l'Observatoire de Paris | SYRTE

Systèmes de Référence Temps-Espace

Testing gravitation in the Solar System with radio-science experiments

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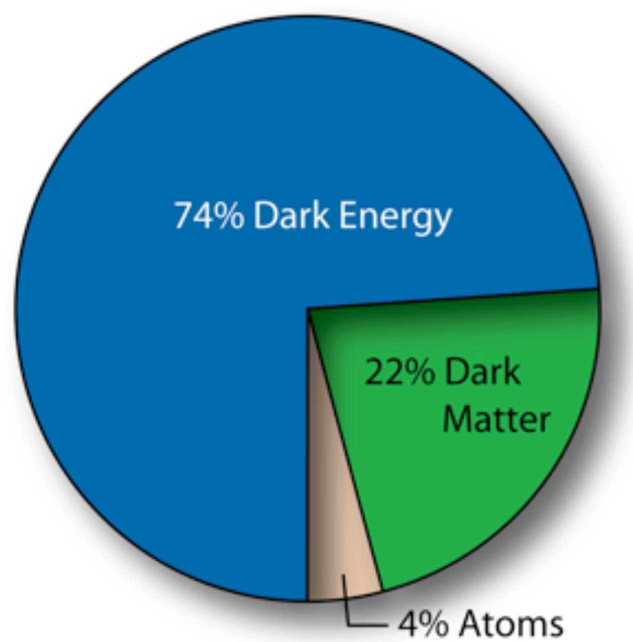
Motivations

- General Relativity (GR) is very well tested in the solar system (light deflection, radio science experiments, ephemerides).
- But... still a lot of interest to perform test of General Relativity:
  - theoretical problem: GR is not the ultimate theory of gravity: quantum theory of gravity, unification with other interactions
  - cosmological problem: no direct detection of Dark Matter and Dark Energy → alternative theory of gravity to explain cosmological observations



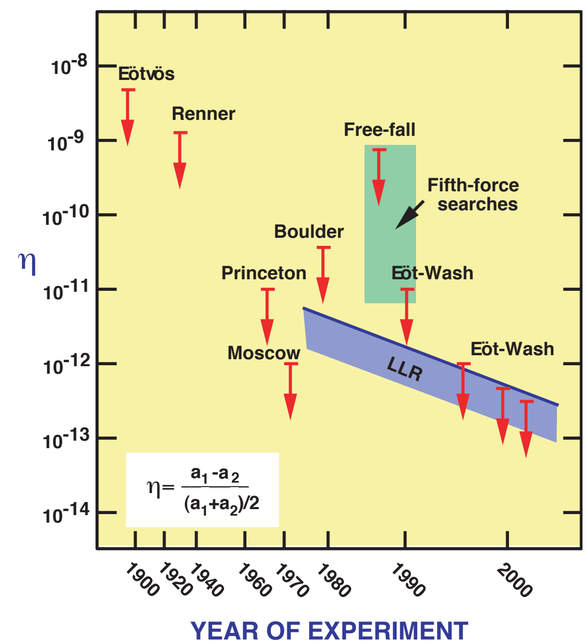
- search for small deviations of GR (smaller than present constraint) or exploration of new situations

Basic principles of GR

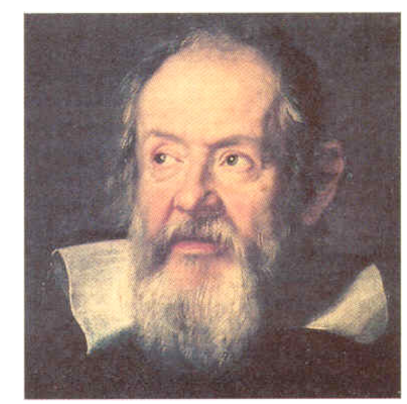
1) Equivalence Principle:

- very well tested (up to 10^{-13} with Eötwash experiments and with Lunar Laser Ranging)[1]

- more accurate measurement needed: alternative (string) theories predict violation smaller[2] → MICROSCOPE accuracy 10^{-15}



C. Will, LRR, 9, 2006



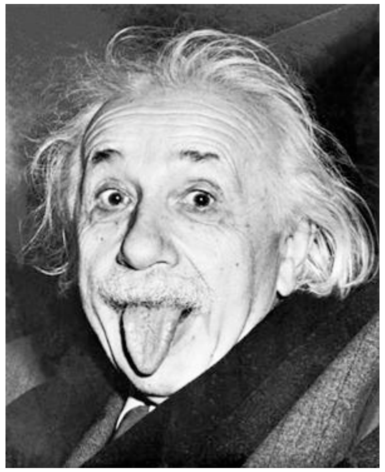
- Gravitation ⇔ space-time curvature (described by a metric $g_{\mu\nu}$)

- free-falling masses follow geodesics of this metric and ideal clocks measure proper time $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$

[1] C. Will, LRR, 9, 2006

[2] T. Damour, A.M. Polyakov, Nucl. Phys B, 423/532,1994

Basic principles of GR



II) Field equations (determination of the metric):

- Einstein Equations: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

  space-time curvature (metric) ⇔ matter-energy content

- In the solar system:
  - sun modeled as a spherical source
  - solution in isotropic coordinates

$$ds^2 = (1 + 2\phi_N + 2\phi_N^2 + \dots)dt^2 - (1 - 2\phi_N + \dots)d\vec{x}^2$$

  with ϕ_N the Newtonian potential

- important effects for space-mission:
  - dynamics ≠ from Newton (ex.: advance of the perihelion)
  - proper time (measured by ideal clocks) ≠ coordinate time
  - coordinate time delay for light propagation (Range/Doppler)
  - light deflection (VLBI)

