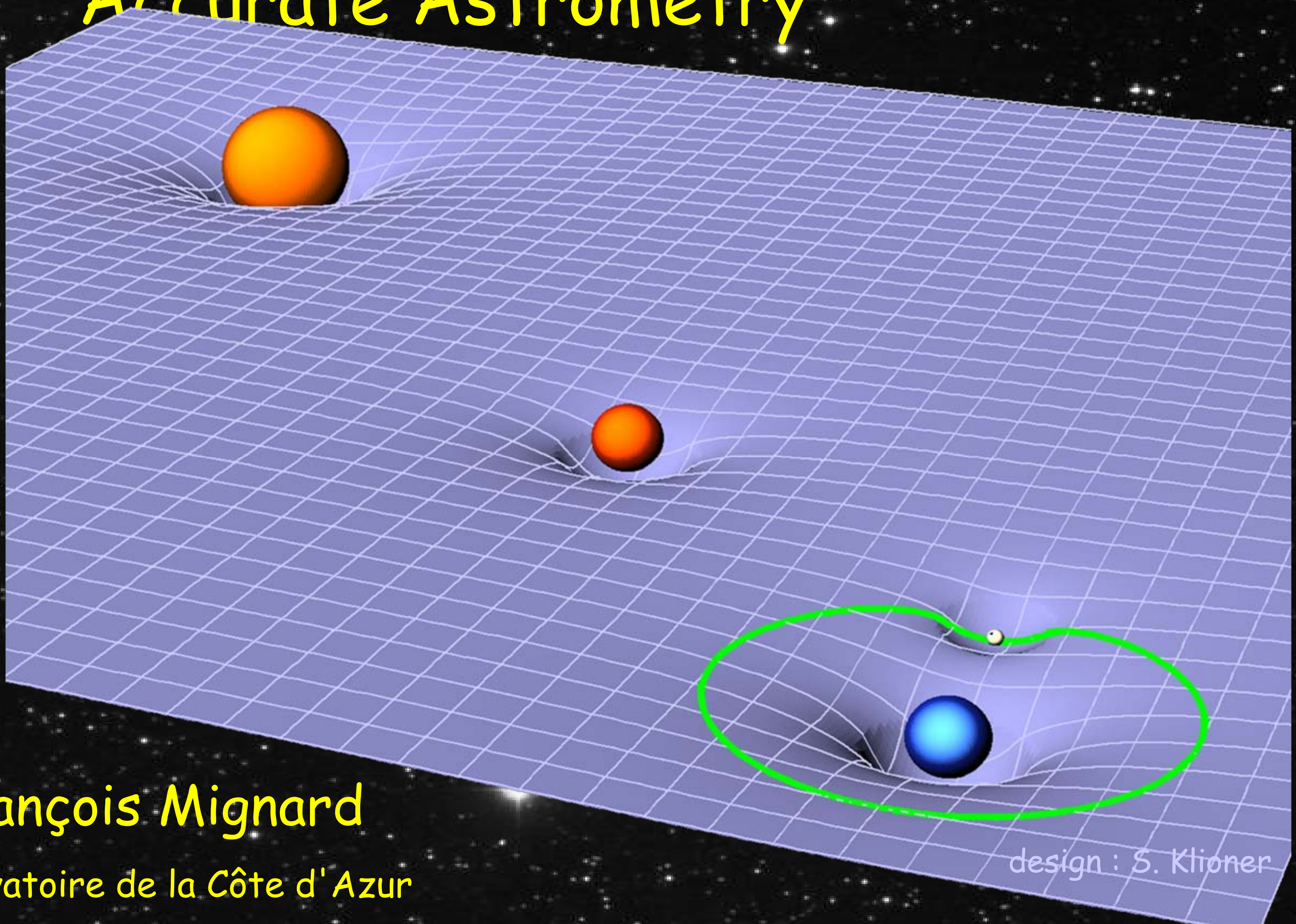
Fundamental Physics and Accurate Astrometry

