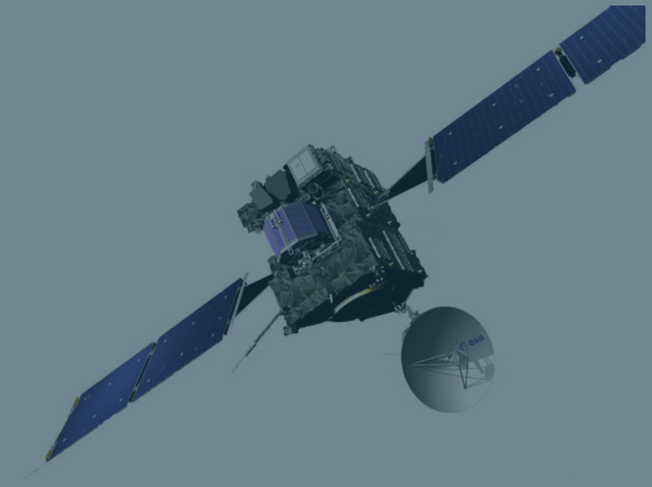
A Relativistic and Autonomous Navigation Satellite System



Semaine de l’Astrophysique française

Session GRAM

Gravitation, Références, Astronomie, Métrologie

Paris

20-23 juin 2011

Pacôme Delva

SYRTE / Observatoire de Paris

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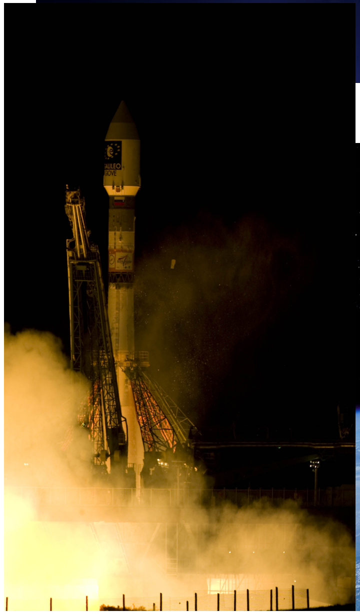
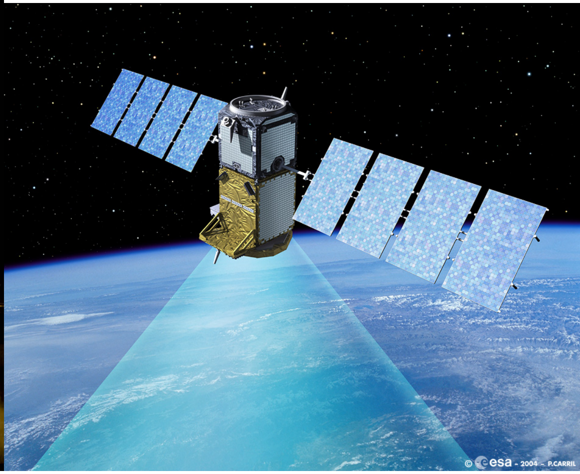
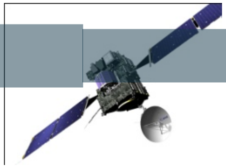
ESA Advanced Concepts

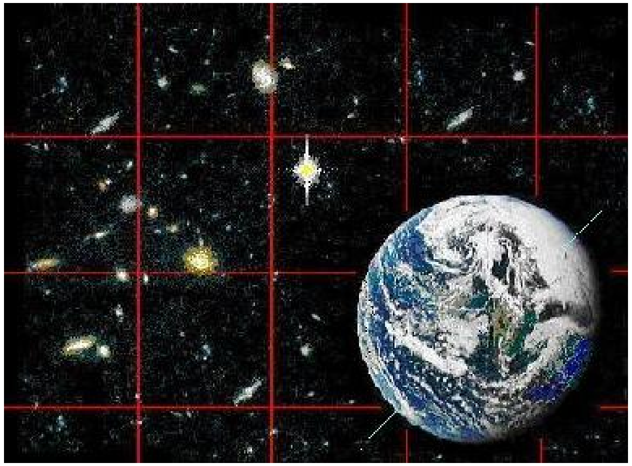
- **December 2005**: **Giove-A** → validate necessary technologies, such as the atomic clocks.
- **April 2008**: **Giove-B** launch
- **May 2009:** Galileo Geodetic Service Provider prototype final review → **Galileo Terrestrial Reference Frame**
- **26 January 2010**: ESA signs three contracts with industry → **Galileo operational infrastructure**
- **25 October 2010**: ESA contract with Spaceopal → **ground-based services**
- **20 October 2011**: first two satellites launch from French Guiana with a Soyuz rocket



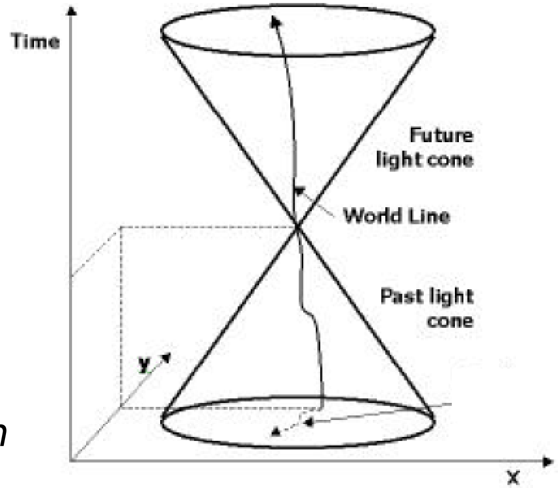




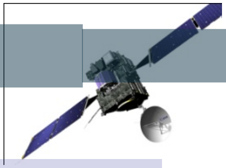
- **Reference system**: determinate the position of an object in space and/or in time