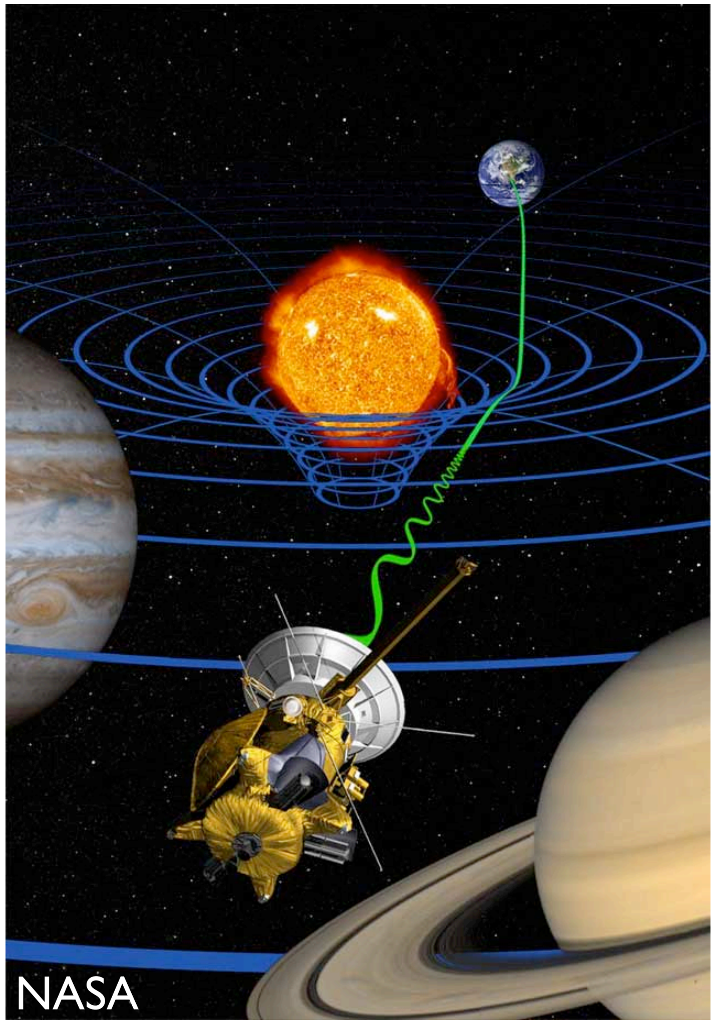
PPN tests of GR

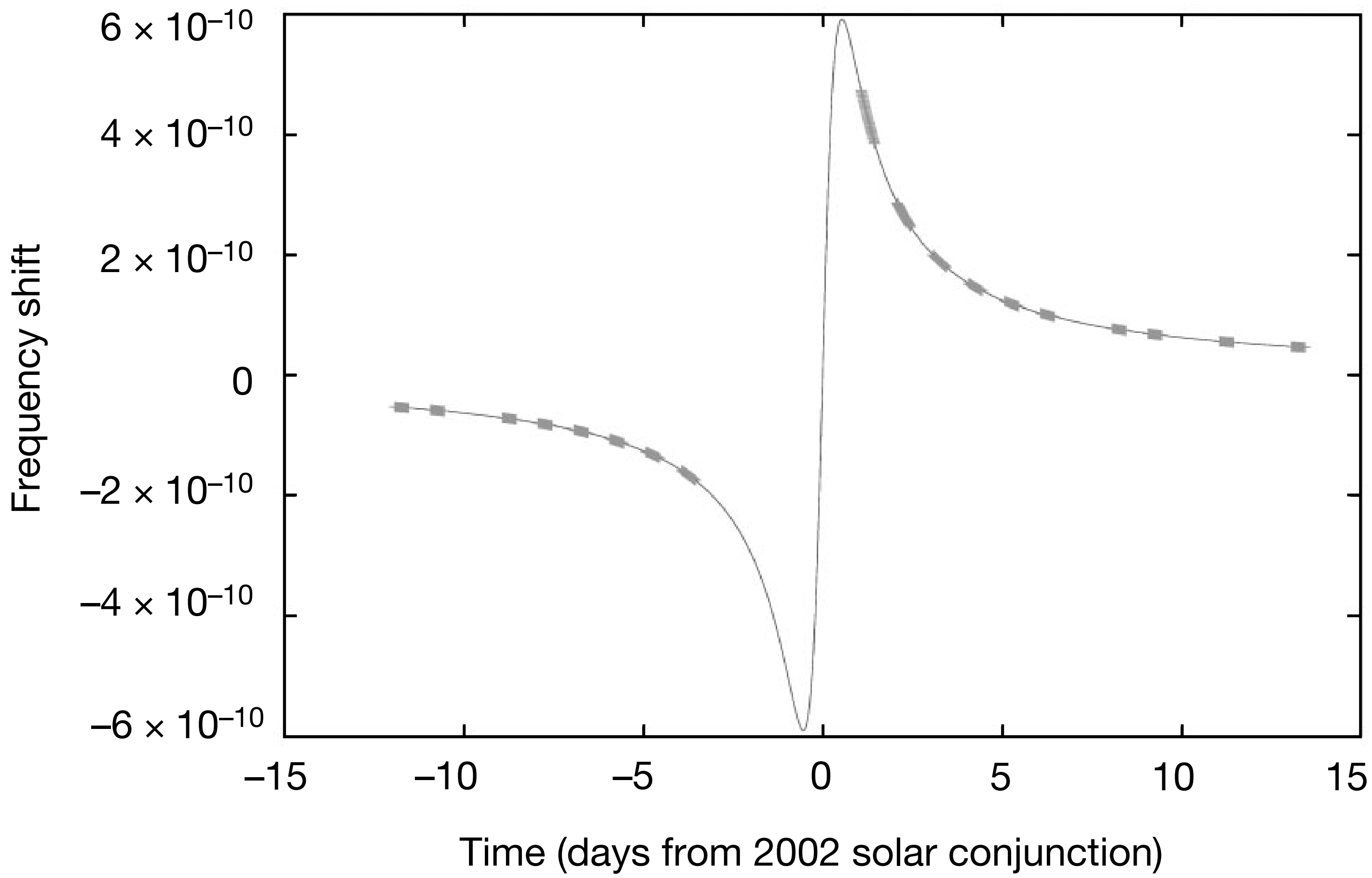
- Post-Newtonian Parametrization of the metric (famous γ and β).

$$ds^2 = (1 + 2\phi_N + 2\beta\phi_N^2 + \dots)dt^2 - (1 - 2\gamma\phi_N + \dots)d\vec{x}^2$$

- 30 years of precise experiments have constrained PN parameters very closely around GR

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$ Doppler during a solar conjunction[1]





[1] B. Bertotti, L. Iess, P. Tortora, Nature, 425/374, 2003

[2] A. Fienga, Moriond Conference, 2011

PPN tests of GR

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- 30 years of precise experiments have constrained PN parameters very closely around GR

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$ Doppler during a solar conjunction[1]

$$\beta - 1 = (-4.1 \pm 7.8) \times 10^{-5}$$ INPOP10a ephemerides [2]

- Confirmed by many other experiments: VLBI (light deflection), Lunar Laser Ranging, Mars orbiters, ...

... and expected to be improved in the future (GAIA, BepiColombo around Mercury, ...)

[1] B. Bertotti, L. Iess, P. Tortora, Nature, 425/374, 2003

[2] A. Fienga, Moriond Conference, 2011

Is it necessary to go beyond ?

- More accurate constraints needed: theoretical models predict smaller PPN deviations: theory attracted towards GR[1] or Chameleon[2]
- Extend PPN framework: not all theories can enter in the PPN framework !! 2 examples: Post-Einsteinian Gravity and MOND External Field Effect
  - PEG theory[3]: phenomenology based on a quantum theory of gravity (1-loop correction) by considering a non-local Einstein field equation

$$
\begin{aligned}
g_{00} &= [g_{00}]_{GR} + 2\delta\Phi_N(r) \\
g_{rr} &= [g_{rr}]_{GR} + 2\delta\Phi_N(r) - 2\delta\Phi_p(r)
\end{aligned}
$$

  - MOND External Field Effect (EFE)[4]: modification of the Newtonian potential due to the external field in which the solar system is embedded

$$
\phi = \frac{GM}{r} + \frac{Q_2}{2} x^i x^j \left(e_i e_j - \frac{1}{3}\delta_{ij} \right)
$$

- What are the effects of these theories on Range/Doppler signals ? Can they be observed ? Simulations performed directly from metric!

[1] T. Damour, K. Nordvedt, Phys. Rev. D, 48/3436, 1993
[2] J. Khoury, A. Weltman, Phys. Rev. D, 69/044026, 2004
[3] M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 22/2135, 2005
[4] L. Blanchet, J. Novak, MNRAS, 2011

Idea of the work

- What is the impact of the gravitation theory on the radio-science measurement (for different space missions) ?
- New tool that performs Range/Doppler simulations from a specific space-time metric (GR, PPN or other alternative theories of gravity) and fits of the orbital initial conditions in GR
- Possible to have quick idea of the order of magnitude/signature of the gravitation theory on Range/Doppler signals
  - order of magnitude of relativistic corrections
  - order of magnitude of expected deviations induced by an hypothetical alternative theory
  - correlations of these deviations with the initial conditions
- In this presentation: Cassini (between Jupiter and Saturn) in Post-Einsteinian Gravity[1] (PEG) or with "MOND" External Field Effect [2]

[1] M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 22/2135, 2005

[2] L. Blanchet, J. Novak, MNRAS, 2011

Covariant Range/Doppler

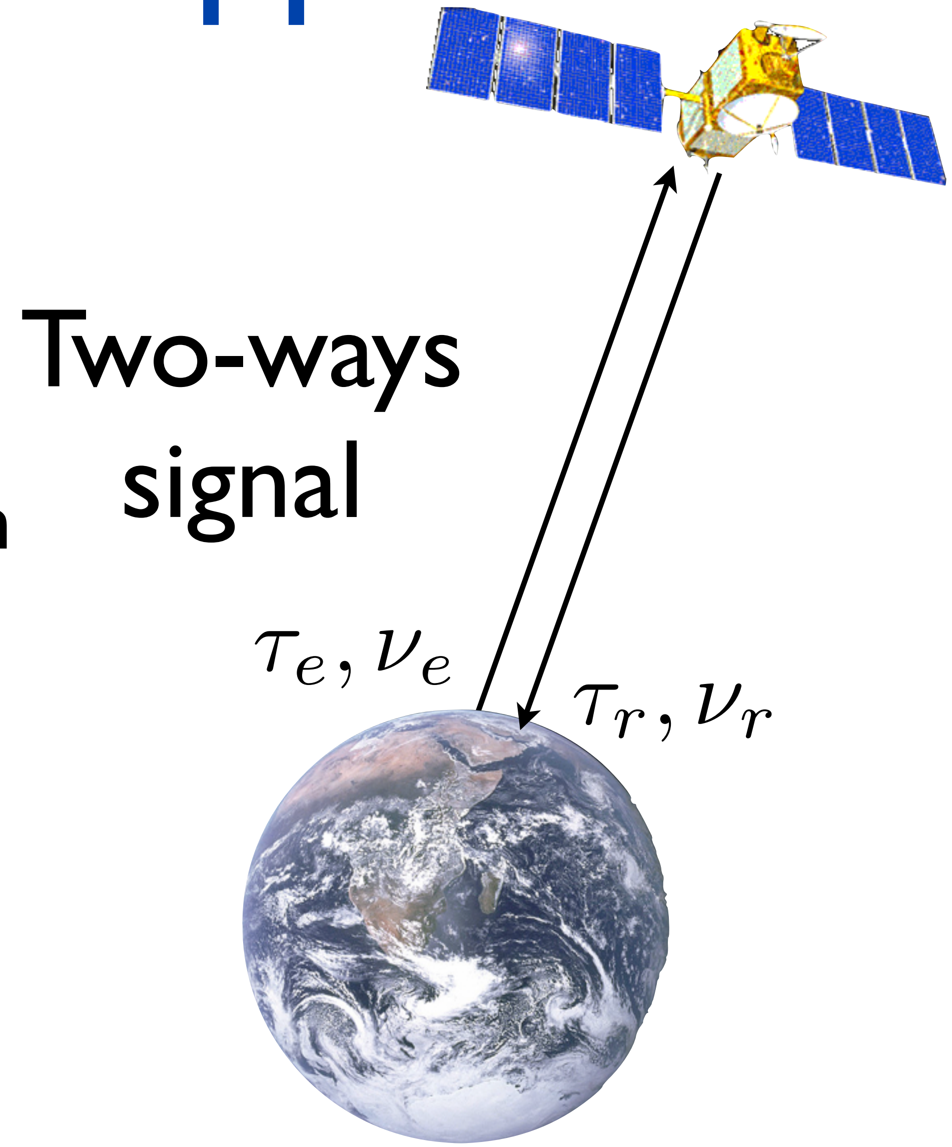
- Range is related to propagation time: difference between receptor proper time and emitter proper time

