Cosmological constant versus astrophysical scale effects

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Context :   Accelerated expansion of the universe interpreted in

General Relativity with cosmological constant framework

→ Concordance LambdaCDM (LCDM) model

LCDM advantages … :

- well known & tested physics : gravitation / general relativity
  (but with a cosmological constant ↔ vacuum as perfect fluid $P = -\varepsilon$)

- **the model works very well !** (SN1a, CMBR, …)

- vacuum energy in physics (Casimir effect, ….)

… & inconvenients :

- bad interface with quantum field theory : **120 orders between the measured Lambda & its QFT expected value** (vacuum energy)

- … (coïncidence pb, …)

→ some authors prefer other options

- alternative gravity (scalar-tensor, $f(R)$, ….)

- matter content

- inhomogeneities (voids, ….)

- ….

Discarding this controversy, the fact the interpretation in terms of Λ results in a valuable cosmological scenario raises the question :

could Λ results in observable **effects at astrophysical scales** (<< cosmology) ?

no Λ clustering effets ➔ cosmo amplitude → astrophys amplitude (in some sense…)

Works made in these lines (LambdaGR) :

matter

- motions about black holes → incidences on accretion disks (?) [refs …]
- gravitational equilibrium [refs …]
- solar system : periastron shift, … [refs …]
- weak local value of the Hubble parameter (~ 60 km/s/Mpc vs ~ 70) [refs …]

light

- lensing [refs …]

- …… (?)

Often expected local effect :

« the cosmolocal constant acts as a radially repulsive force proportional to the distance »

A <u>general proof</u> of this claim ??????

Supported by **<u>Schwarzschild-de Sitter</u>** solution …

$$ds^2 = -\left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

weak field

$$\vec{g}_{eff} = -m\frac{\vec{r}}{r^3} + \frac{\Lambda}{3}\vec{r}$$

… and by <u>RW-cosmological models</u> (including the Einstein static universe) ...

… but in all these models, the **spherical symmetry is present from the very start !!!**

Another solution

$$ds^2 = -\frac{\cos^2 \bar{x}}{\left|\sin \bar{x}\right|^{2/3}} d\bar{t}^2 + d\bar{x}^2 \pm \left|\sin \bar{x}\right|^{4/3} \left(d\bar{y}^2 + d\bar{z}^2\right) \quad \text{with} \quad \bar{x}^\alpha \equiv \frac{\sqrt{3\Lambda}}{2} x^\alpha$$

solves $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$ (vacuum LGR)

(+ : static non-spherical spacetime)

(- : spacetime in anisotropic expansion/contraction)

➔ (a priori) Λ could result in non-spherical effects

<u>How determining the general local effect ?</u> Just do what you do in ($\Lambda = 0$)-GR …

… ie (1) consider the LGR equation

$$R_{\alpha\beta} = 8\pi\left(T_{\alpha\beta} - \frac{1}{2}Tg_{\alpha\beta}\right) + \Lambda g_{\alpha\beta}$$

& (2) expand :

$$g_{\alpha\beta} = m_{\alpha\beta} + h_{\alpha\beta} \quad \text{with} \quad |h_{\alpha\beta}| << 1$$

- about Minkowski
- without any prior symmetry assumption

& (3) determine the solution at the linearized order

& (4) write the geodesic equation → identify Λ effects on the free fall problem

!!!!! Problem : Minkowski is NOT a vacuum LGR solution …

… but all <u>LGR solution is locally minkowskian</u>

→ OK for getting just local effects

One gets $$-\Box h_{\alpha\beta} + \partial_\alpha \partial_\sigma \overline{h}^\sigma_\beta + \partial_\beta \partial_\sigma \overline{h}^\sigma_\alpha = 16\pi \overline{T}_{\alpha\beta} + 2\Lambda m_{\alpha\beta} + O(h^2) + O(\Lambda h)$$

where $$\overline{X}_{\alpha\beta} \equiv X_{\alpha\beta} - \frac{1}{2} X m_{\alpha\beta} \qquad (X \equiv m^{\mu\nu} X_{\mu\nu})$$

→ This equation was already adressed by **several authors**, who considered :

- Nowakowski & Arraut (2008, 2010) → Λ impact on gravitational radiation
- Bernabéu, Espinoza & Mavromatos (2011) → one solution (SdS in deguise …) & impact on gravitational radiation

… but **did not consider** the possible **impact on free fall dynamics** (that we are interested in)

Separate matter and Lambda (linearity) :

→ $h'_{\alpha\beta} \equiv h_{\alpha\beta} - \overset{(m)}{h}_{\alpha\beta}$ where $\overset{(m)}{h}_{\alpha\beta}$ is a solution for $\Lambda = 0$ (matter)

$$\partial_\alpha \partial_\sigma h'^\sigma_\beta + \partial_\beta \partial_\sigma h'^\sigma_\alpha - \partial_\alpha \partial_\beta h' - \Box h'_{\alpha\beta} = 2\Lambda m_{\alpha\beta}$$

$\partial\partial\ldots = cste$ → look for solutions $h'_{\alpha\beta} = \Lambda K_{\alpha\beta\mu\nu} x^\mu x^\nu$ with $\underbrace{K_{\alpha\beta\mu\nu} = K_{\alpha\beta\nu\mu} = K_{\beta\alpha\mu\nu}}$ → 100 cstes

Must satisfy

field equation → 10 constraints on these 100 constants

$$m^{\lambda\sigma}\left(K_{\beta\lambda\alpha\sigma} + K_{\alpha\lambda\beta\sigma} - K_{\lambda\sigma\alpha\beta} - K_{\alpha\beta\lambda\sigma}\right) = m_{\alpha\beta}$$