- Time arises from motion
- Relativity: events localized in **SPACETIME**

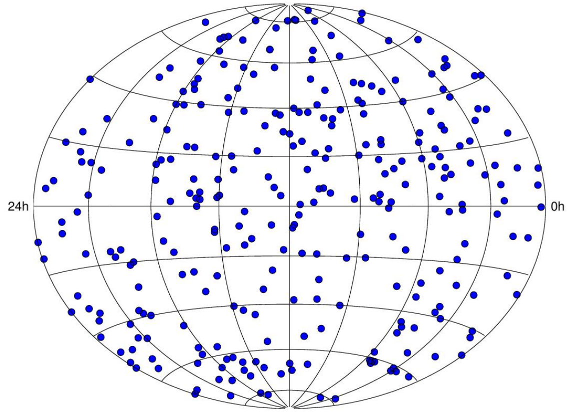


- Define a reference system (Kovalevsky 1989)
  - **Concept**: *ideal reference system*
  - **Physical structure**: *reference system*
  - **Modeling the structure**: *conventional reference system*
  - **Realize the reference system**: *conventional reference frame*



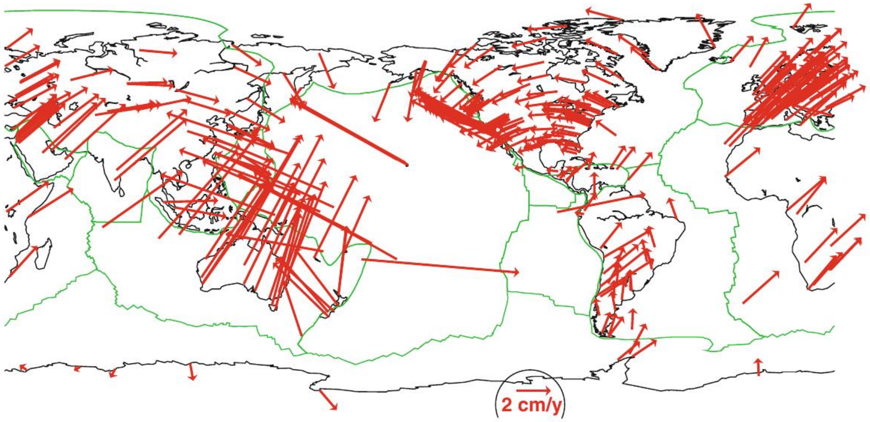
International Celestial Reference System

- **Concept**: such as objects with no proper motion (far away) remain fixed (inertial)
- **Physical structure**: extragalactic radio sources
- **Model**: conventions for transformations between frames for observations
- **Re**... urces ob...



International Terrestrial Reference System

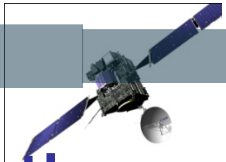
- **Concept**: such as the terrestrial crust shows no residual rotation wrt system
- **Physical structure**: represented by a set of fiducial points on the surface of the Earth
- **Model**: tectonic plates + deformations
- **Realization**: set of positions and

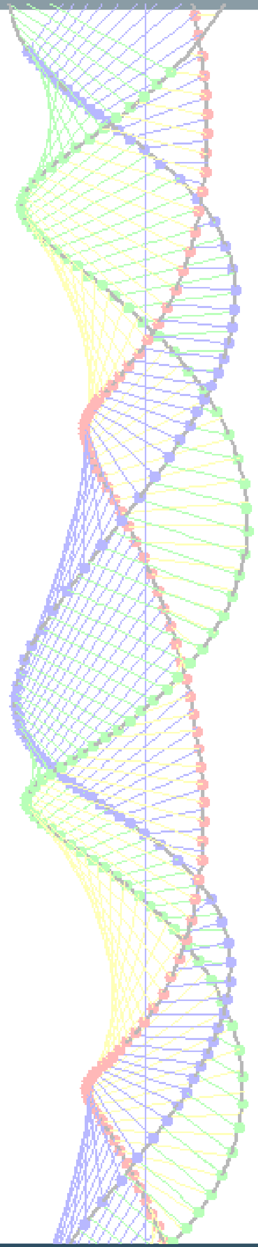


Autonomous Basis of Coordinates (ABC) reference system

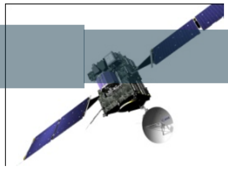
- **Concept**: coordinate system based on dynamics given by a hamiltonian (describes the spacetime geometry & non gravitational forces)
- **Physical structure**: satellites in Earth orbit & electromagnetic signals between the satellites (create a physical spacetime web)
- **Modeling**: choice of the hamiltonian (Minkowski, Kepler, Schwarzschild, ...)
- **Realization**: numerical simulation of the satellite constellation and signals