$$R(\tau_r) = \tau_r - \tau_e$$

- Doppler is proper frequency shift between emission and reception

$$D(\tau_e) = \frac{\nu_r}{\nu_e}$$

- These definitions are covariant: do not depend on the coordinates system

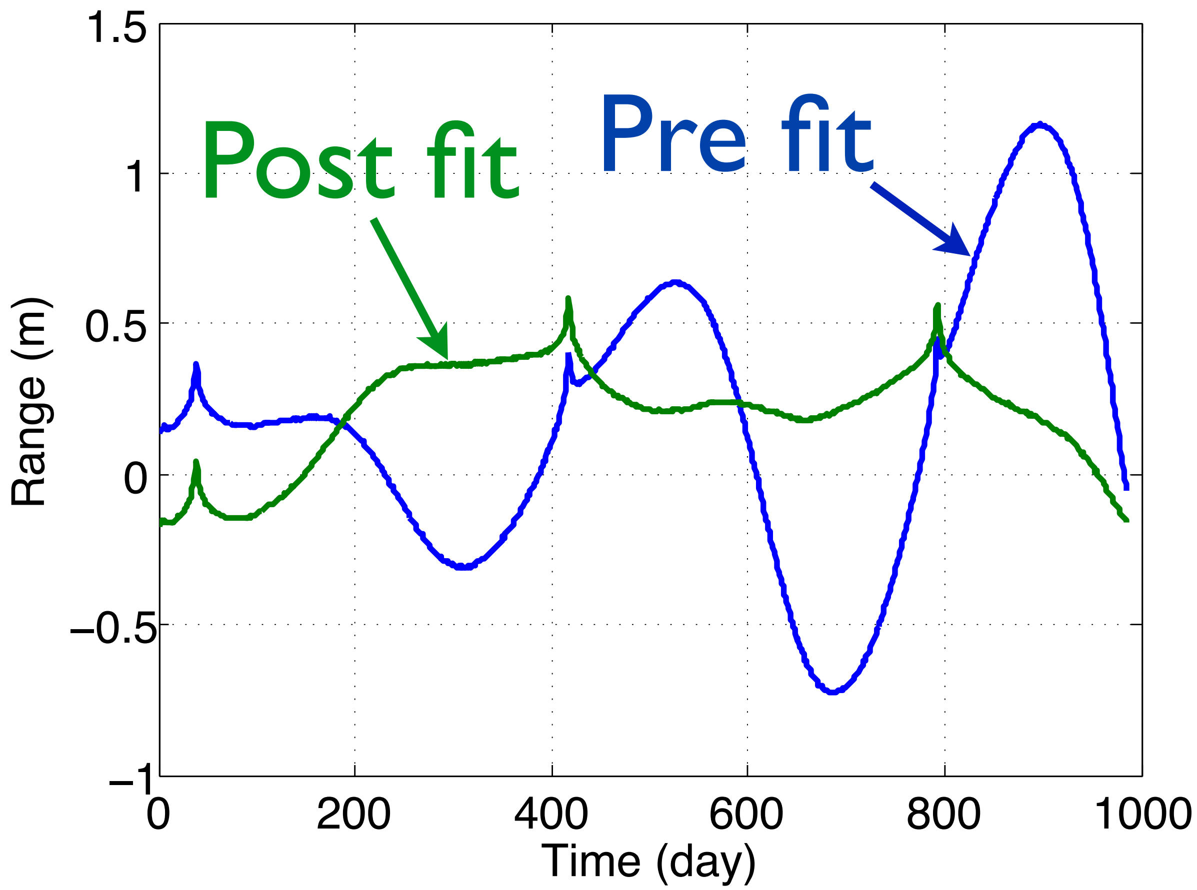


- Simulations: orbit of spacecraft/planets, clock behavior, light propagation directly from space-time metric
- Comparison with GR: fit of the initial conditions needed
  - to avoid effects due to the choice of coordinates
  - this fit is always performed in practice

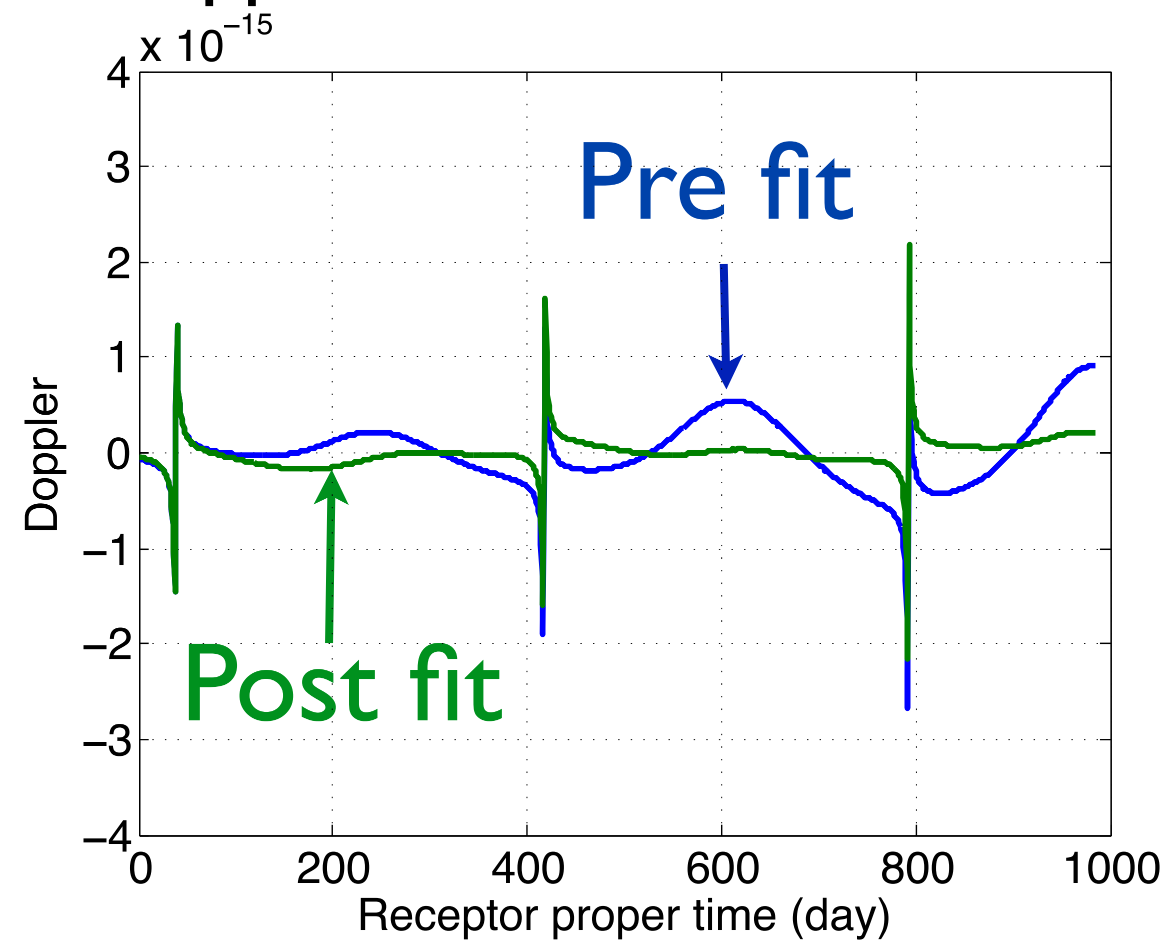
Example 1: PPN effects on Cassini

- Cassini Range/Doppler simulations with γ-1=10^{-5}.
- Modification of the metric $g_{rr} = [g_{rr}]_{GR} - 2(\gamma - 1)\frac{GM}{rc^2}$
- Fit of the initial conditions of Cassini in GR