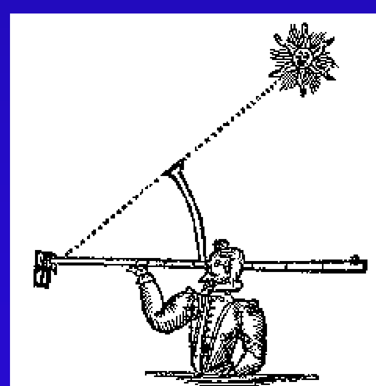


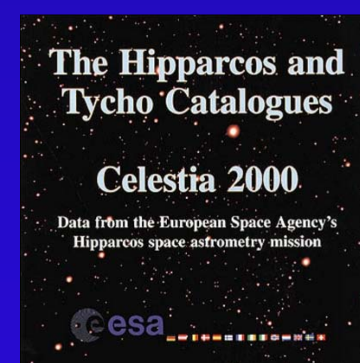
François Mignard

Observatoire de la Côte d'Azur

What is meant by Astrometry ?

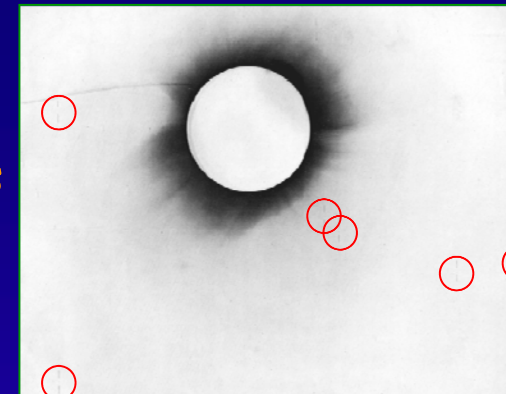
- Astrometry deals with the measurement of the positions and motions of astronomical objects on the celestial sphere.
  - Global or wide field astrometry
  - Local or small field astrometry
- Astrometry relies on specialized instrumentation and observational and analysis techniques.
- It is fundamental to all other fields of astronomy.





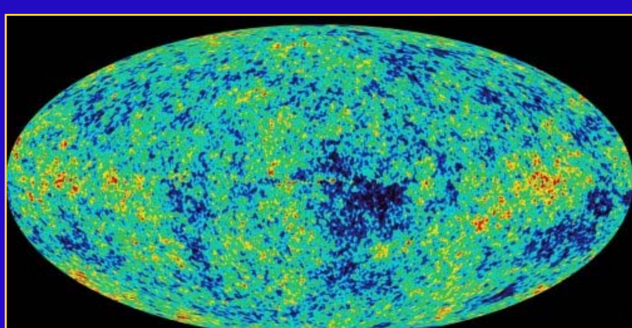
Fundamental Physics

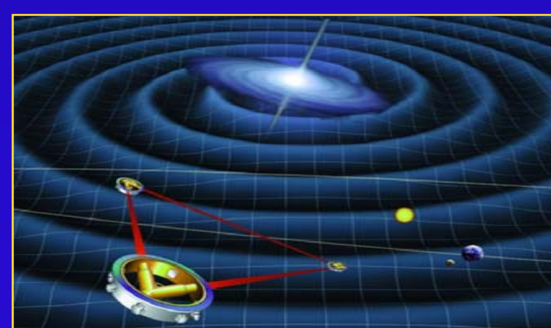
- Relevant topics
  - Very variable according to historical periods
    - dominated by the law of motion, covariance of physical laws under reference frame transformation
  - Closely associated to astrometric accuracy
    - but not only ➔ eg COBE/WMAPS/PLANCK

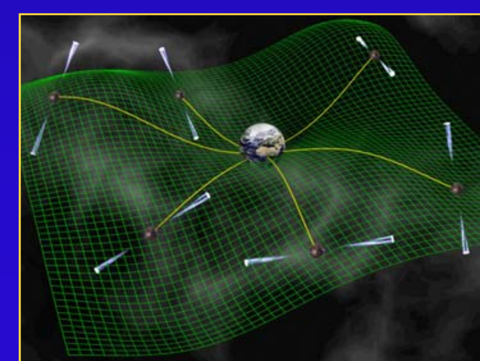


- Astronomy can provide clues only on large distance scale

- 100 -1000 km	Earth satellites
- 10^8 – 10^9 km	Solar System
- pc – kpc	Stellar system in the MW
- Mpc	Local group
- Gpc	QSOs, CMB, SN1a