based on an idea of Bartolomé Coll (see SYPOR project 2003

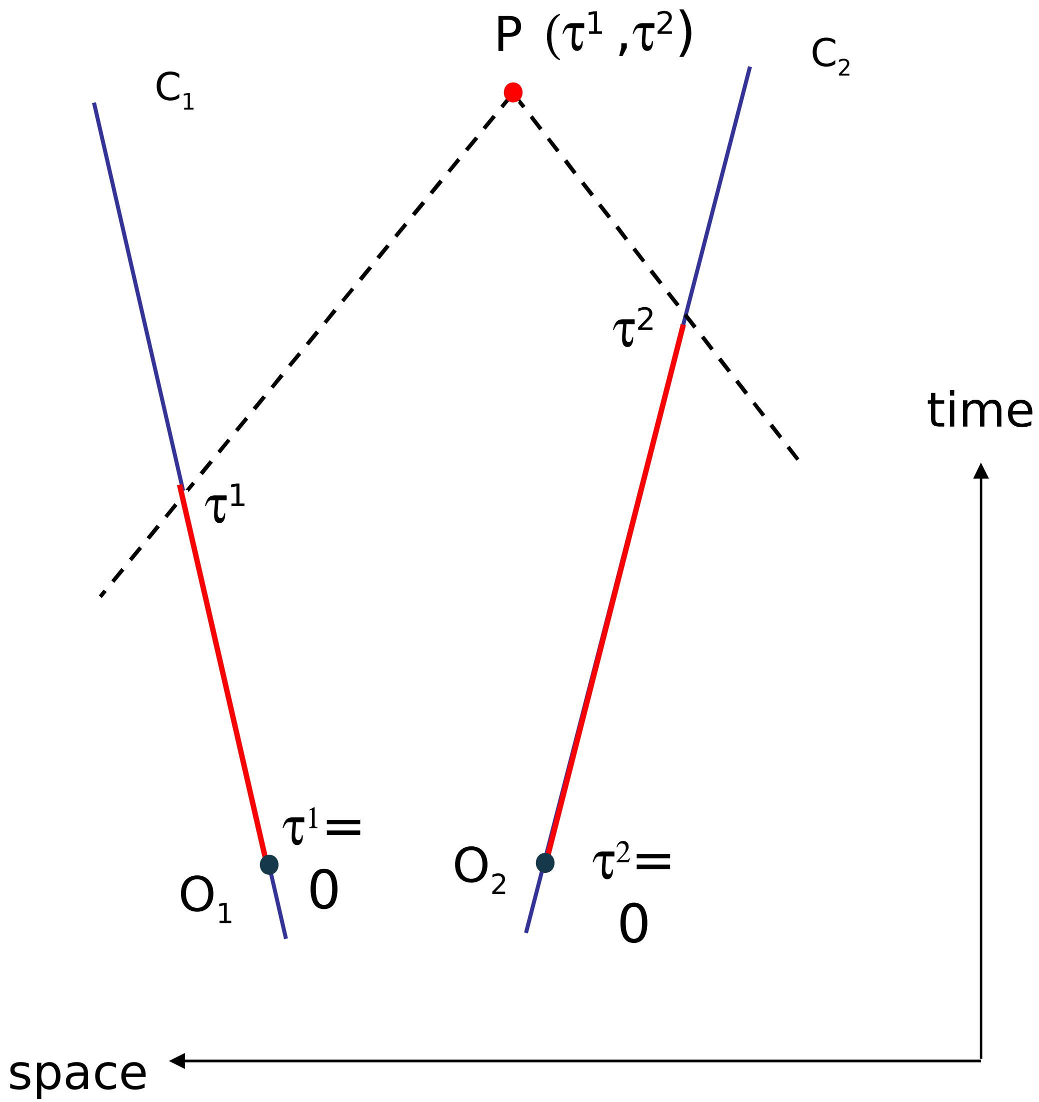




- Its realization **does not rely on observations from Earth**
  - No entanglement with Earth internal dynamics
  - No Earth stations for maintaining of the frame
- **Stability** and **accuracy**
  - Based on well-known satellite dynamics
  - Satellite orbits are very stable in time, and can be accurately described
- **Positioning system**
  - Observation of the signals sent by 4 satellites allows anyone to know its proper coordinates
- Full **GR concepts**
  - Up-to-date conception of spacetime
  - Deep understanding of localization in spacetime
- Applications in **geophysics** and **relativistic gravitation**



- General relativity + 4 test particles, whose time-like trajectories C_α are exactly known and parameterized with proper times τ^α.
- Given a point P, its past light cone intersects the four trajectories at proper times τ^1, τ^2, τ^3 and τ^4.
- Then $(\tau^1, \tau^2, \tau^3, \tau^4)$ are the coordinates of point P → **emission coordinates**



Rovelli, PRD **65** (2002)
Coll & Pozo, CQG **23** (2006)