Range Difference PEG - GR



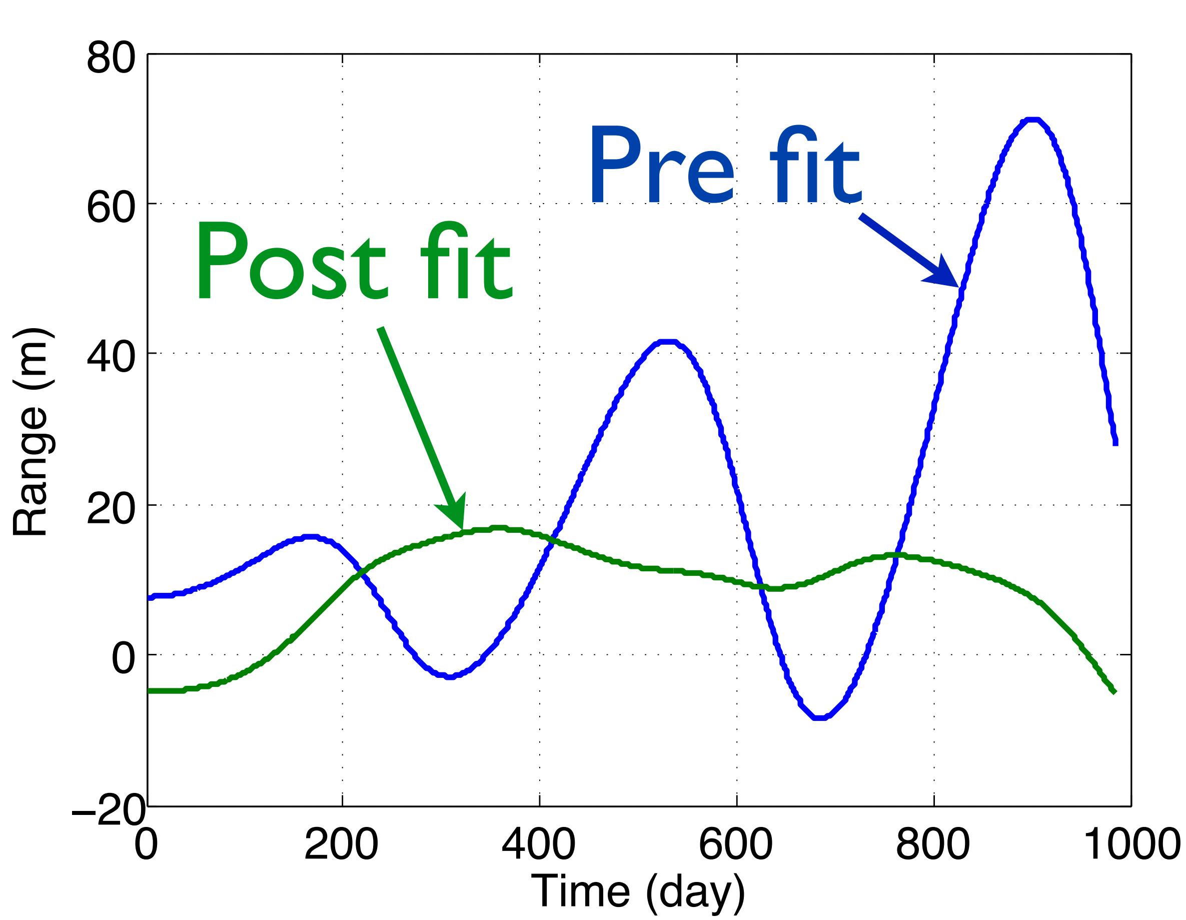
Doppler Difference PEG - GR



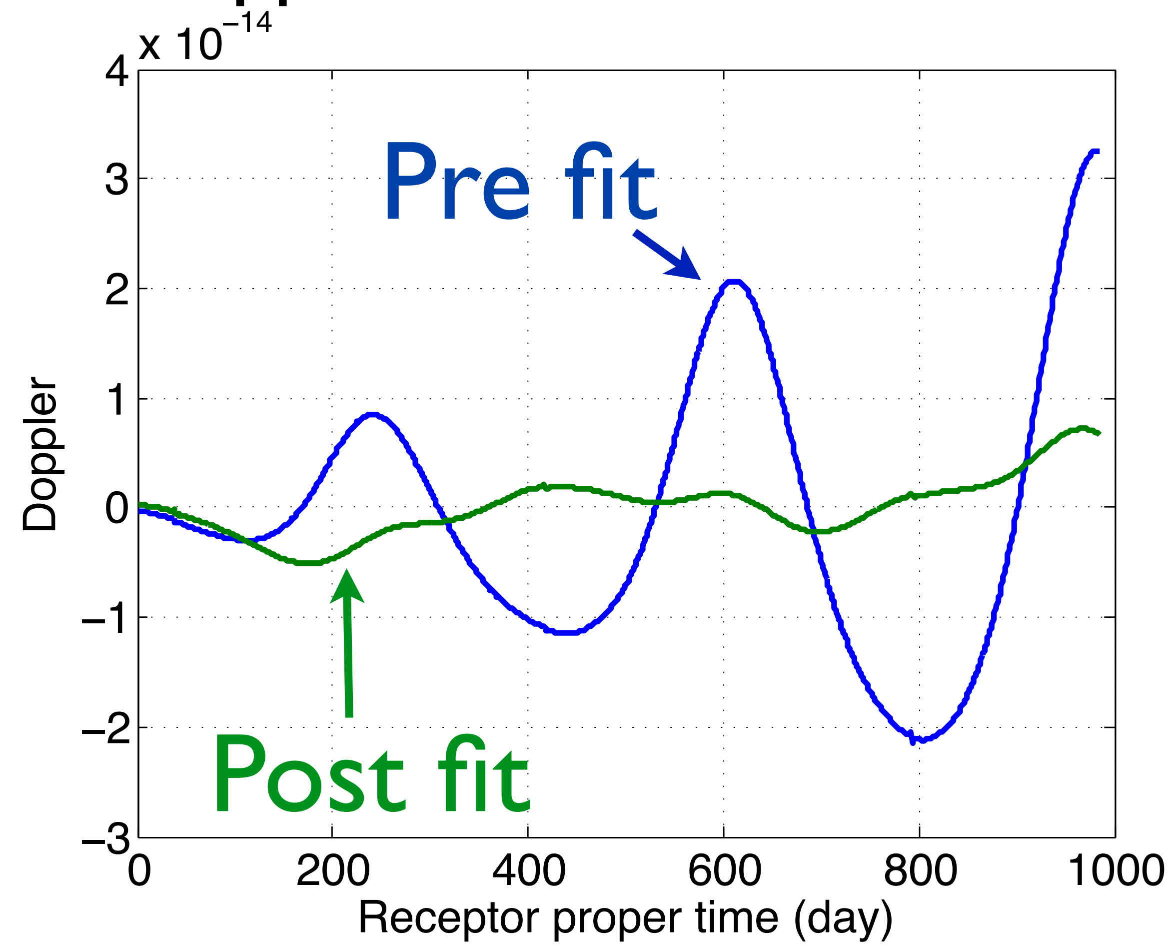
Example 2: PEG effects on Cassini

- Alternative theory: PEG in the second sector[1]: $g_{rr} = [g_{rr}]_{GR} - 2\delta\Phi_p(r)$ with $\delta\Phi_p(r) = \sum_i \chi_i r^i$
- Cassini Range/Doppler simulations with $\chi_1 = 10^{-23}\ \mathrm{m}^{-1}$.
- Fit of Cassini initial conditions in GR

Range Difference PEG - GR



Doppler Difference PEG - GR

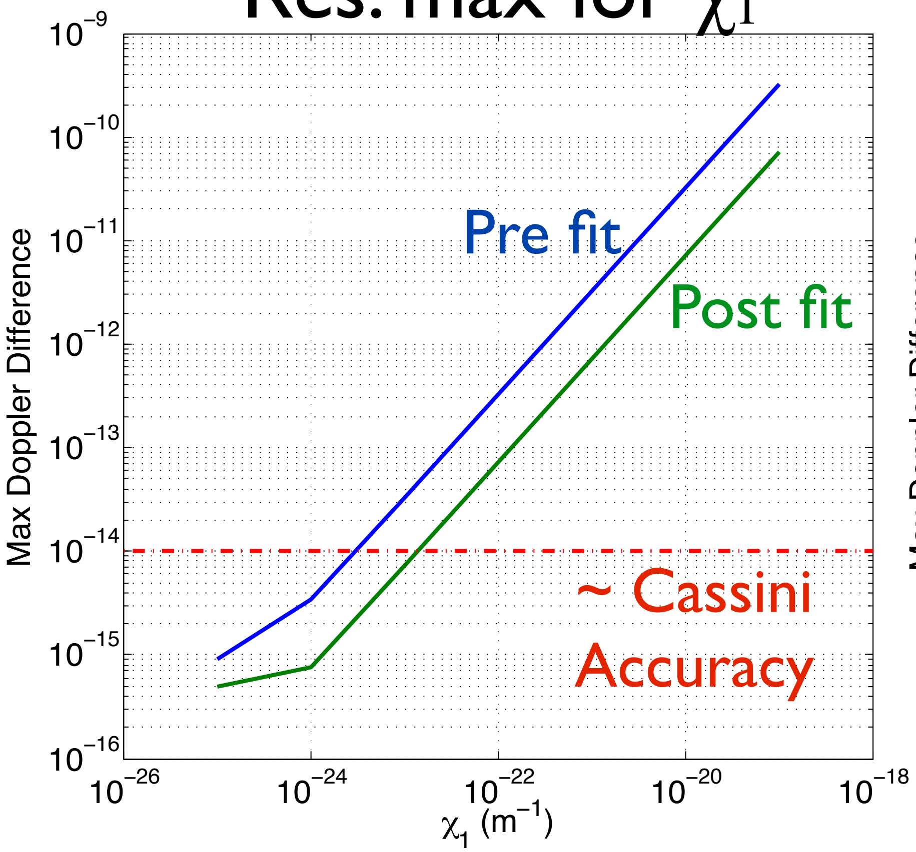


[1] M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 22/2135, 2005
M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 23/777, 2006

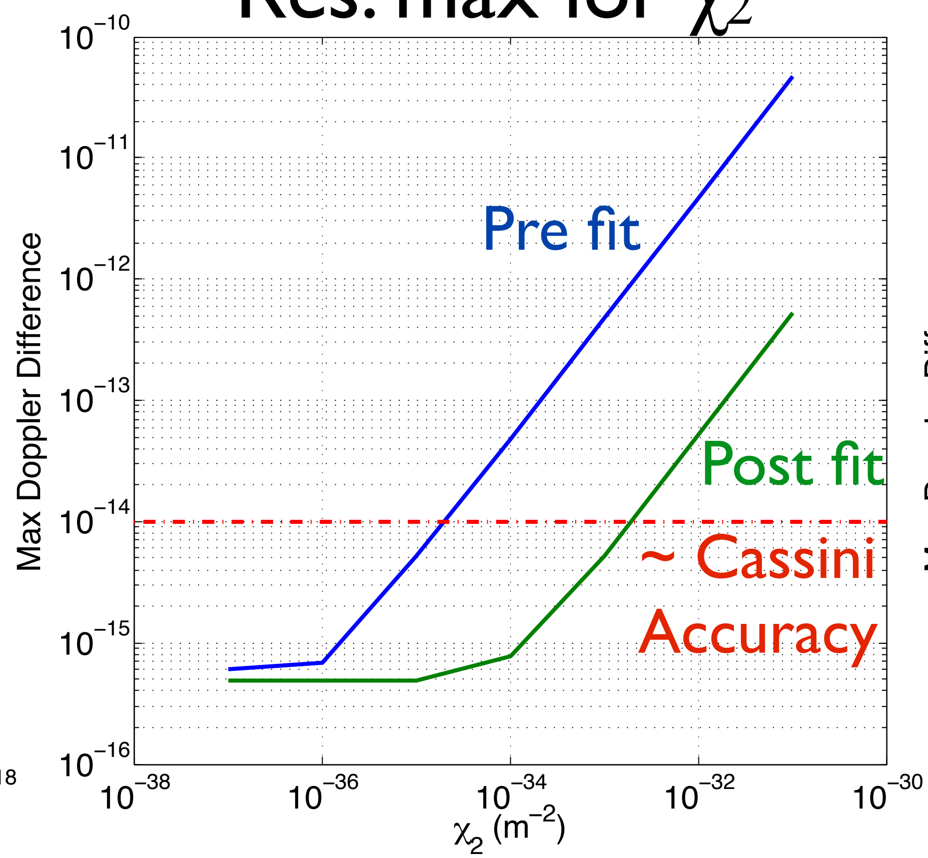
Example: 3 PEG parameters

- Cassini Doppler simulations with χ_1, χ_2, $\delta\gamma = \gamma$-1
- Modification of the metric $g_{rr} = [g_{rr}]_{GR} - 2\chi_1 r - 2\chi_2 r^2 - 2\delta\gamma \frac{GM}{c^2 r}$
- Maximum of the residuals for different values of PEG parameters
- Comparison with Cassini precision ($\sim 10^{-14}$ on Doppler) gives constraints on parameters: $\chi_1 \sim 10^{-23} m^{-1}$, $\chi_2 \sim 2\ 10^{-33} m^{-2}$, γ-1~3 10^{-5} (similar to Bertotti et al[1]).

