Relativistic effects in radioscience experiments

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ABSTRACT

Due to considerable increase of the accuracy level in modern space missions in the recent years, or expected in close-future missions, relativistic gravitational effects must be considered when computing spacecraft ephemerides and observables. Since General Relativity is invariant under coordinate transformations, we must consider the observables, i.e.: Range and Doppler signals modeled completely within general relativity.

In this poster, we present a software that integrates the equations of motion and determine the Doppler/Range signal directly from the space-time metric. This means that all the relativistic effects are taken into account. This software is based upon the integration of the geodesic equation [1] and the computation of time transfer thanks to Synge World function formalism [2, 3]. The distinction between proper time and coordinate time is also clearly made in order to have a consistent definition of the observables.

With such a software, it is possible to simulate Range/Dopper signals for any space mission in any metric theory of gravitation. This can be useful to quantify relativistic effects in space mission but also to quantify effects due to a possible deviation from General Relativity. In this poster, as an example, the case of BepiColombo is studied within General Relativity.

I. Simulations of observables

Our software directly generates Doppler/Range signal from the metric considered in a coherent way. First, we need to give a covariant definition of the Range/Doppler signals. Our model includes the integration of the relativistic equations of motion, the determination of time transfer and the clock modeling.

A. Observables

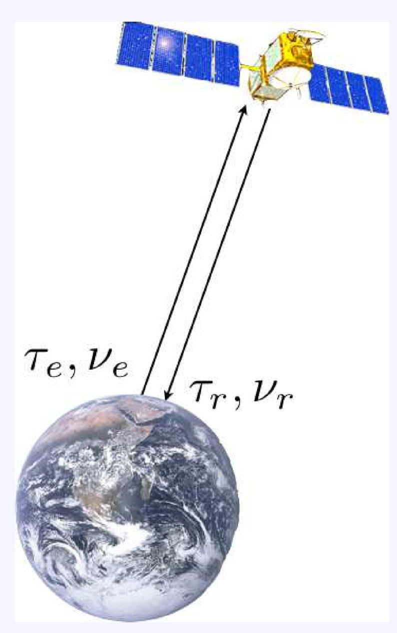
Since General Relativity is invariant under coordinates transformations, it is necessary to have a covariant definitions of the Range/Doppler signal.

- Range signal: difference between receptor proper time and emitter proper time

$$R(\tau_r) = \tau_r - \tau_e$$

- Doppler signal: proper frequency shift between emission and reception

$$D(\tau_r) = \frac{\nu_r}{\nu_e}$$



B. Equations of motion

The equations of motion are directly derived from the metric thanks to the geodesic equation [1] integrated with respect to coordinate time (in a particular coordinates system)

$$\frac{d^2x^i}{dt^2} = -\Gamma^i_{00} - 2\Gamma^i_{0j}v^j - \Gamma^i_{jk}v^jv^k + \Gamma^0_{00}v^i + 2\Gamma^0_{0j}v^iv^j + \Gamma^0_{jk}v^iv^jv^k \qquad (1)$$

where x^i (v^i) are the spatial coordinates (velocities) of the spacecraft, $\Gamma^\alpha_{\beta\gamma}$ are the Christoffel symbols of the considered metric and t is coordinate time.

C. Clock behavior

In order to make a clear distinction between proper time (time given by a clock) and coordinate time, proper time equations are also derived from the metric and integrated along the clock trajectory. The equation of proper time is

$$\frac{d\tau}{dt} = \sqrt{g_{00} - 2g_{0i}v^i - g_{ij}v^iv^j}$$

where $g_{\mu\nu}$ is the space-time metric and τ the proper time.

D. Range simulation

Steps followed to compute Range $R(\tau_r) = \tau_r - \tau_e$

- Transformation from receptor proper time τ_r to reception coordinate time t_r (using clock behavior)
- computation of coordinate propagation time $T = t_r - t_e$ using Synge World Function formalism [2, 3]:

Minkowski contribution

$$T = \frac{R_{er}}{c} + \frac{R_{er}}{c}\int_0^1 f(z^\alpha(\mu))d\mu + O(c^{-4})$$

Post-Minkowskian correction

Distance between emitter and receiver (found iteratively) $R_{er} = \|\mathbf{x}_e(t_e) - \mathbf{x}_r(t_r)\|$

Function f defined by the metric $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$

$$f = -h_{00} - 2N^i_{er}h_{0i} - N^i_{er}N^j_{er}h_{ij}$$

with $N^i_{er} = \left(x^i_r(t_r) - x^i_e(t_e)\right)/R_{er}$

Integral perfomed over Minkoski path (straight line) which is simpler than to find the photon trajectory

- Transformation from emission coordinate time $t_e = t_r - T$ to emitter proper time τ_e (using clock behavior)

E. Doppler Simulation

Frequency shift can be expressed from the metric

$$D(\tau_r) = \frac{\nu_r}{\nu_e} = \frac{d\tau_e}{dt_e}\frac{dt_e}{dt_r}\frac{dt_r}{d\tau_r} = \frac{\left[\sqrt{g_{00} + 2g_{0i}v^i/c + g_{ij}v^iv^j/c^2}\right]_e}{\left[\sqrt{g_{00} + 2g_{0i}v^i/c + g_{ij}v^iv^j/c^2}\right]_r}\frac{dt_e}{dt_r}$$

In the same spirit as for the Range, $\frac{dt_e}{dt_r}$ can be expressed thanks to integrals defined by the metric and its first derivatives over the Minkowski path.