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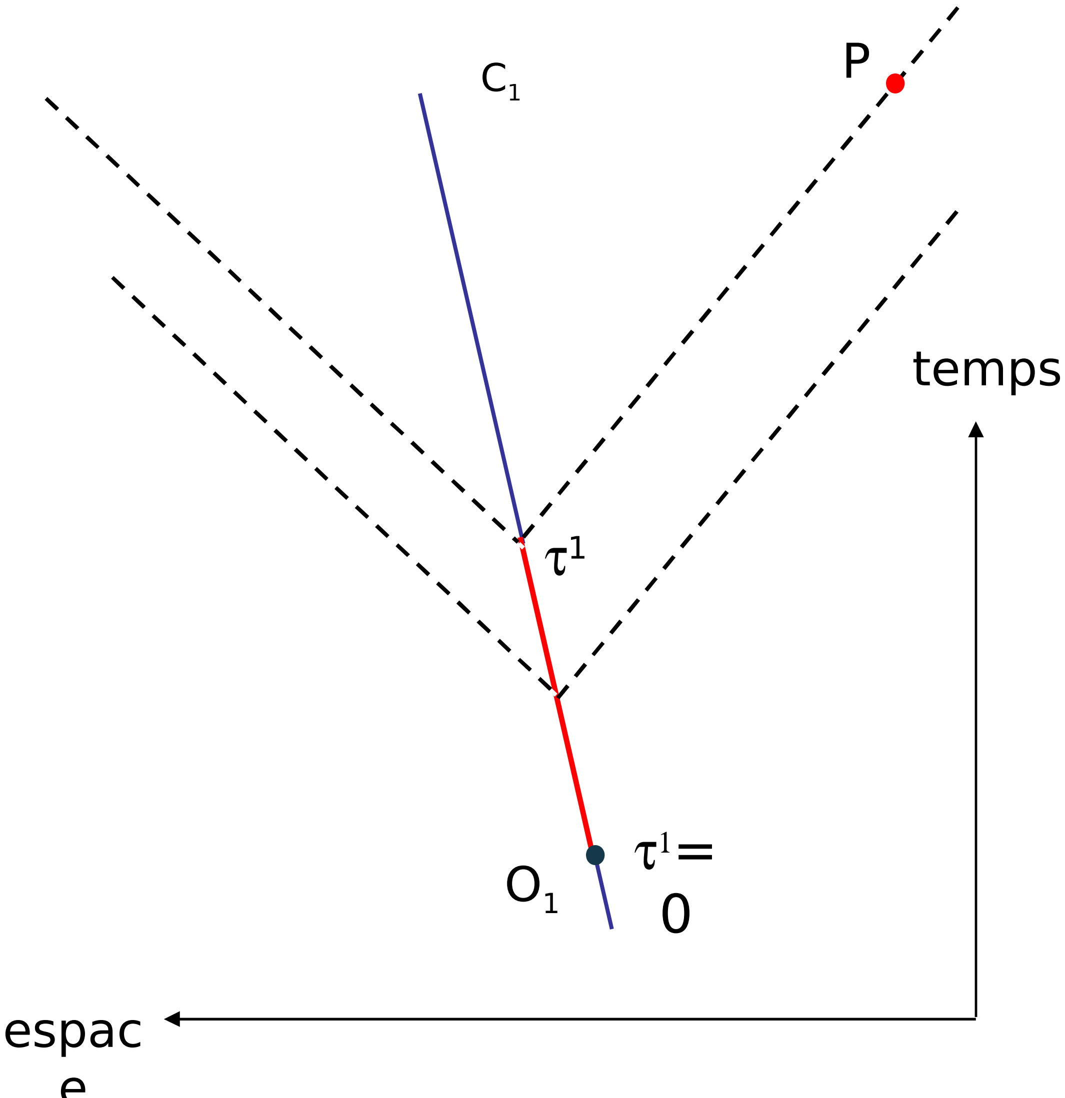

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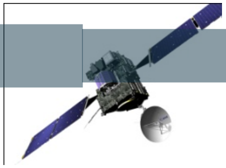
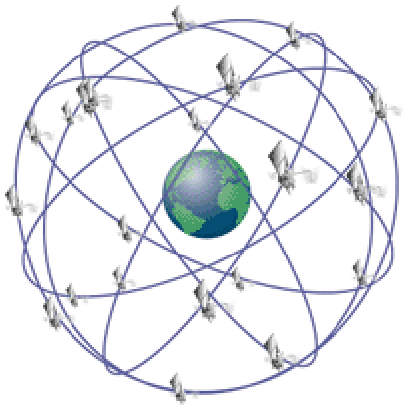