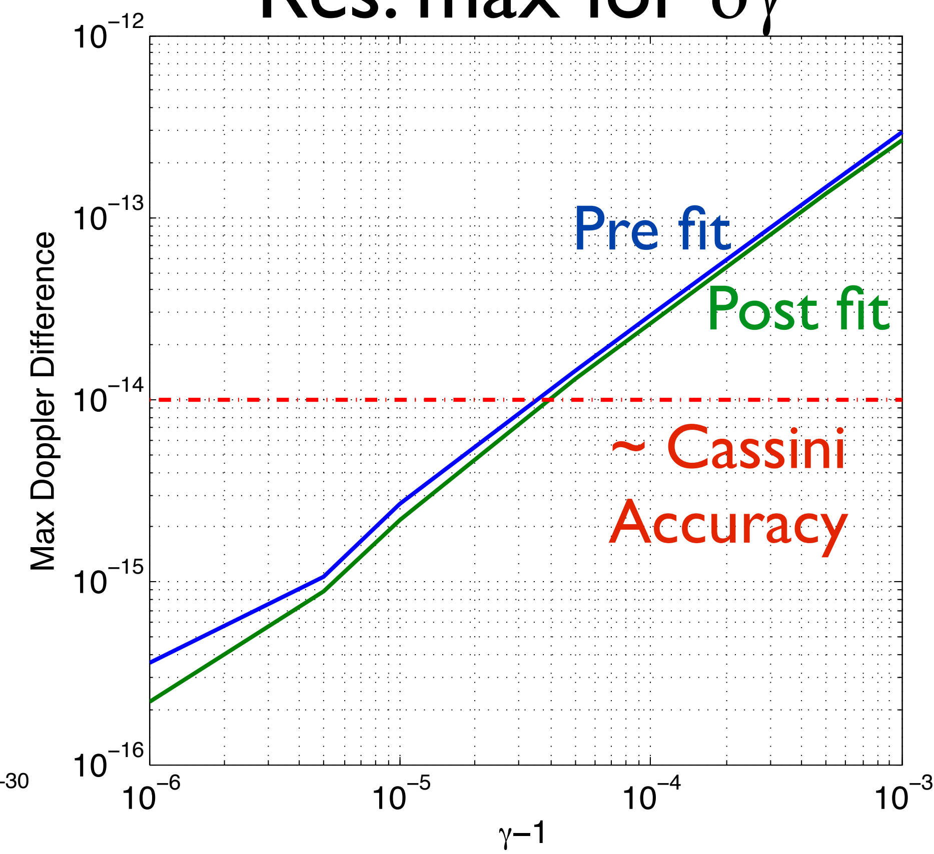
Res. max for χ_1



Res. max for χ_2



Res. max for $\delta\gamma$



[1] B. Bertotti, L. Iess, P. Tortora, Nature, 425/374, 2003

Example: MOND field

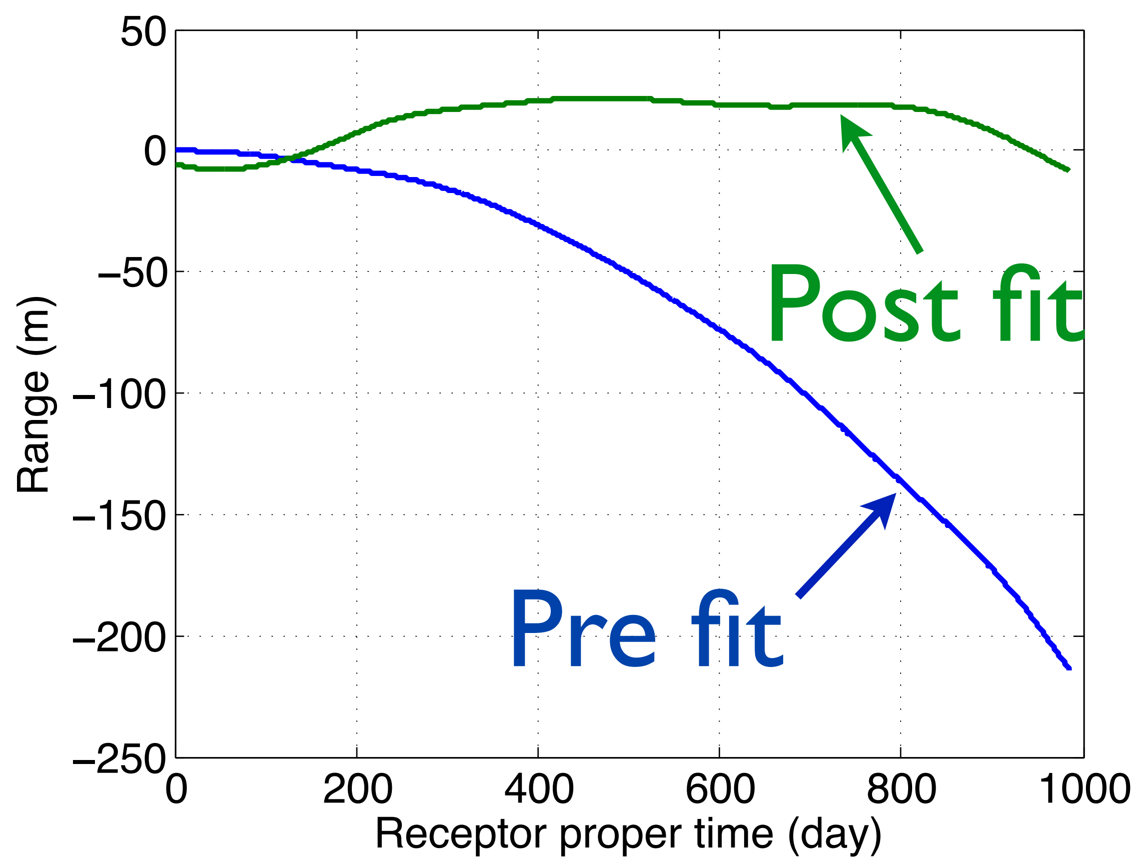
- The dominant effect (External Field Effect) of MOND around Sun is a quadrupole[1]

$$U = \frac{GM}{r} + \frac{Q_2}{2} x^i x^j \left(e_i e_j - \frac{1}{3}\delta_{ij} \right)$$

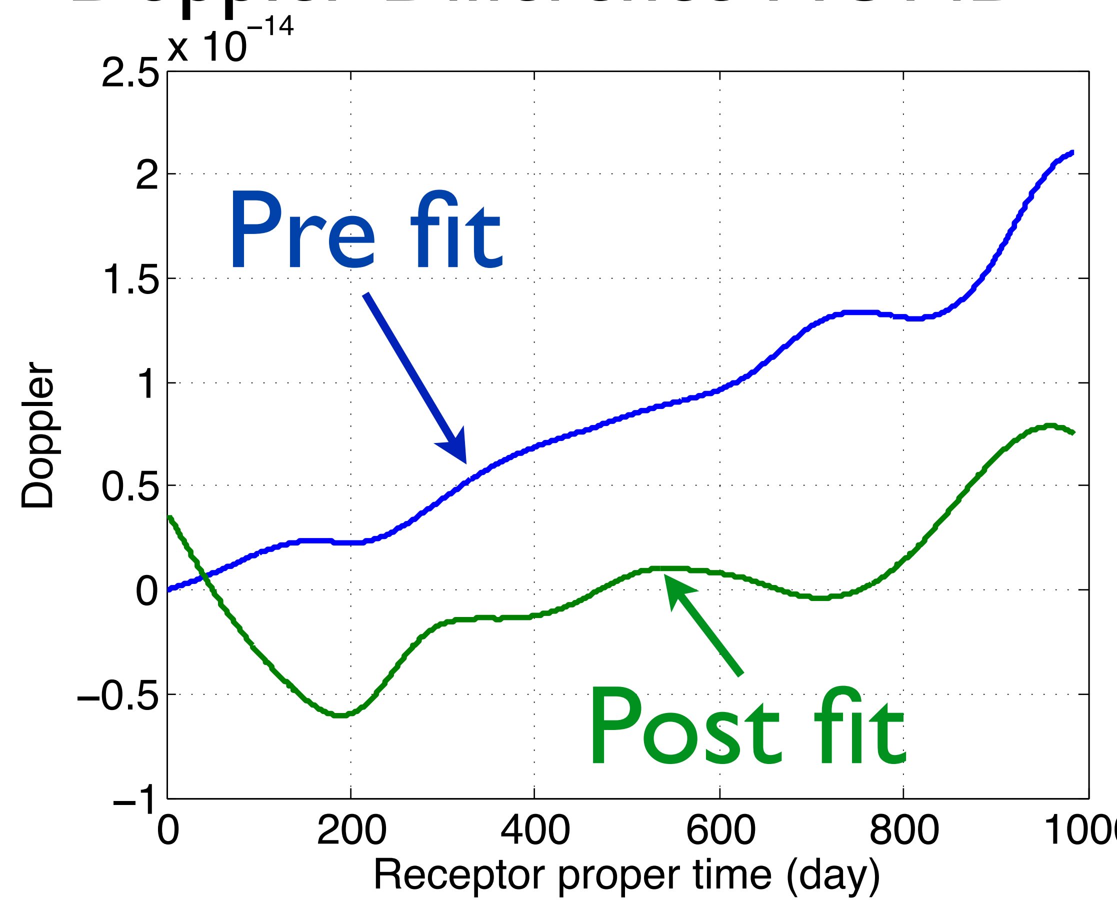
with $2.1\ 10^{-27}\ s^{-2} \leq Q_2 \leq 4.1\ 10^{-26}\ s^{-2}$ for different MOND function

- Range/Doppler simulations and fit with upper bound on Q_2
- Signals and residuals below Cassini accuracy: Cassini not useful to test MOND theory

Range Difference MOND - GR



Doppler Difference MOND - GR



[1] L. Blanchet, J. Novak, MNRAS, 2011

Conclusion

- Testing GR in the solar system is very challenging but very important:
  - search for small deviations (smaller than present PPN accuracy)
  - search for deviations in an extended framework

- Software that simulates Range/Doppler observables directly from the space-time metric

- We can answer the question: Can a particular alternative theory of gravity be seen in Range/Doppler measurements of a specific mission? What is the order of magnitude/signature of the signal ?

