U_1 = \frac{k^2 M'}{r^3}\left[\frac{2C - A - B}{2} P_2(\sin\delta) + \frac{A - B}{4} P_2^2(\sin\delta)\cos 2\alpha\right]$$

Semi-major Axis

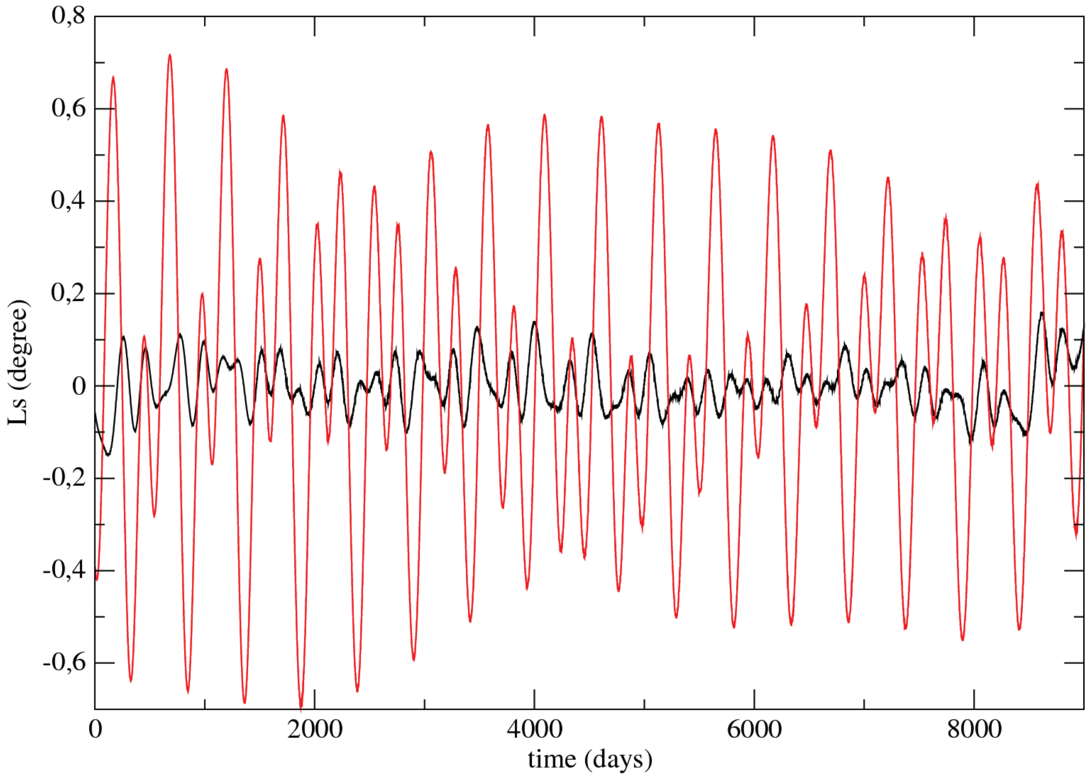


Leading amplitudes and periods characterizing the semi-major axis

Period days	Amplitude sin AU	Amplitude cos AU
261.98	$-2.319\ 10^{-4}$	2.609×10^{-4}
498.93	$-3.309\ 10^{-5}$	-1.756×10^{-4}
177.54	$-5.715\ 10^{-5}$	-9.971×10^{-6}