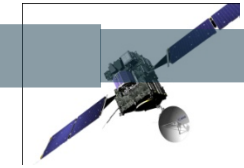


Emission coordinates

User

Satellites constants of motion

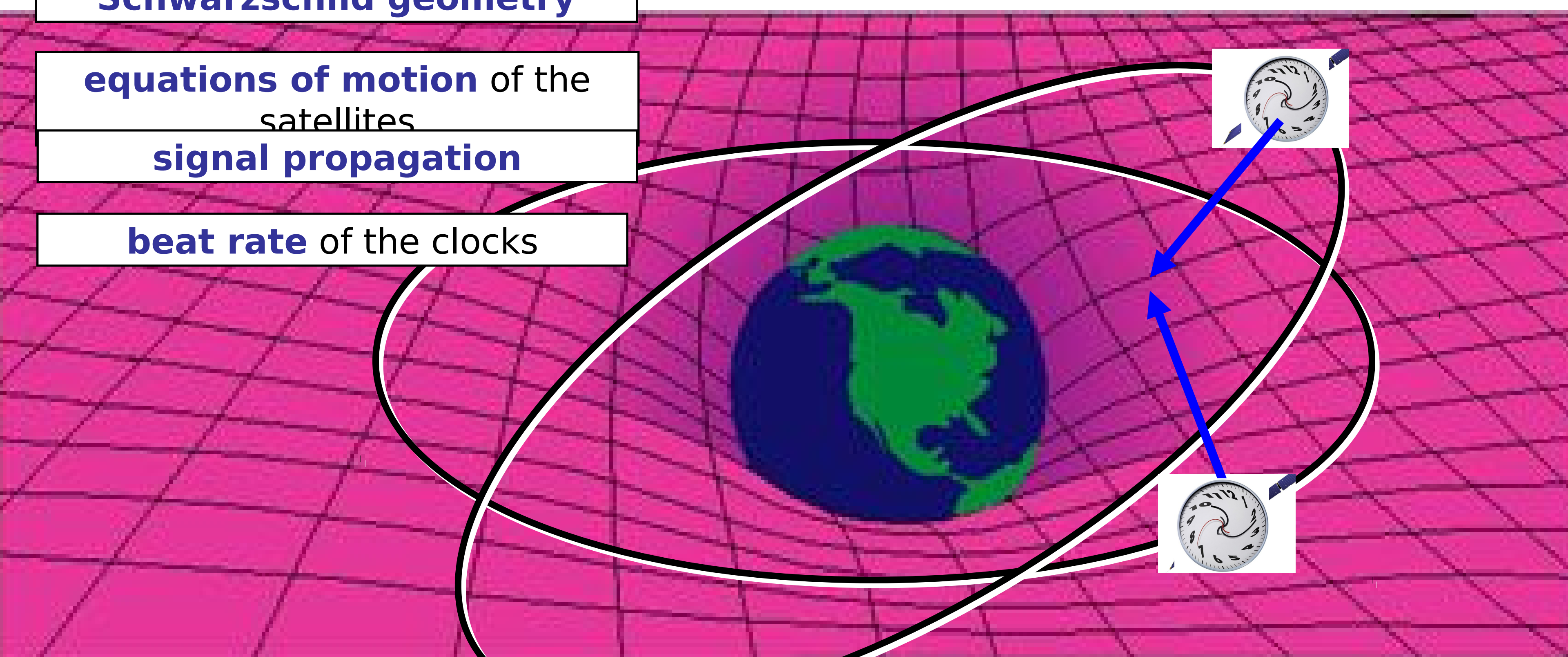
Coordinate transformation



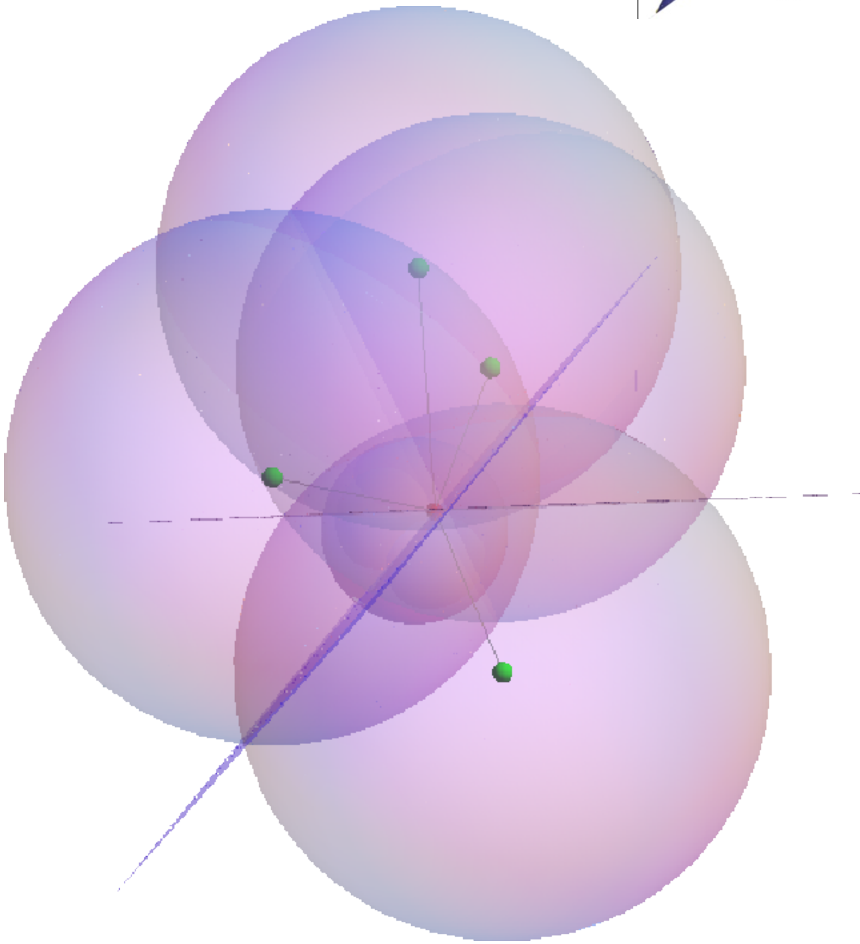
- Main ingredient: Hamiltonian to describe dynamics

- A first step towards the modelization of a GNSS in a relativistic framework and the understanding of a relativistic positioning system:

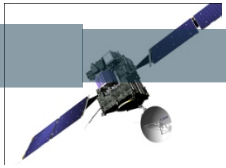
$$H = \frac{1}{2}\left[-\frac{1}{1-\frac{2M}{r}}\, p_t^2 + \left(1-\frac{2M}{r}\right) p_r^2 + \frac{1}{r^2}\left(p_\theta^2 + \frac{1}{\sin^2\theta}\, p_\phi^2\right)\right]$$



- **Solve analytically** the set of non-linear differential equations → elliptical functions and integrals.
- **Numerical code**
  - coordinate transformations from emission to Schwarzschild coordinates (and the inverse problem)
  - constellation of N satellites, known constants of motions
- Effects of **non-gravitational perturbations** (clocks errors, drag, micro-meteorites) on the positioning system