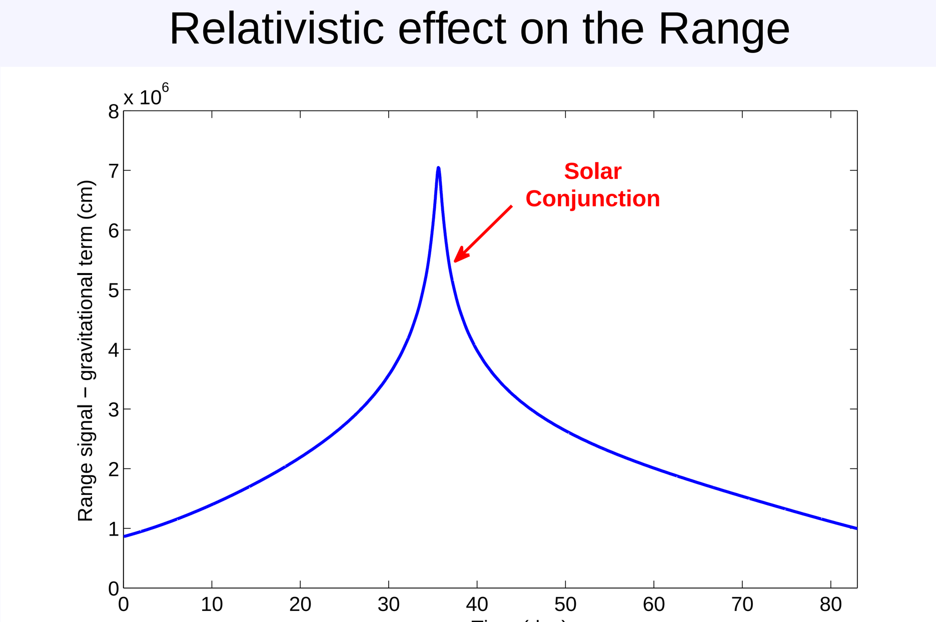
II. Simulations for the BepiColombo mission

BepiColombo is a space mission planned to be launched in 2014 to Mercury [4]. The main goals of this mission are the study of the origin of Mercury, its internal structure, its form, its composition... Another goal of this mission is to test Einstein's theory of gravitation [5].

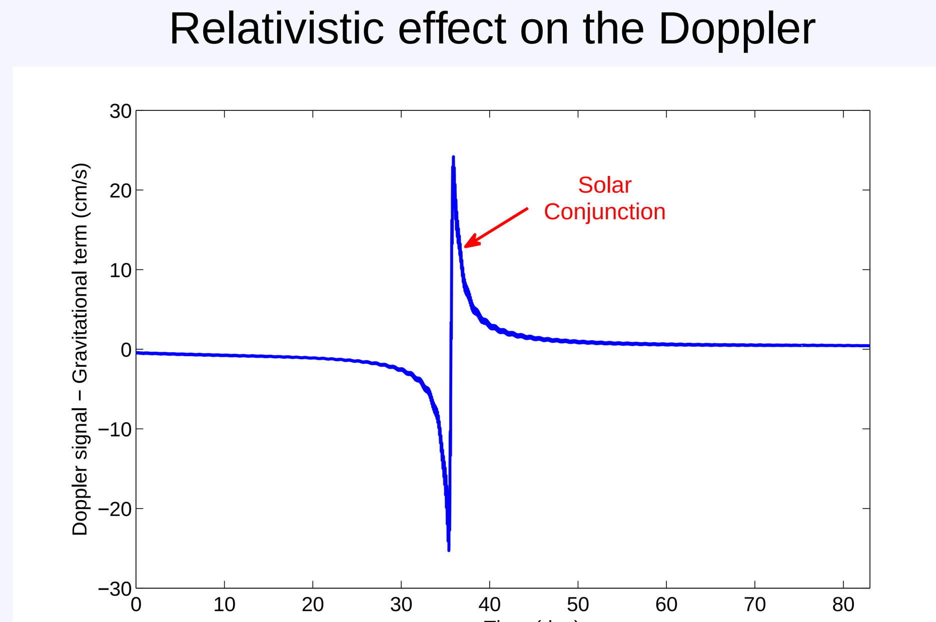
We simulate Doppler/Range two-way signal for this mission. This includes:

- orbits simulation of the Sun, Mercury, Earth and spacecraft
- integration of proper time for the Earth and spacecraft
- simulation of the Range/Doppler between the Earth and spacecraft as described above
- the metric used is the point masses metric in harmonic coordinates [6]
- similar simulations are done using metric theories different from GR, in order to study potential deviations. This is not discussed in this poster where we concentrate purely on GR.

Relativistic effect on the Range



Relativistic effect on the Doppler



These effects have to be compared to the expected precision for the BepiColombo mission: $10\ cm$ for the Range and $2\ \mu m/s$ for the Doppler [7].

To check our software, we compare our results with usual post-newtonian approach: we find a good agreement.

III. Conclusion

- new software that generates Range/Doppler signal directly from the space-time metric –> possible to simulate signals in GR and in alternative theories of gravitation.
- software validated by comparison with usual approach
- software useful to determine the magnitude of relativistic effects on space mission. For example: $8\ 10^6 cm$ effect in Range and $30 cm/s$ in Doppler for BepiColombo.
- software useful to quantify possible effects of an alternative theory of gravity (than GR) on the signals.

IV. Perspectives

- a complete understanding of the physically observable effects requires the adjustment of parameters (initial conditions) in the standard theory (GR) to the observables simulated in a different theory
- objective: detection of effects on the residuals and determination of correlations between fundamental effects and adjusted parameters.

Acknowledgments

A. Hees is a research fellow from FRS-FNRS (Belgian Fund for Scientific Research) for his PhD thesis at ROB-UCL (Université Catholique de Louvain) and SYRTE. Numerical simulations were made on the local computing resources (cluster URBM-SysDyn) at the University of Namur (FUNDP).

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