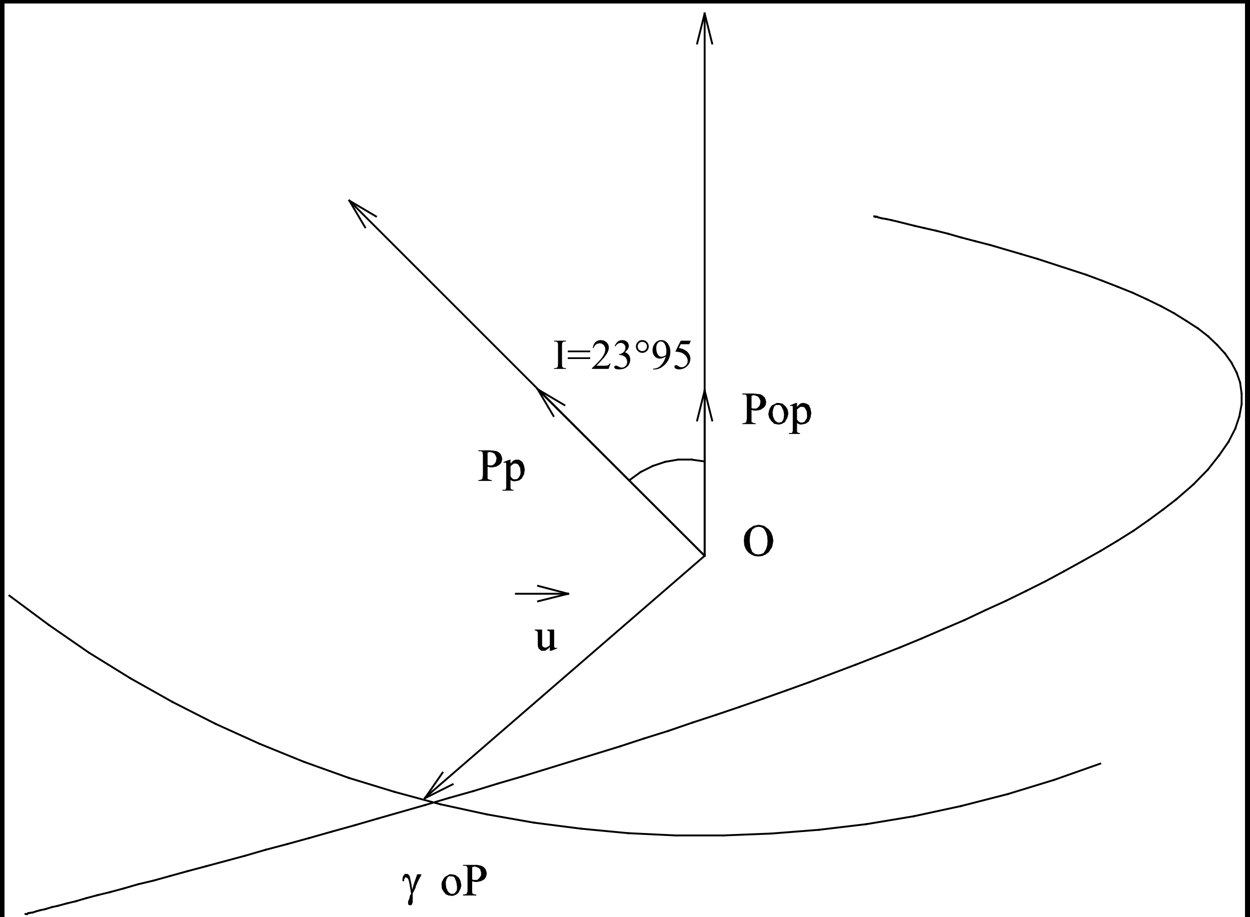
Relative variation of a around the mean value a=0.0864273 in 9000 days times span.

The mean elements of the orbital parameters : the mean longitude Ls



Obliquity of Phoebe

$$\overrightarrow{P_{\mathrm{op}}} = \begin{pmatrix} \sin i_0 & \sin\Omega_0 \\ -\sin i_0 & \cos\Omega_0 \\ \cos i_0 & \end{pmatrix}, \overrightarrow{P_{\mathrm{p}}} = \begin{pmatrix} \cos\delta_{\mathrm{p}}^0 & \cos\alpha_{\mathrm{p}}^0 \\ \sin\alpha_{\mathrm{p}}^0 & \cos\delta_{\mathrm{p}}^0 \\ \sin\delta_{\mathrm{p}}^0. & \end{pmatrix}$$