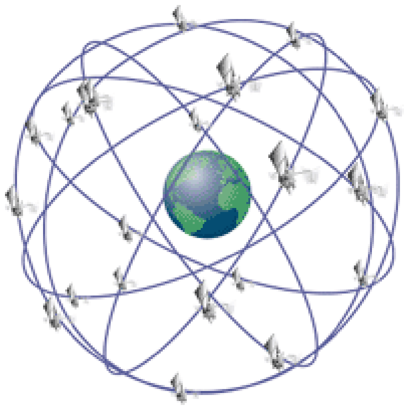
Delva, P., Kostić, U., Čadež, A., *Numerical modeling of a Global Navigation Satellite System in a general relativistic framework*, **Advances in Space Research**, **2011**, Special Issue on Galileo



Receive 4 emission coordinates	uses	Calculation speed (PC computer)	comment
Calculate satellites space-time positions	Relativistic orbit	0.0502 sec	30 digit accuracy
Solve transformation equations from emission to Minkowski coordinates	Transform. eq. from null to Minkowski	0.00058 sec	Up to 5 steps 27-30 digit accuracy
Calculate corrections from Minkowski to local Schwarzschild coordinates	Relativistic time of flight, linear interpolation	0.00130 sec 0.01055 sec	2 steps of iteration 27-30 digit

- 3 different languages to check for numerical errors and newtonian limits

Find global coordinates from emission coordinates



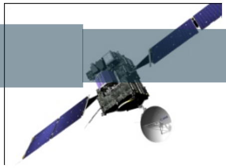
Emission coordinates

Satellites constants of motion

User

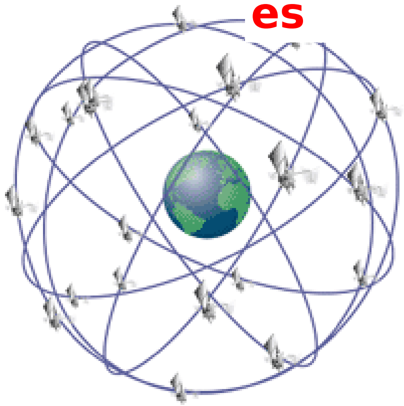
Coordinate transformation

Realize the dynamical ABC reference frame



Inter-Satellite Links (ISL)

Emission coordinates



Clock correction and data reduction

Realization of the ABC reference frame

Satellites constants of motion

Emission coordinates

User

Coordinate transformation to the ABC reference frame

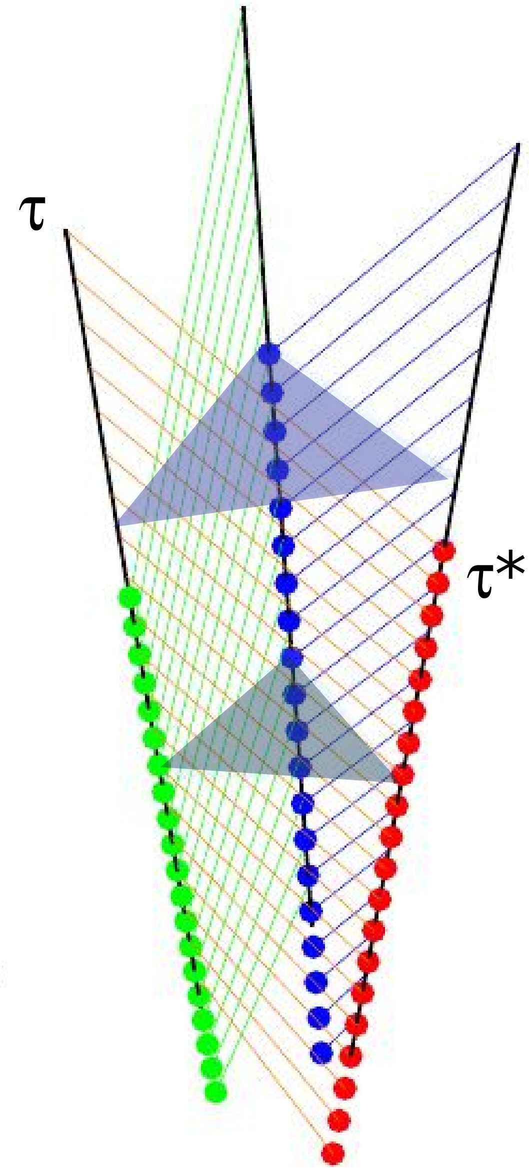
- Set of data (τ*,τ)_{ij}

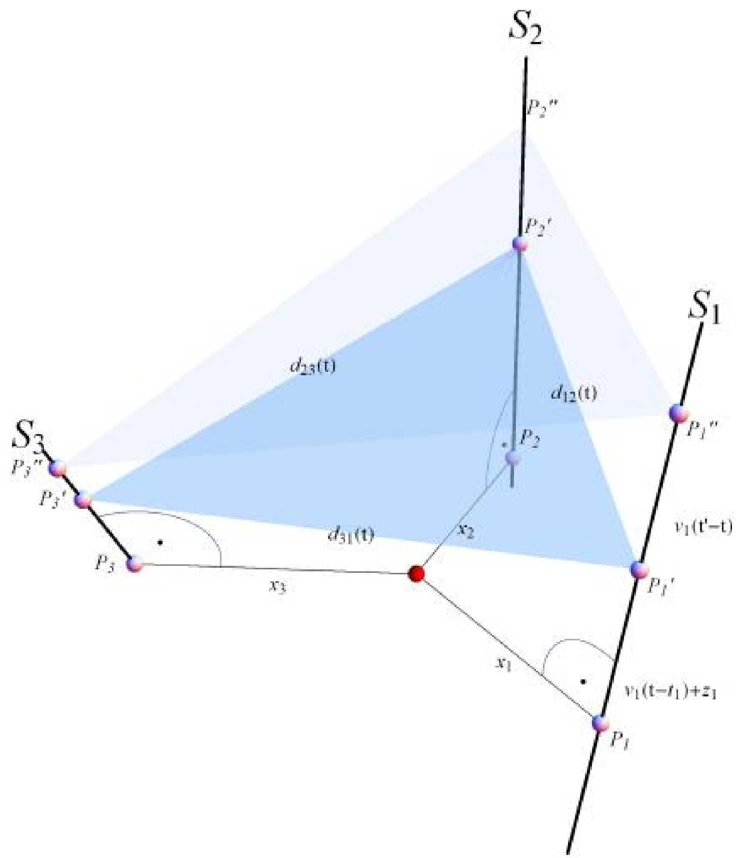
- Reconstruct the dynamics of the satellites by fitting data to the model → **mutual constants of motion**