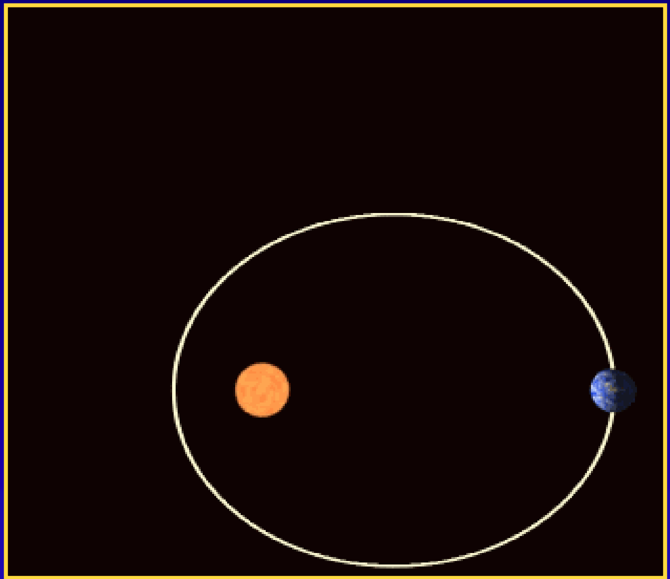
Astrometry → Fundamental laws

Kepler Laws	1610	Kepler
Finite speed of light	1676	Roemer
Gravitation theory – $1/r^2$ law	1700	Newton
Aberration of Light	1727	Bradley
Universal Gravitation	1827	Savary
Orbit of Mercury	1850	LeVerrier
Light deflection by the Sun	1919	Eddington
Recession of galaxies	1925	Hubble
Radar echo delay	1970	
Superluminuous radiation	1980	
Einstein rings and lensing.	1980	
Orbital evolution of the binary pulsar	1982	
Strong Equivalence Principle (LLR)	1990	
Dark matter in Galactic clusters	1990	

Assumptions in Newtonian Gravity

- Laws of motion

$$m_a \frac{d^2 \mathrm{x}_a}{dt^2} = -\sum_{b \neq a} G m_a m_b \frac{\mathrm{x}_a - \mathrm{x}_b}{\left|\mathrm{x}_a - \mathrm{x}_b\right|^3}$$



- … few subtleties

$$m_a^{I} \frac{d^2 \mathrm{x}_a}{dt^2} = -\sum_{b \neq a} G\, m_a^{G}\, m_b^{G} \frac{\mathrm{x}_a - \mathrm{x}_b}{\left|\mathrm{x}_a - \mathrm{x}_b\right|^3}$$

Assumptions in Newtonian Gravity

- There is an inertial frame
  - F = mg
- There is an absolute time
  - t is absolute and 'flows uniformly'
- Equivalence principle