- As an example: PPN/PEG simulations were presented on Cassini spacecraft → constraint on PEG parameters derived

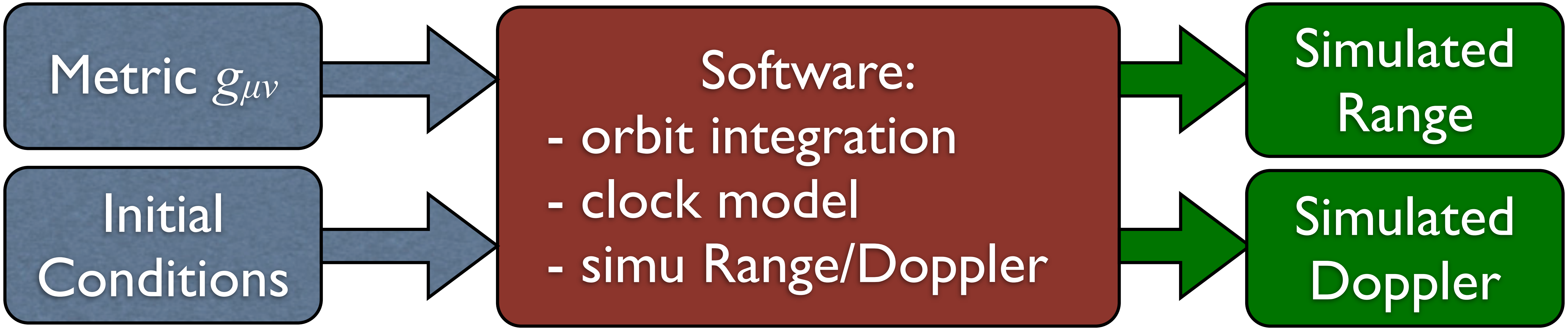
- Other Ex.: MOND theory signature on Cassini just too small to be detected with this arc.

Numerical simulations were made on the local computing resources (cluster URBM-SysDyn) at the University of Namur (FUNDP)

BACKUP SLIDES

Strategy

1. Simulation of covariant Doppler/Range (alternative theory)



Strategy

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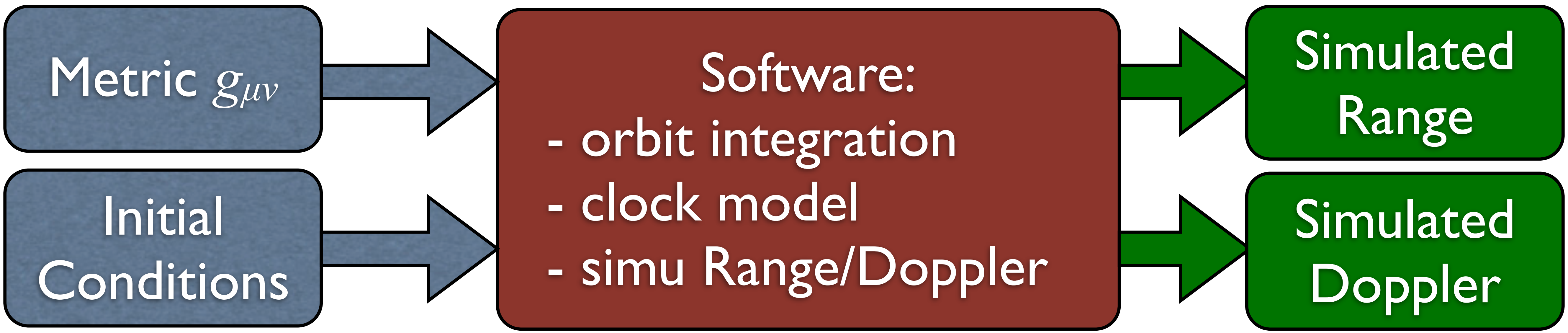
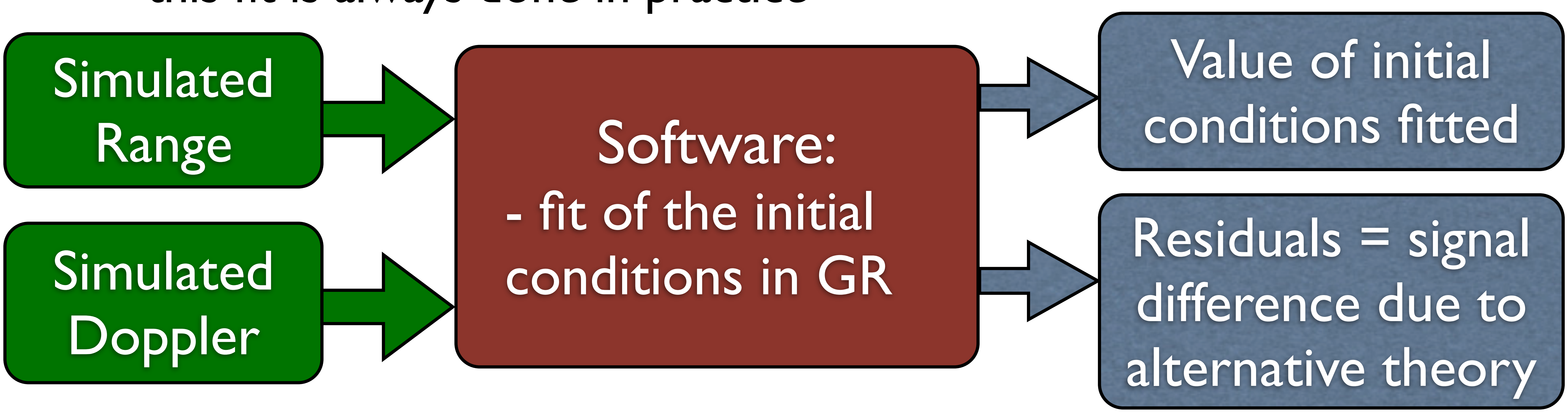
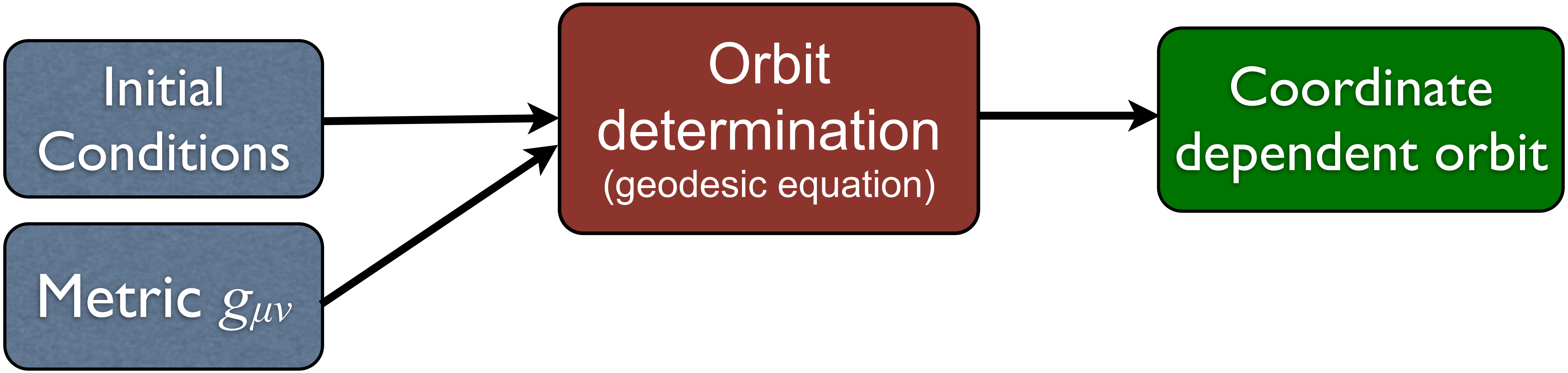


2. Comparison with GR: fit of the initial conditions needed
   - to avoid effects due to the choice of coordinates
   - this fit is always done in practice