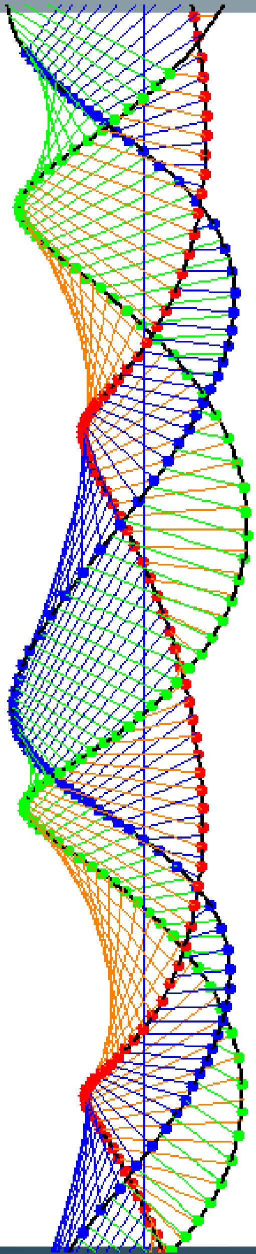
- Define a procedure to build the frame from mutual constants of motion → geometrical structure to attach the **Autonomous Basis of Coordinate**

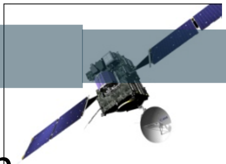
- Curved worldlines → curved spacetime

- Components of **Riemann tensor** can in principle be measured trough accelerations



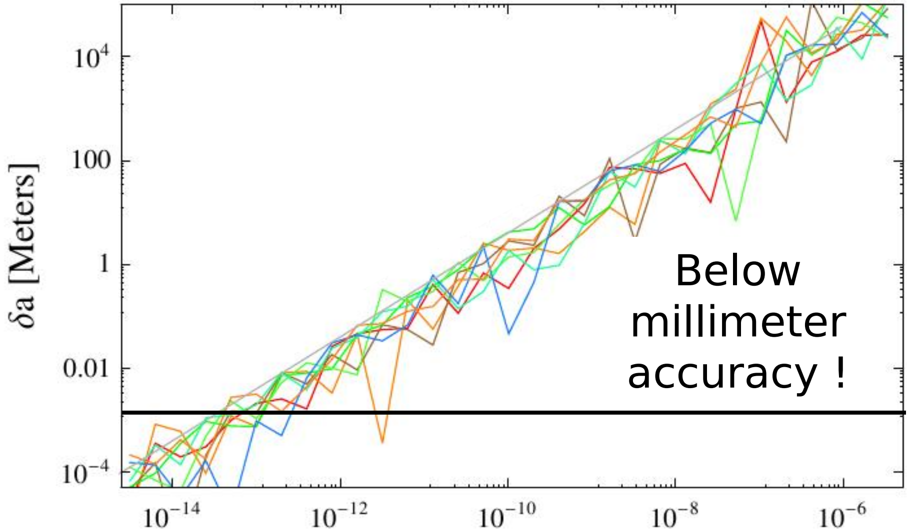






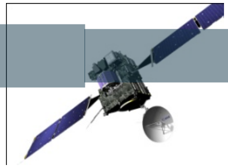
- **Simulation** of data pairs (τ*,τ)_{ij} with additional random noise δT
- **Robustness** of recovering constants of motion with respect to noise in the data
- **Consistency** of description with redundant number of satellites
- Possibility to use the constellation as a clock with **long term stability**
- Possibility to use perturb toward a better **long te**

GALILEO ~1 ns/day



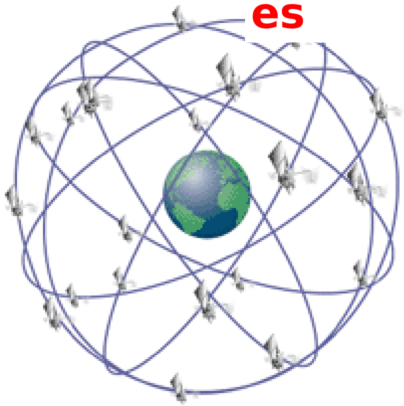
Allan deviation of the clocks @ 1 day

Refine the spacetime geometry description



Inter-Satellite Links (ISL)