$$m_a^I = m_a^G$$

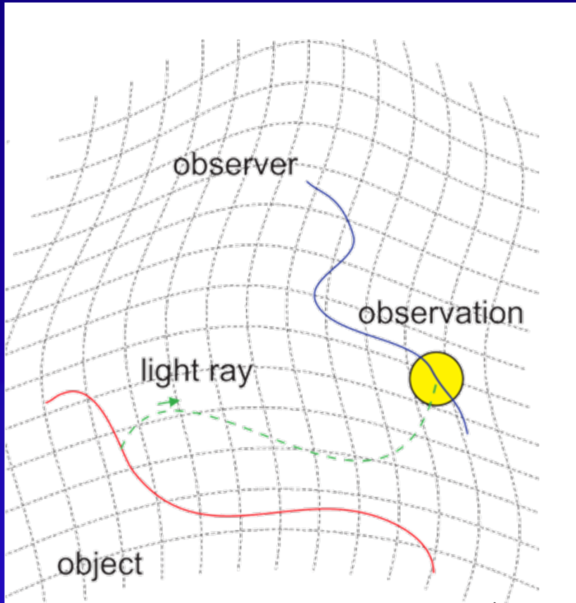
- G is a fundamental coupling constant

$$G \neq G(t) \qquad G \neq G(x)$$

Astronomy can help check these assumptions in the large scale domain

A dual relationship with mutual benefit

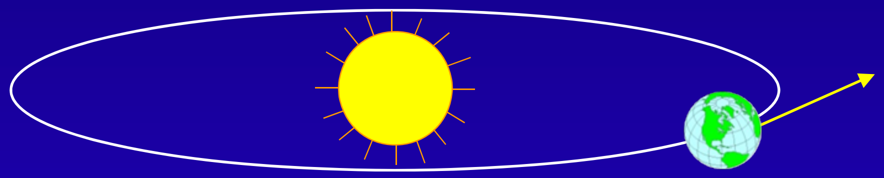
- Astronomy has been the source of early thinking about space and time fundamental properties
- Fundamental physics provides astronomers with tools to model space-time observations
- Accurate astronomy is a playground to put physical theories under tests