Range and Doppler from the metric



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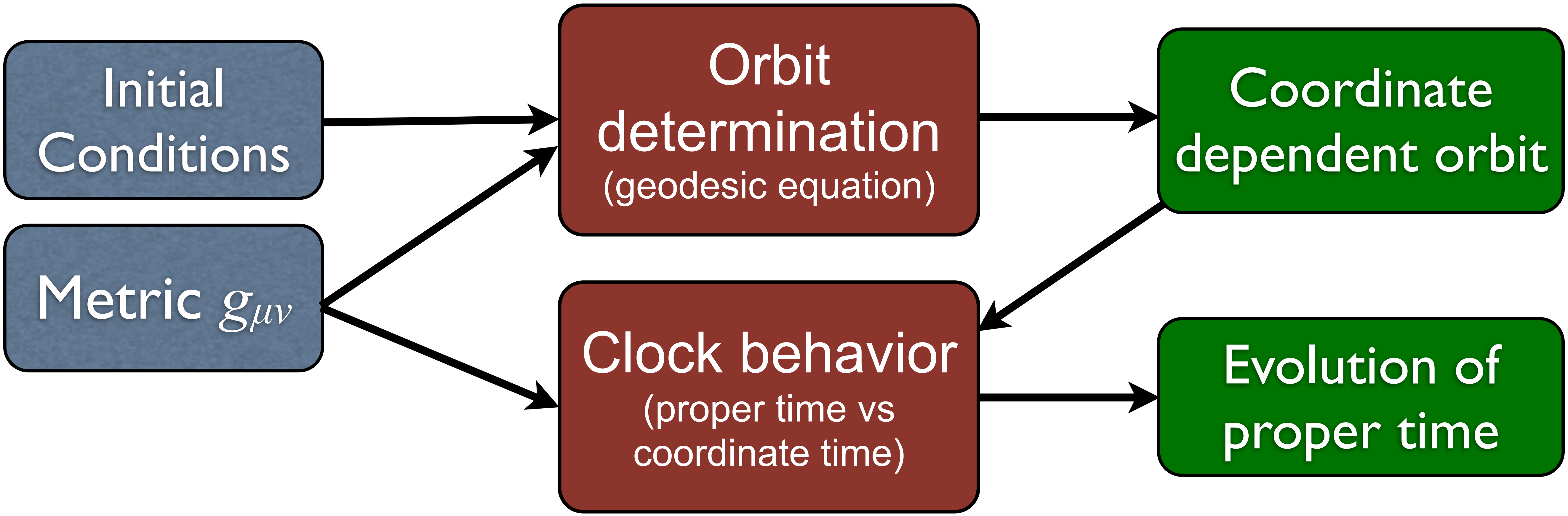

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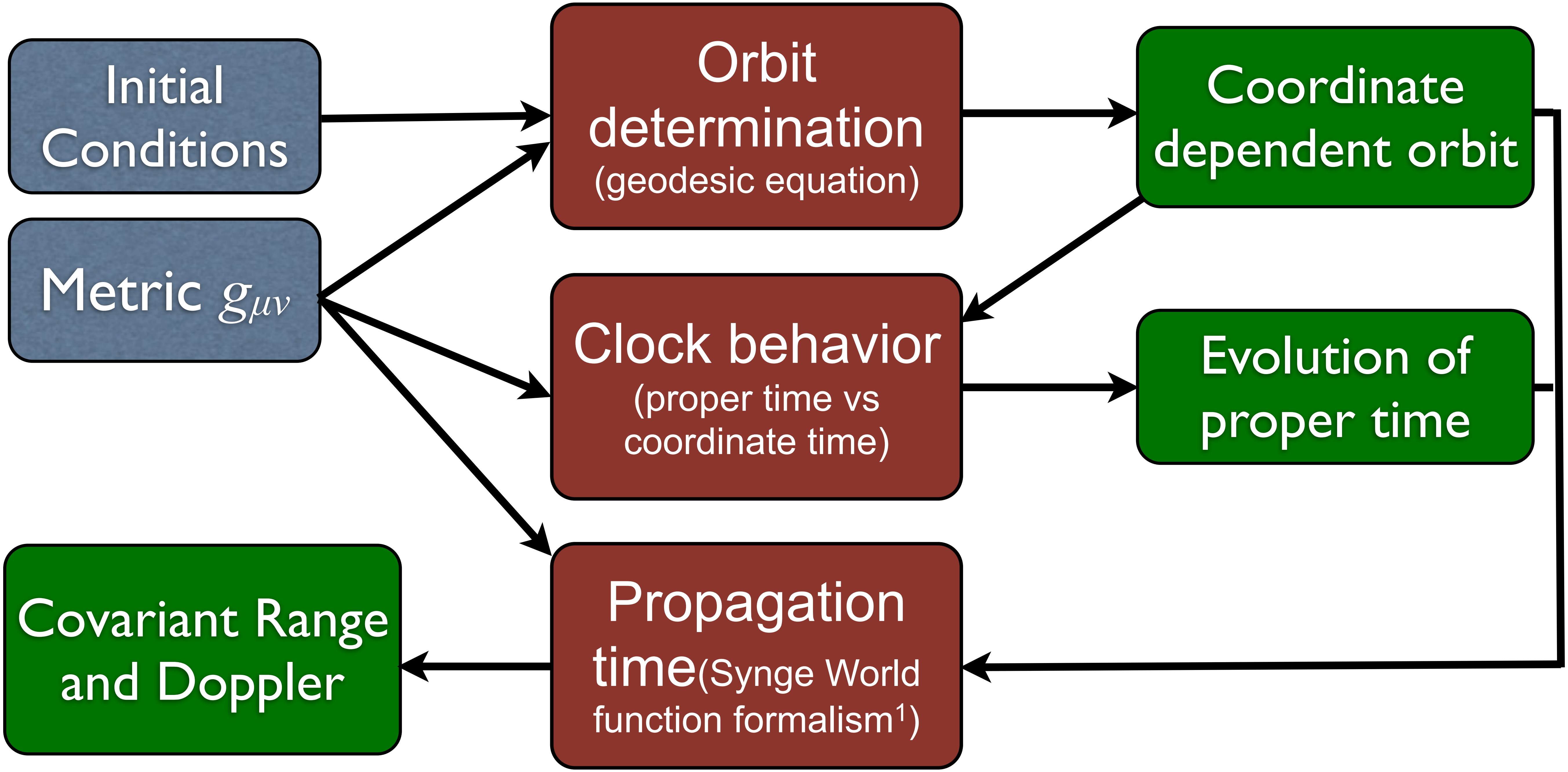
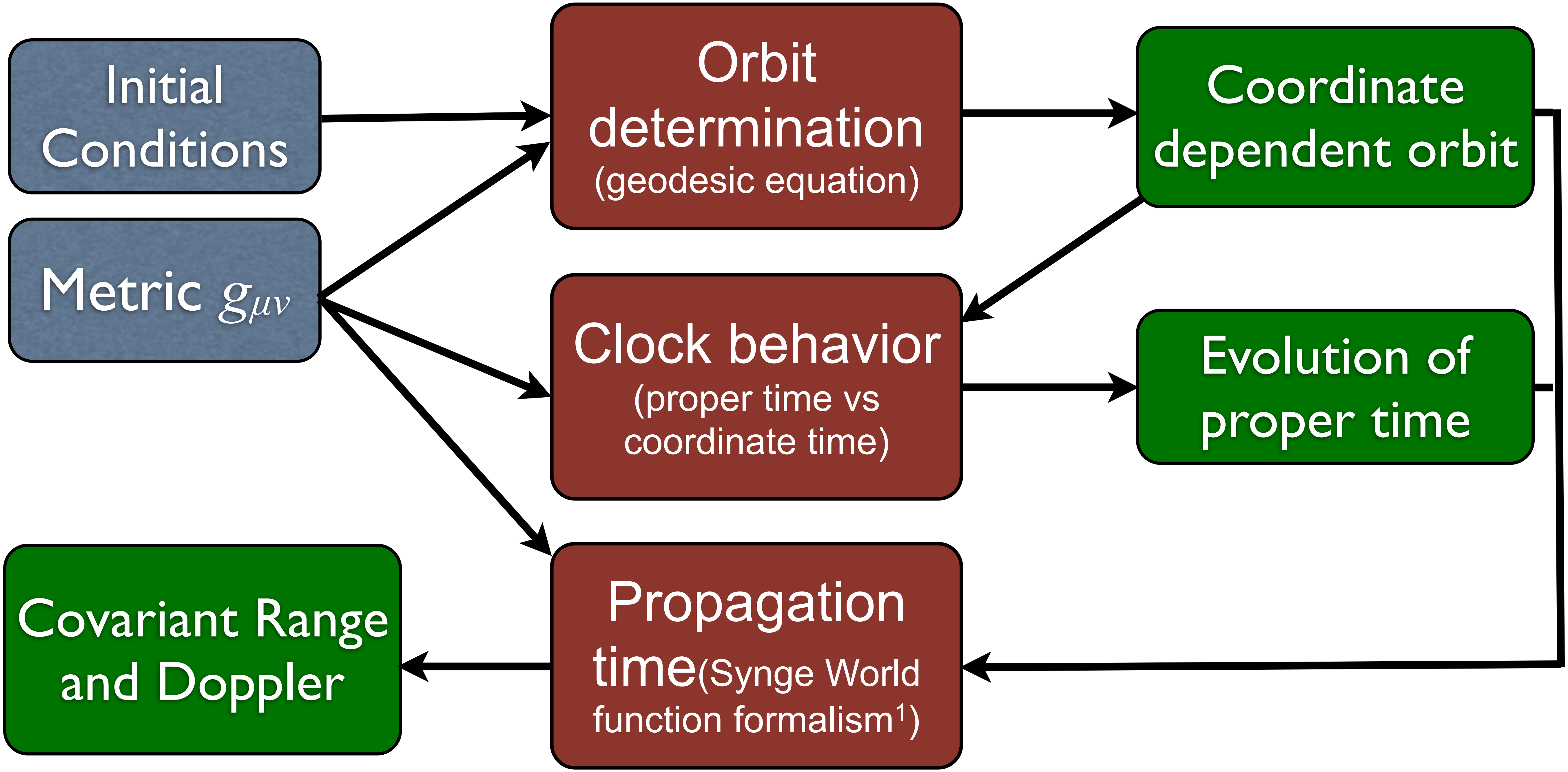