Emission coordinates



Clock correction and data reduction

Realization of the ABC reference frame

Satellites constants of motion

Coordinate transformation to the ABC reference frame

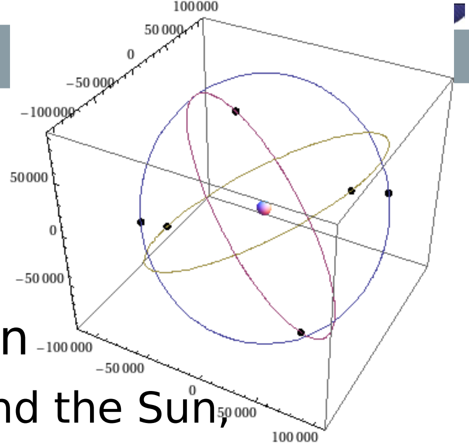
Hamiltonian:

Geometry & non-gravitational forces

Emission coordinates

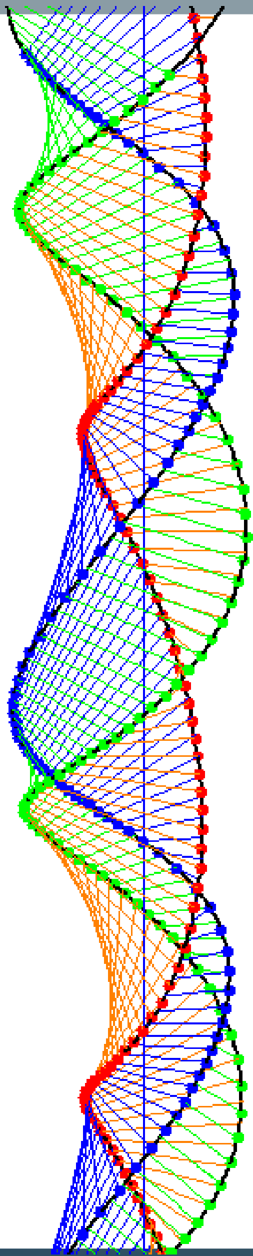
User

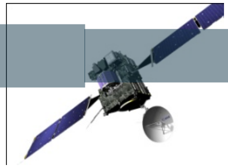
The PECS project



1. Add **gravitational perturbations** to the hamiltonian
    - multipoles of the Earth gravitational field, of the Moon and the Sun, and potentially of other planets, and to the Earth rotation
1. Solve the perturbed geodesic equations
    - Hamiltonian formalism → **perturbation theory** (give time evolution of 0th order constants)
    - Ariadna study: analytic solutions of 0th order
1. Find **accurate constants of motion**
    - Using inter-satellite distances measured over many periods
    - stability and degeneracies
    - Ariadna study: done for 0th order
1. **Refine** the values of gravitational perturbation coefficients
    - Use residual errors between orbit prediction and orbit determination through inter-satellite communication
    - accuracy of position

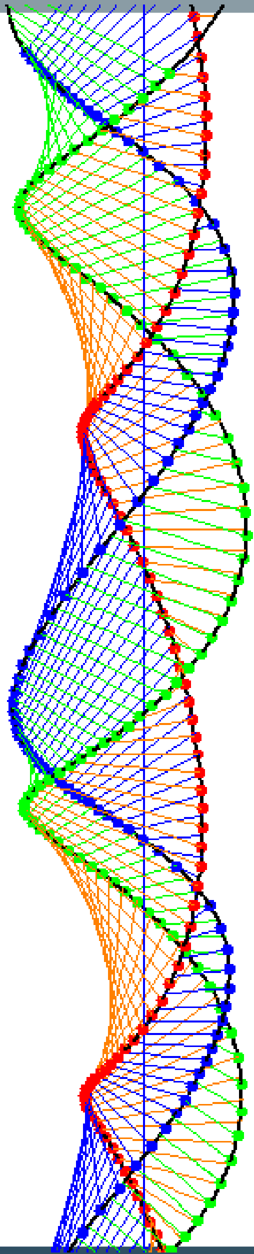
5 - Applications

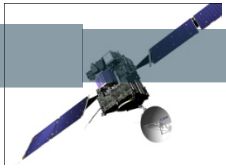




- **Experimental Gravitation**
  - **“Riemannian gravimeter”:** satellites and inter-satellite links create a space-time web that “probe” its geometry
  - Test of the **equivalence principle** by modelling or measuring non-gravitational perturbations accurately
- **Reference frames**
  - Comparison between the ABC reference frame and the International Celestial Reference Frame → how the local geometry is integrated into the global arena of space-time
- **Geophysics**
  - Absolute positions of markers on the ground with **sub-millimeter accuracy**
  - Interior structure of the Earth, continental drift, earthquake prediction…
  - Gravitational potential difference, ocean currents

Conclusion





- Turn non-dedicated satellites to a **powerful scientific instrument** for experimental gravitation (“**Riemannian gravimeter**”), reference systems, geophysics, …

- Implementation of **inter-satellites links** on second generation GALILEO satellites under study (GNSSPLUS, GNSS evolution, ADVISE) → non-scientific motives

- Data sets and dynamics treated in a coherent frame, independent of Earth internal dynamics → **stability** and **accuracy**

- Operation of the experiment for many decades, with continuous data flow, constantly refining the Hamiltonian

- Concept applicable to other configurations and objects: Pulsars (tartaglia et al.), very precise clocks (optical clocks), …

Cadez, A., Kostic, U., Delva, P., and Carloni, S., 2011, Mapping the Spacetime Metric with a Global Navigation Satellite System - Extension of study: Recovering of orbital constants using inter-satellites links, Advanced Concepts Team, European Space Agency

Delva, P., Cadez, A., Kostic, U., and Carloni, S., A relativistic and autonomous navigation system, Proceedings of the Rencontres de Moriond and GPHYS colloquium, March 13th-20th, 2011, **http://arxiv.org/abs/1106.3168**