(linearized) Schw-de Sitter in isotropic coord is solution → OK

Could satisfy

Hilbert gauge (usual, coord system choice) $\partial_\sigma \bar{h}^{\alpha\sigma} = 0$

→ $$m^{\lambda\sigma}\left(K_{\alpha\lambda\beta\sigma} - \frac{1}{2}K_{\lambda\sigma\alpha\beta}\right) = 0$$ → 16 constraints (→ total = 26 constraints)

(linearized) Schw-de Sitter in harmonic coord is solution → OK

Local dynamics

Aim : write geodesic equation (free particles motion)

Hypothesis (usual) :

- slow particles motions
- slow sources motions → $\left|\partial_0 \overset{(m)}{h}_{\alpha\beta}\right| << \left|\partial_i \overset{(m)}{h}_{\alpha\beta}\right|$ (but not for $\left|\partial_0 h'_{\alpha\beta}\right|$ & $\left|\partial_i h'_{\alpha\beta}\right|$)

One gets

$$\frac{dv^k}{dt} = \underbrace{N^k t + M^{kl} x^l}_{\text{cosmological terms}} + \underbrace{\sum_{(A)} \partial_k \left(\frac{M_A}{r_A}\right)}_{\text{usual newtonian terms}}$$

cosmological terms

usual newtonian terms

$$\begin{cases} N^k = \Lambda\left(K_{000k} - 2K_{0k00}\right) & (3 \text{ cstes}) \quad \text{de Sitter} \rightarrow N^k = 0 \\ M^{kl} = \Lambda\left(K_{00kl} - 2K_{0k0l}\right) & (9 \text{ cstes}) \quad \text{de Sitter} \rightarrow M^{kl} = \ldots \ (\propto \delta_{kl}) \end{cases}$$

Local effects of Lambda

- there are solutions such that $N=M=0$ → no Λ effect in this case

- cases $N=0$ & $M^{kl} \propto \delta_{kl}$ → effective radial force propto the distance (includes de Sitter)

- (generic case) : $N^k \neq 0$ and $M^{kl} \neq 0$ (and not $\propto \delta_{kl}$)
  - N → (locally) **uniform acceleration field** (varies on cosmological timescales)
  - M → acceleration field propto $\vec{r}$ but not colinear

(*B. Chauvineau & T. Regimbau*, PRD, 2012)

Effects of M :

consider $$\frac{dv^k}{dt} = M^{kl} x^l$$ → in the case where there is Q such that $Q^{kl} Q^{lm} = M^{km}$

a solution reads $v^k = Q^{kl} x^l$

→ (if Q symmetric) **quadrupole-like term + contribution to expansion** (if Q not traceless)

Role in clusters dynamics ?

local dynamics : very complex ! Fitting data requires :

- one (or even two) attractor(s)
- quadrupole terms
- …

→ **could** the « local cosmological fields » ***N* & *M* contribute ?**

Expected (?) effect of N :

no known exact non-sym LGR solution & 90 (or 74) free parameters !!!

→ no obvious « natural » way to get an order of magnitude ….

Expected (from de Sitter ???) : $K \sim 1/10$ or less … → $\frac{dv_N}{dt} \sim \frac{1}{5} cHM^2 t$

→ $\mathrm{v} \sim \frac{1}{10}(Ht)^2 c$ → some 10^2 or 10^3 km/s for $Ht \sim \frac{1}{10}$ (→ ???)

→ going further ???

- developments including higher order terms (*h**L*, h**h*, ...)

- first order on a (L=0)-solution (instead of Minkowski)

→ (for instance) de Sitter (vacuum)

→ potentially includes non-sph local effects on a globally spher solution

$$g_{\alpha\beta} = \gamma_{\alpha\beta} + h_{\alpha\beta} \quad \text{with} \quad \gamma_{\alpha\beta} = diag\left(-1, a^2\right) , \quad a = e^{Kt} \quad , \quad K = \sqrt{\frac{\Lambda}{3}}$$

$$ds^2 = -dt^2 + e^{2Kt}\left(dx^2 + dy^2 + dz^2\right) + h_{\alpha\beta}dx^\alpha dx^\beta \quad \text{with} \quad \left|h_{\alpha\beta}\right| << 1$$

Gauge choice : **synchronous gauge** $h_{0\alpha} = 0$

Geodesic equation : $$\frac{d}{dt}\left(a^2 \frac{dx^k}{dt}\right) = 0$$

→ as unperturbed de Sitter, but the link coordinates vs physics depends on *h* ….

Field equations

$$h_{ii} = U + a^2 V$$ and $$\partial_k h_{ik} = \partial_i U + a^2 \Psi_i$$

with $U(x,y,z)$ & $V(x,y,z)$ & $\Psi_i(x,y,z)$

and

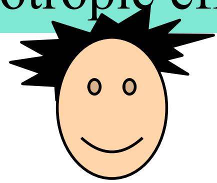
$$a^{-2}\left(\partial_i \partial_k h_{jk} + \dots\right) + \partial_0 H_{ij} + K\left(H_{ij} + \delta_{ij} H\right) = 0$$ with $$H_{ij} = a^2 \partial_0 \left(\frac{h_{ij}}{a^2}\right)$$

$$\rightarrow \quad \partial_i \partial_i V - \partial_i \Psi_i + 4K^2 U = 0$$

time-dependent (only…) perturbation → OK

$$ds^2 = -dt^2 + \left(a^2\delta_{ij} + \frac{P_{ij}}{a}\right)dx^i dx^j \quad \text{with} \quad P_{ii} = 1 \quad (\text{for } \frac{|P_{ij}|}{a^3} << 1\,)$$

anisotropic effect (…but non-local …)



Mixed time-(x,y,z) solutions ??? **In progress ….**

Thank you for your attention