Precession

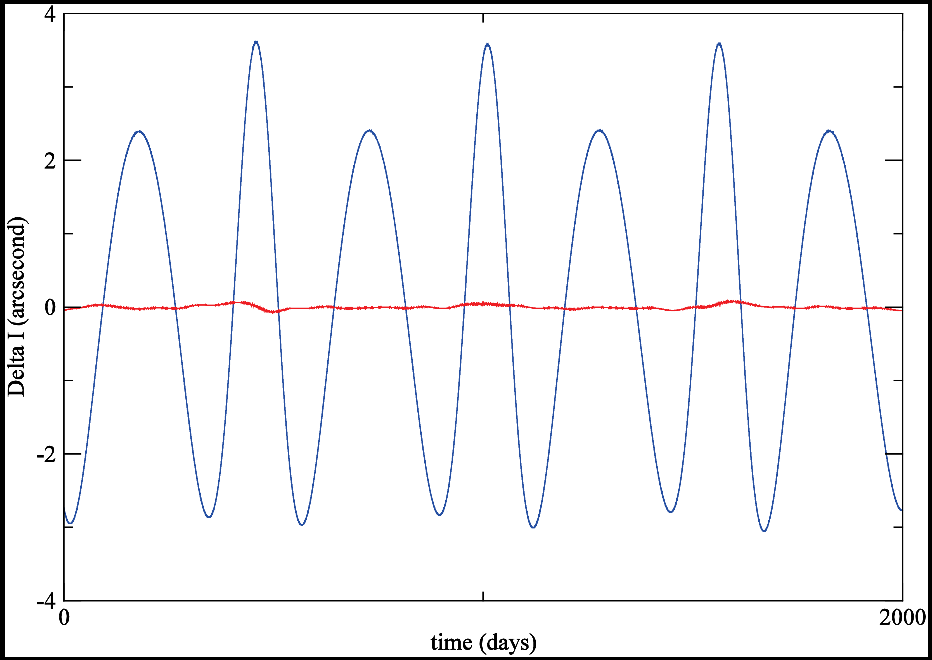
$$\dot{\psi}\ phoebe = 5580''.65/cy$$

$$\dot{\psi}\ terre = 1583''.99/cy$$ **(Sun only)**

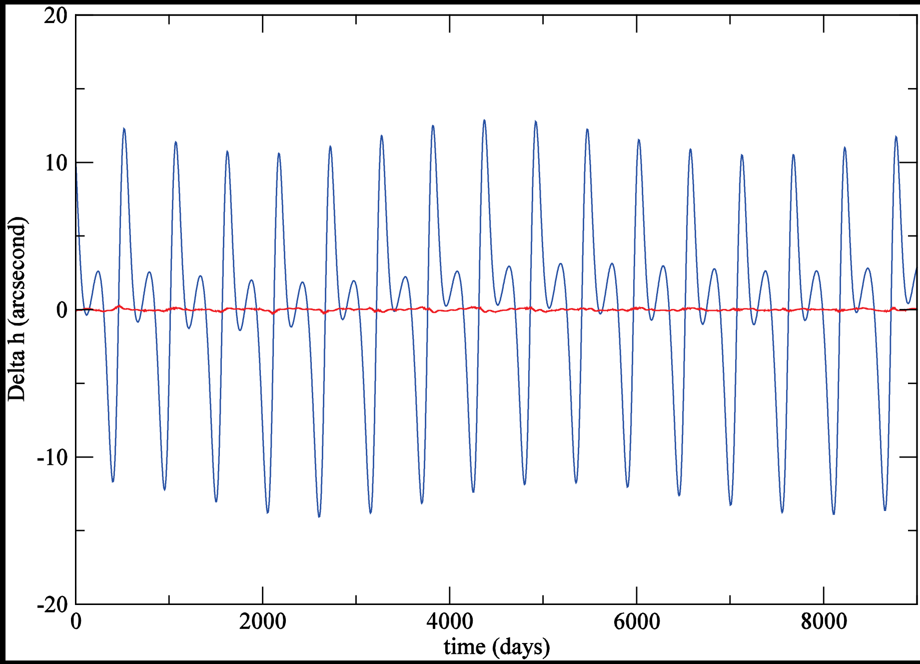
$$\dot{\psi}\ terre = 5000''.3/cy$$ **(Sun + Moon)**