Courtesy: S. Klioner

Astrometric modelling

- Effects due to motion



$v/c = 10^{-4} \sim 20''$

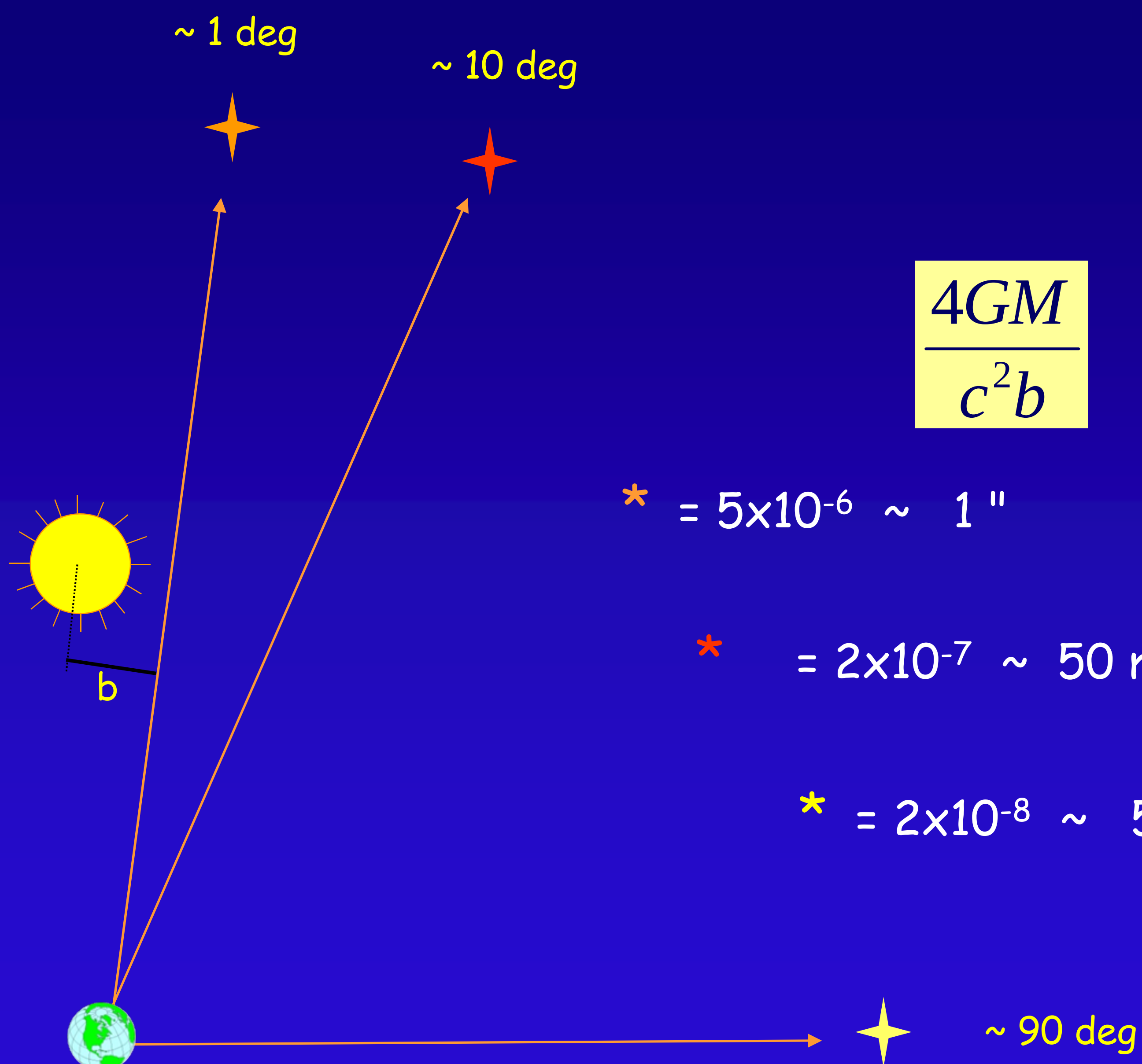
$v^2/c^2 = 10^{-8} \sim 1$ mas

$v^3/c^3 = 10^{-12} \sim 0.1\ \mu$as

- Astrometry ~ 1700 ➔ 20" = discovery of aberration
- Ground based astrometry < 1980 ➔ Newtonian aberration
- Hipparcos (~ 1mas) ➔ v^2/c^2 terms
- Gaia, (~ 1-10 μas) ➔ full relativistic formulation

Test of Local Lorentz Invariance ?

Spacetime curvature effects



$$\frac{4GM}{c^2 b}$$

* = 5×10^{-6} ~ 1 " 1915

* = 2×10^{-7} ~ 50 mas 1960

* = 2×10^{-8} ~ 5 mas 1980

Astrometric modeling

- Newtonian models cannot describe high-accuracy observations:
  - many relativistic effects are several orders of magnitude larger than the observational accuracy
  - space astrometry missions would not work without relativistic modelling
    - both for space and time → 4D modelling
- The simplest theory which successfully describes all available observational data:

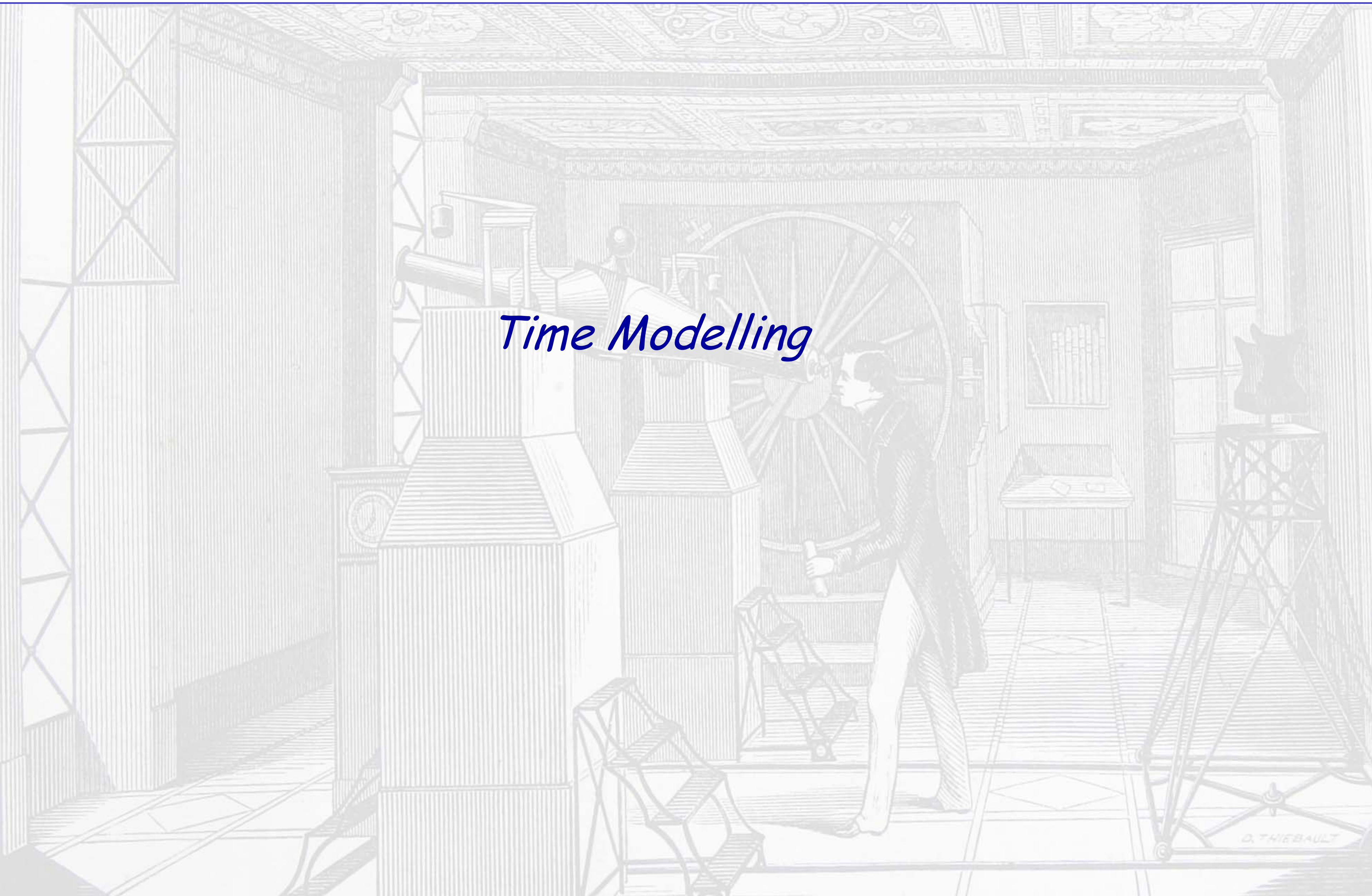
GENERAL RELATIVITY

" Astrometry is the measurement of space-time coordinates of photon events "

A. Murray

Implementation for Gaia

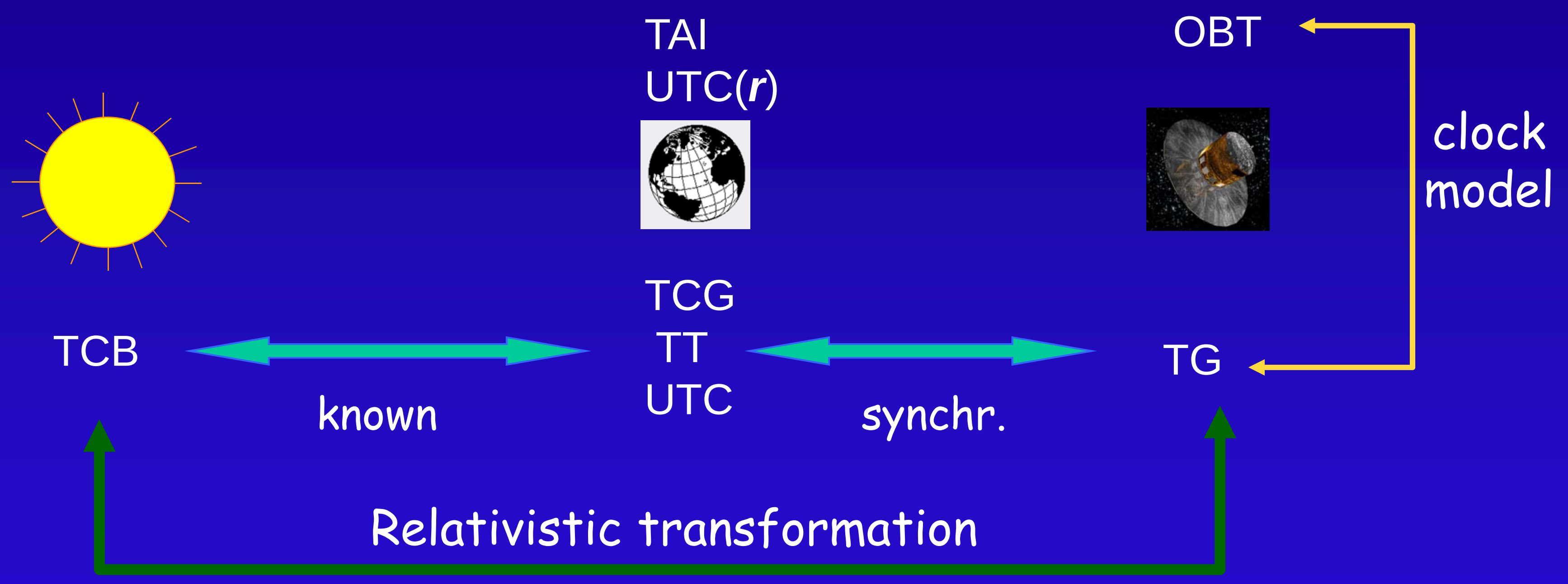
- The astrometric model is a key element in the DP
  - a modeling accuracy of 0.1 μas is the requirement
- Two independent models have been developed
  - GREM by Klioner et al.
  - RAMOD by Vecchiato, Crosta et al.
- They will be used in different context in the data processing
  - GREM is the baseline for the pipeline reduction
  - it is implemented in the Gaia Tool library
  - it has a direct (➔ proper directions) and a reverse mode
  - both stellar and solar system sources
  - accuracy can be controlled by the user ➔ CPU-effective
  - partial derivatives are optional
- Comparisons are under way to check respective properties
- Solar system ephemeris (INPOP) are consistent with the model