[1] P. Teyssandier, C. Le-Poncin-Lafitte, Class. and Quantum Grav. 25/145020, 2008

Range and Doppler from the metric



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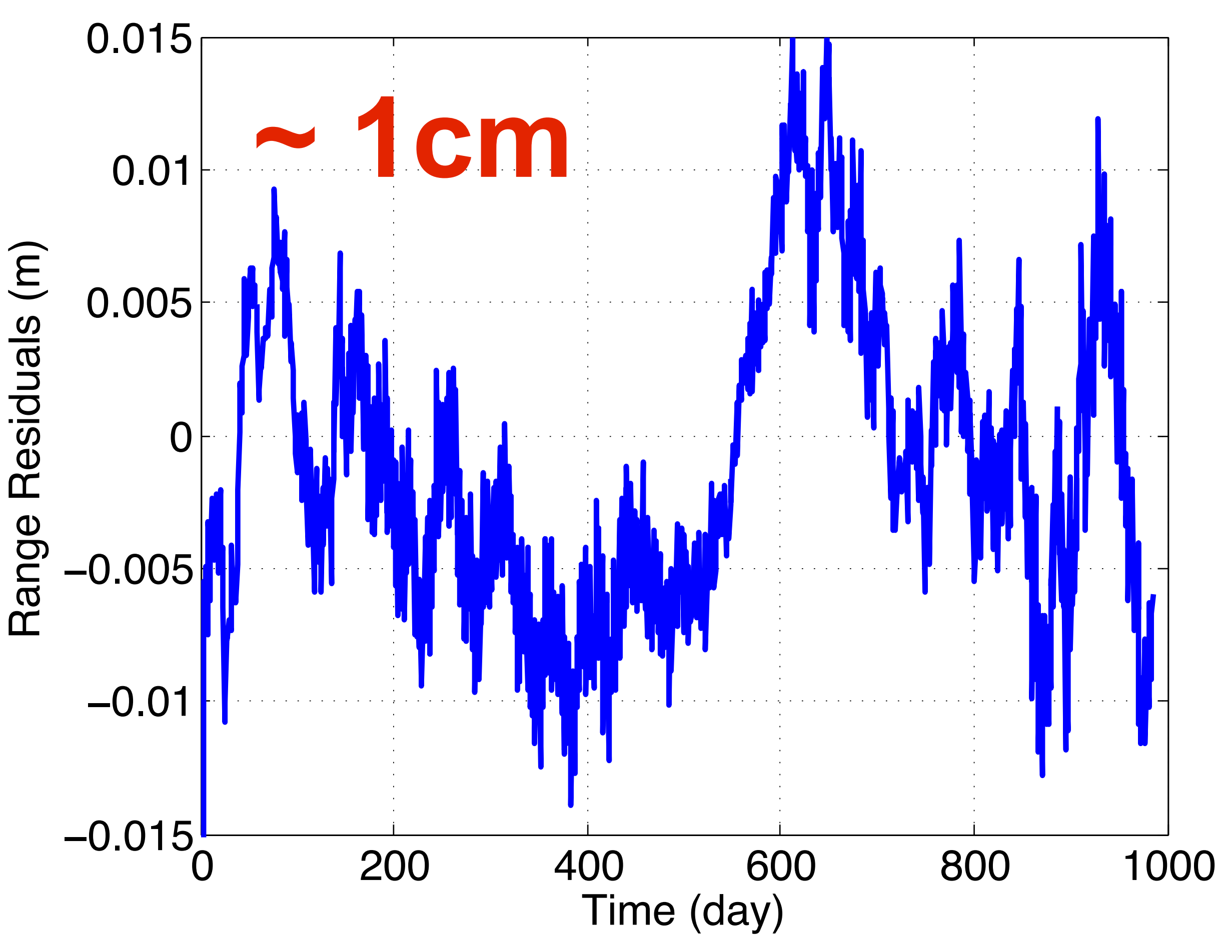
[1] P. Teyssandier, C. Le-Poncin-Lafitte, Class. and Quantum Grav. 25/145020, 2008

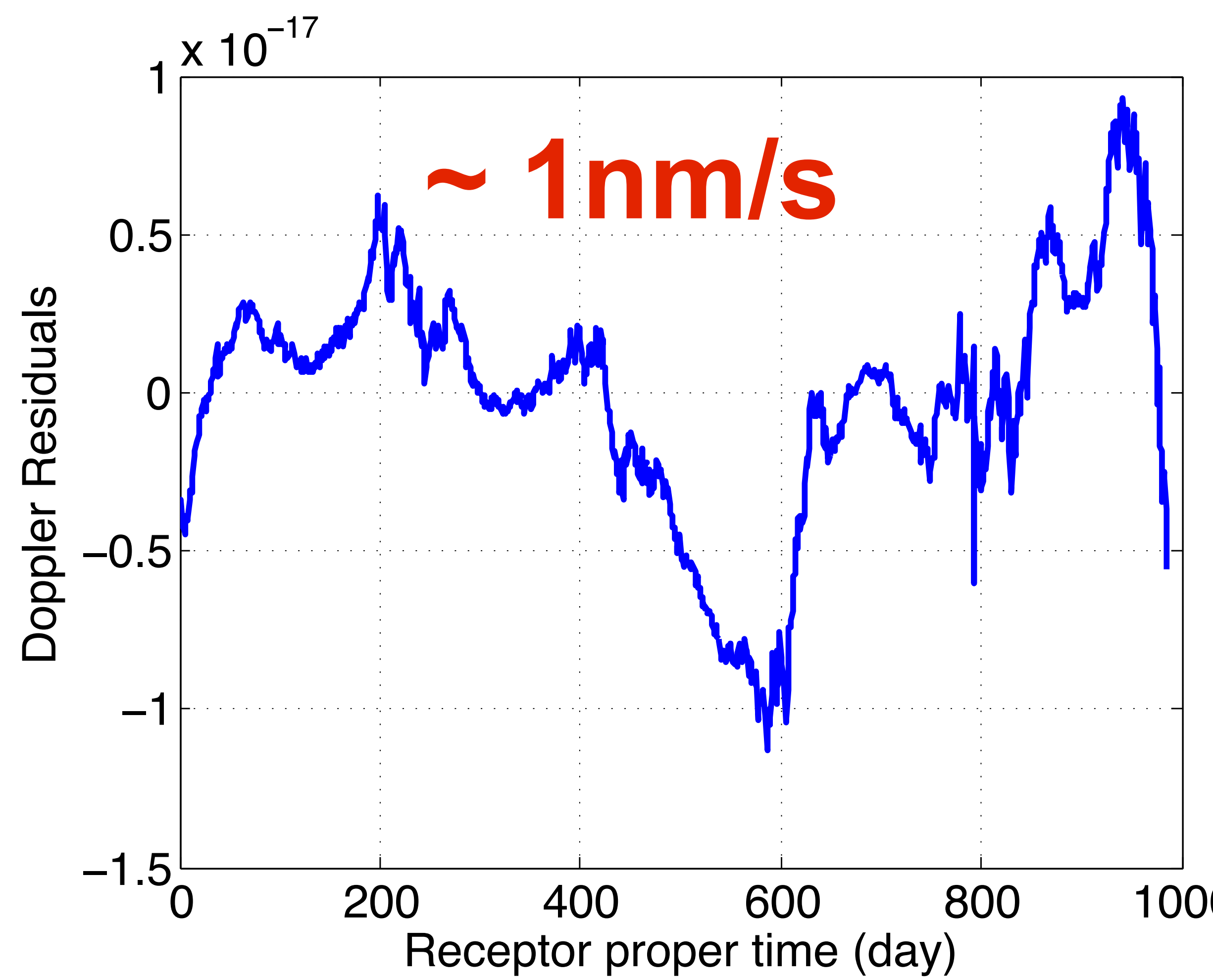
Software accuracy

- Two independent software developed (ROB-SYRTE and LKB): similar methods (except for the fit)

- Check between the two software: ~ 14 digits agreement (pretty good)

- Numerical accuracy of the whole process (simulation + fit): simulation in GR and fit of the initial conditions also in GR. Non-zero residuals are due to numerical errors. Ex. with Cassini:





Example of simulations + fit: PEG

- Alternative theory of gravity: Post-Einsteinian theory of Gravity[1].
  - from a phenomenological point of view: 2 functions are added to GR metric

$$g_{00} = [g_{00}]_{GR} + 2\delta\Phi_N(r)$$
$$g_{rr} = [g_{rr}]_{GR} + 2\delta\Phi_N(r) - 2\delta\Phi_p(r)$$

  - as an example, let's take a series expansion of $\delta\Phi_N$, $\delta\Phi_P$.

$$\delta\Phi_N(r) = \sum_i \alpha_i r^i$$
$$\delta\Phi_P(r) = \sum_i \chi_i r^i$$

  - This extends PPN framework:

$$\gamma - 1 = \chi_{-1} c^2/GM$$
$$\beta - 1 = \alpha_{-2}\left(c^2/GM\right)^2$$

- Simulations of Cassini spacecraft from 6 june 2002 (between Jupiter and Saturn)
  - Simple model: Sun, Earth, Spacecraft

[1] M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 22/2135, 2005
M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 23/777, 2006

Perspectives

- perform a lot of simulations with different gravitation theories on different (future and past) space missions

  Answer the question: can a particular deviation from GR be seen with a selected space mission ?

- include more effects to predict more subtle correlations: asteroid belt, planetary gravitational field, non-gravitational forces on spacecraft...

- extend the work to the another type of measurement done in the solar system: direction of light ray (VLBI)

Numerical simulations were made on the local computing resources (cluster URBM-SysDyn) at the University of Namur (FUNDP)