Numerical results

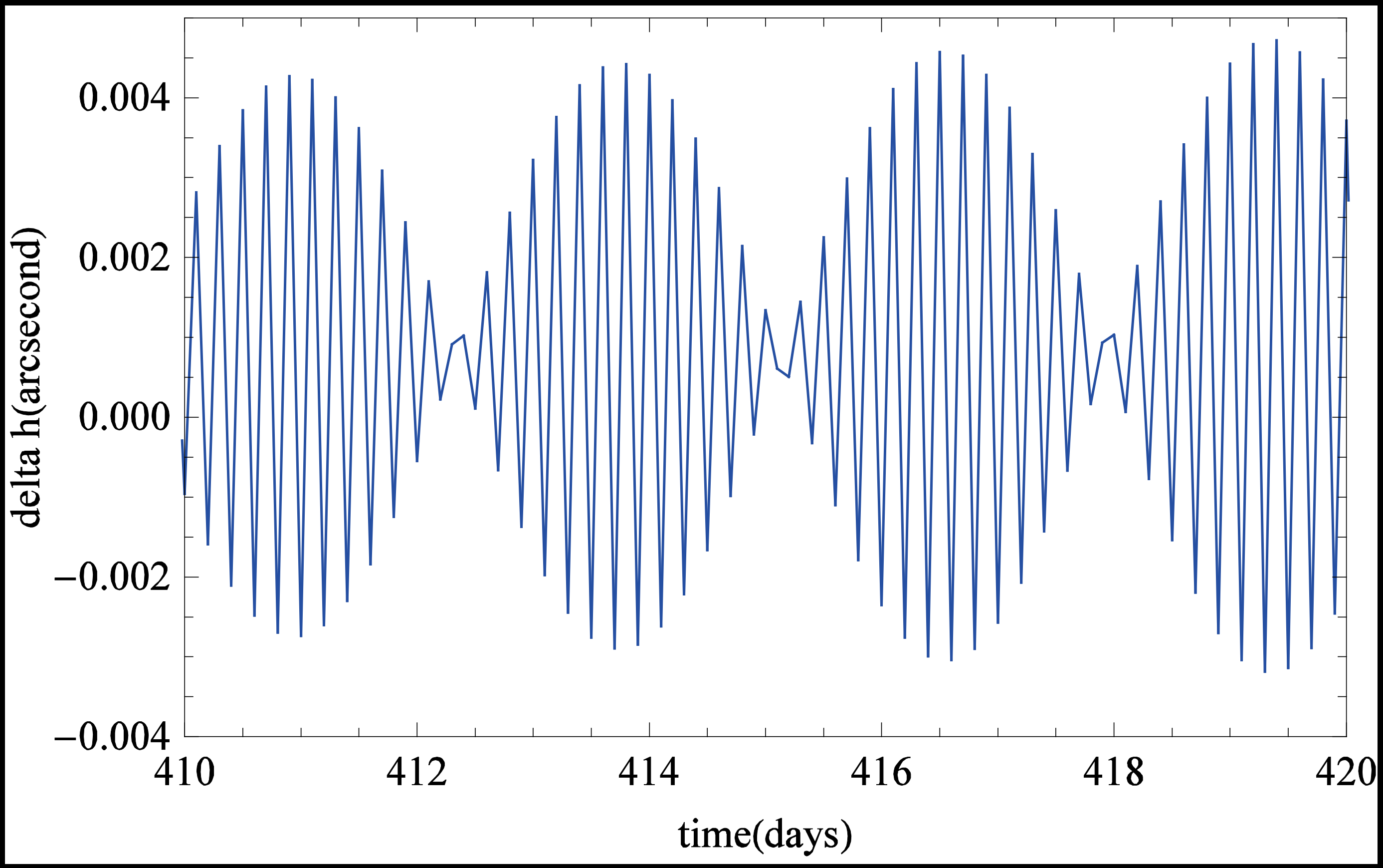
Obliquity of Phoebe



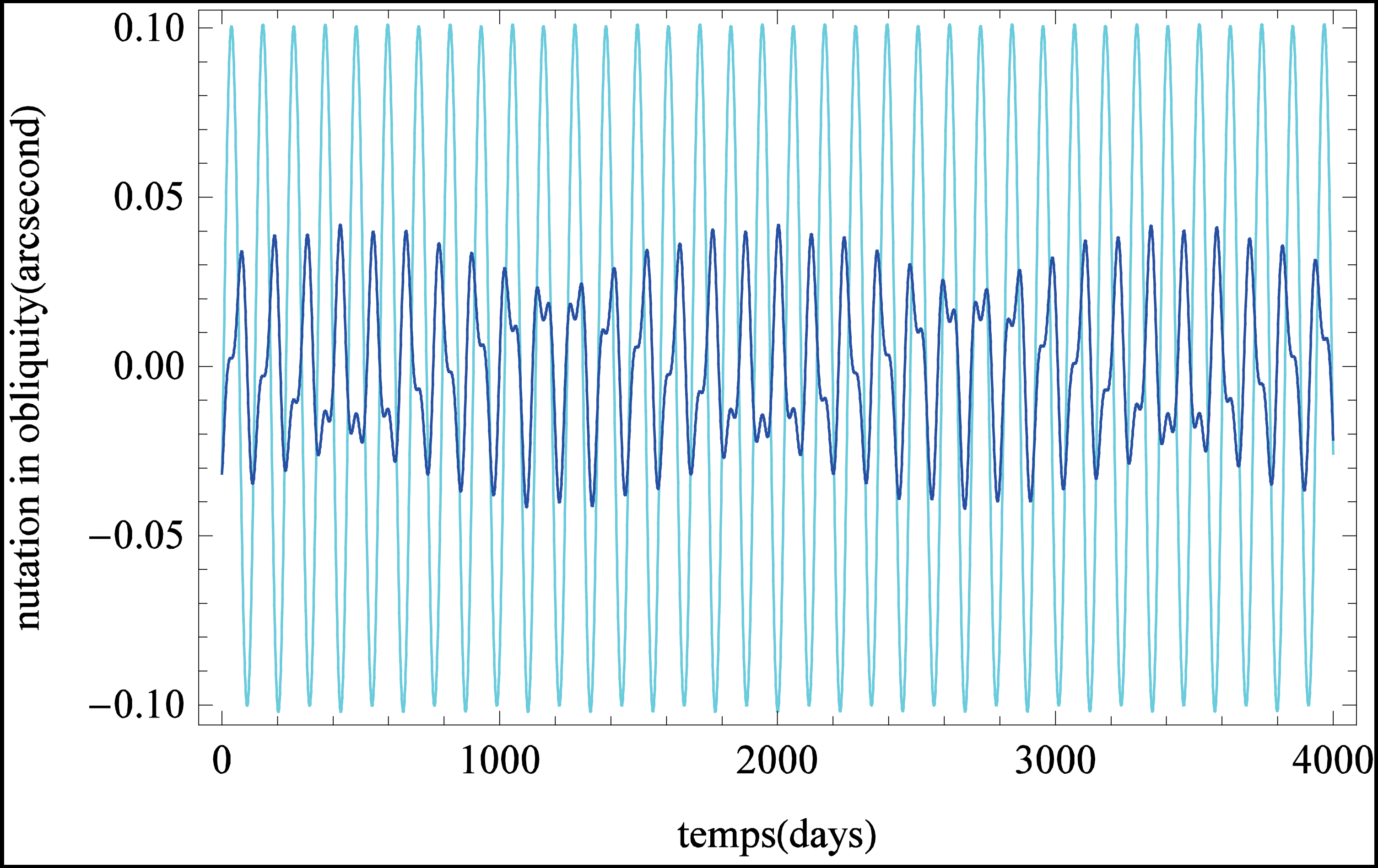
Longitude of Phoebe



Effect of the triaxiality of Phoebe on its nutation in longitude

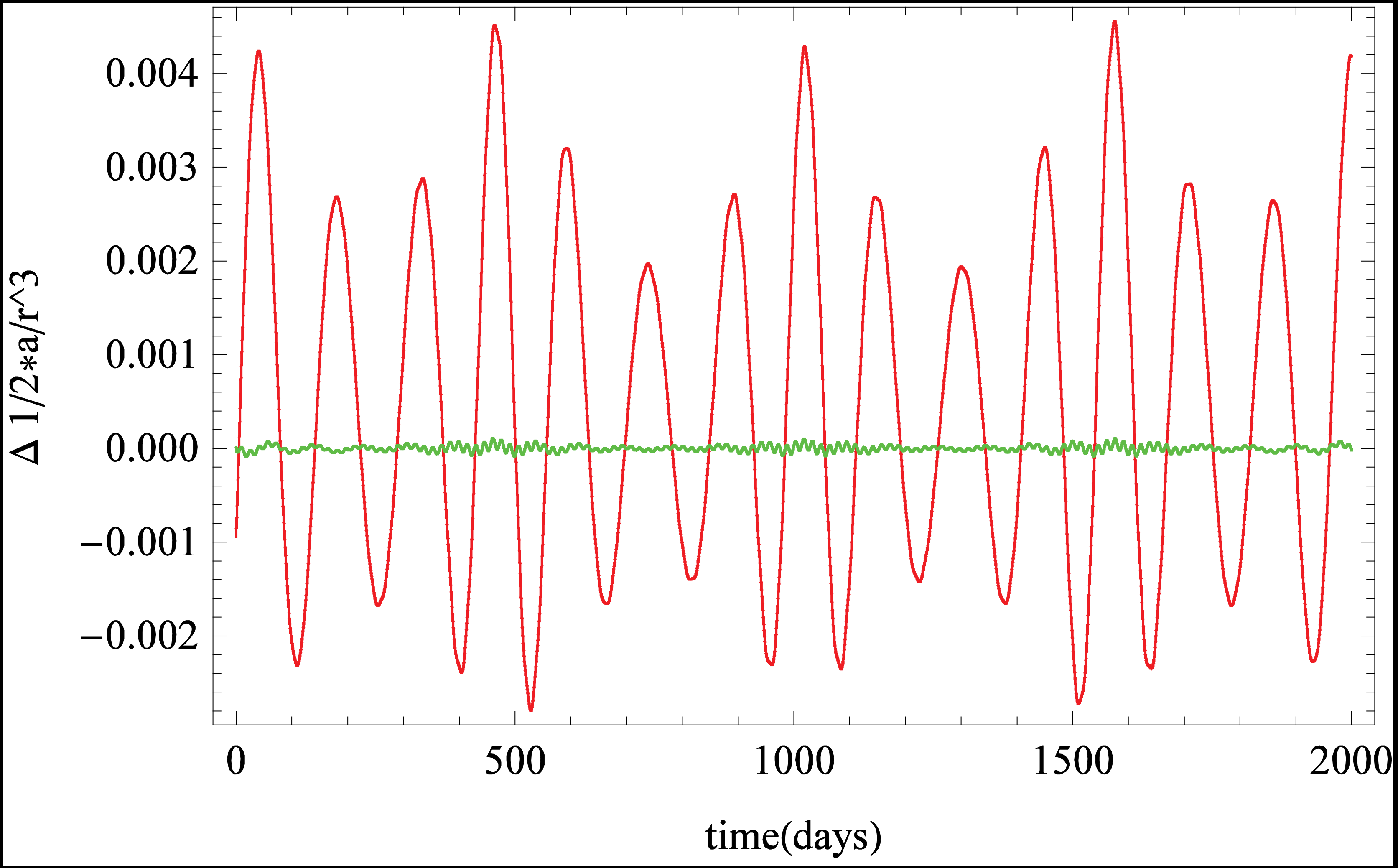


Comparison with Venus

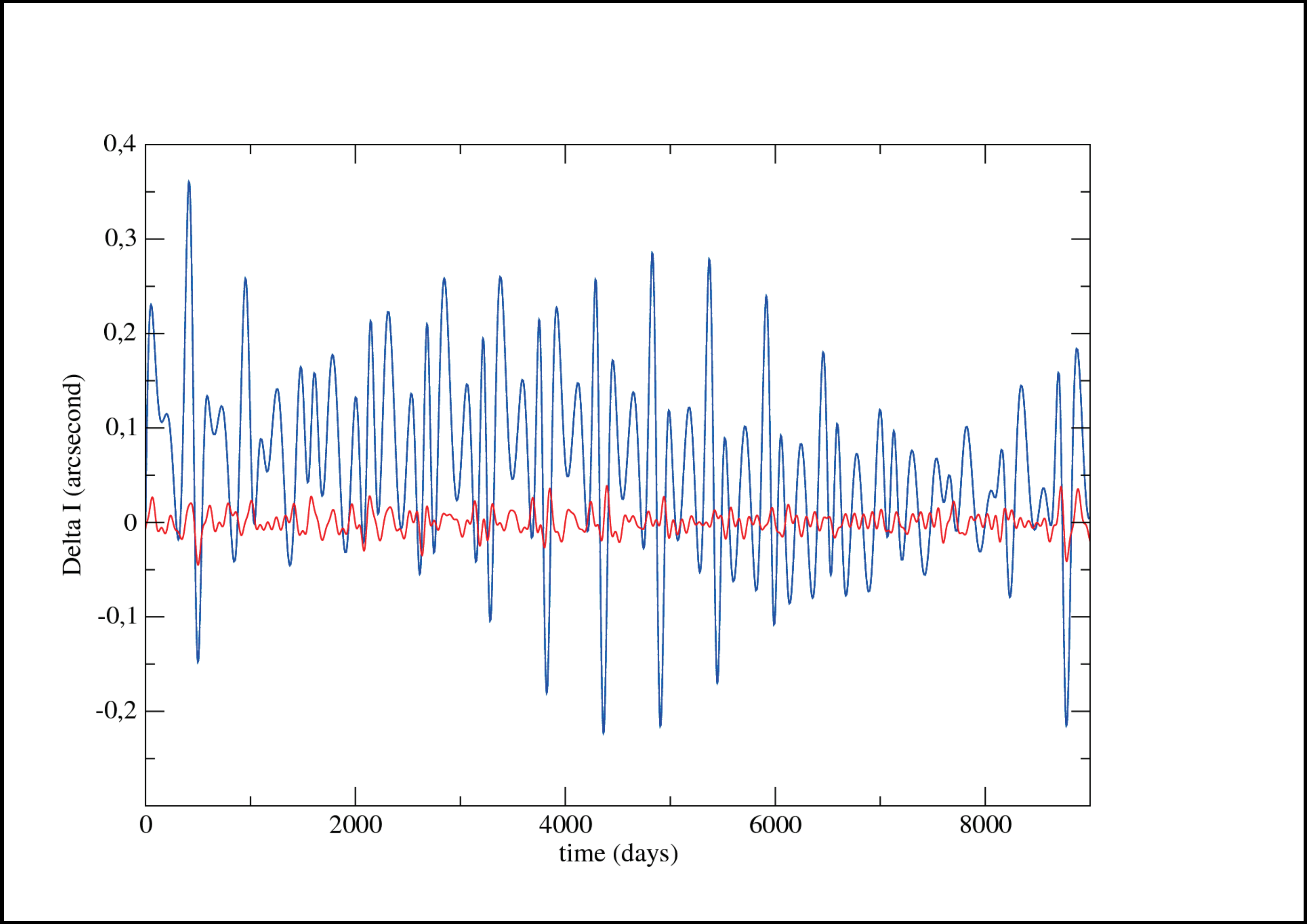


L.Cottereau and J.Souchay A&A (2010)

Analytical developments



Analytical results and problems



Conclusion

- The orbital motion of Phoebe is far from being a Keplerian motion
- The precession nutation motion of Phoebe undergoing the gravitational perturbation of saturn is quite similar to that of the Earth undergoing the gravitational effect of both the Moon and the Sun
- The analytical model set by Kinoshita (1977) gives a good first approximation of the precession nutation motion of Phoebe but further analytical developments are needed to reach the same accuracy than for terrestrial planets.

Prospect

- Study of another very precise analytical model of the rotation of Phoebe
- Evaluated the effect of Titan and the Sun and Saturn’s dynamical flattening on the rotation of Phoebe
- Develop the long’s term orbital ephemerides
- Understand why the rotation of Phoebe is retrograde

Thank You
For your attention