Time Modelling

Relevant timescales for Gaia

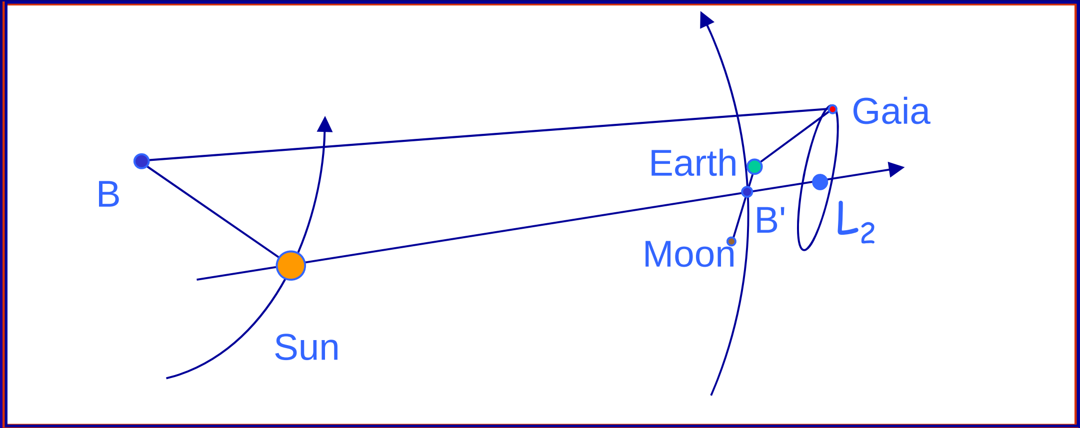
- Modelling and data processing in TCB
- on-board clock delivering a realisation of TG (→ OBT)
- tracking and ground-based timing in UTC



Mignard, Crosta, Klioner, 2004

Motion of Gaia : TCB - TG

- Orbit of Gaia around L2



$$\frac{d\tau}{dt} \approx 1 - \frac{1}{c^2}\left[\frac{V^2}{2} + U\right] + \frac{1}{c^4}\left[-\frac{V^4}{8} - \frac{3}{2}V^2 U + \frac{U^2}{2} + 4\mathbf{V}.\mathbf{W}\right]$$

$$t - \tau = \int\left(\frac{V^2}{2c^2} + \frac{U}{c^2}\right)dt + \int\left(\frac{1}{8}\frac{V^4}{c^4} + \frac{3}{2}\frac{V^2 U}{c^4} - \frac{U^2}{2c^4} - 4\mathbf{V}.\mathbf{W}\right)dt$$

- Numerical quadrature + solar system ephemerides

Gaia DPAC

Secular term

	TCG	Gaia	
L_c	$1.480\ 826\ 867 \times 10^{-8}$	$1.481\ 259\ 949 \times 10^{-8}$	day/day
		$1.481\ 259\ 960 \times 10^{-8}$	with $1/c^4$ terms

Periodic terms

P(yr)	TCG μs	Gaia μs	
1.00	1656.68	1664.74	Sun
0.486	-	121.74	Lissajous
1.09	22.42	22.63	J-S
0.5	13.84	13.83	2S
11.8	4.77	4.76	J
1.04	4.68	4.63	Sa-S
29.5	2.26	2.28	Sa
0.95	-	1.33	Lis- S

Relativity Tests with Gaia

Relativity tests with astrometry

light deflection

spacetime curvature

planetary J2

orbital precession

non linearity of gravitation

Lunar orbit

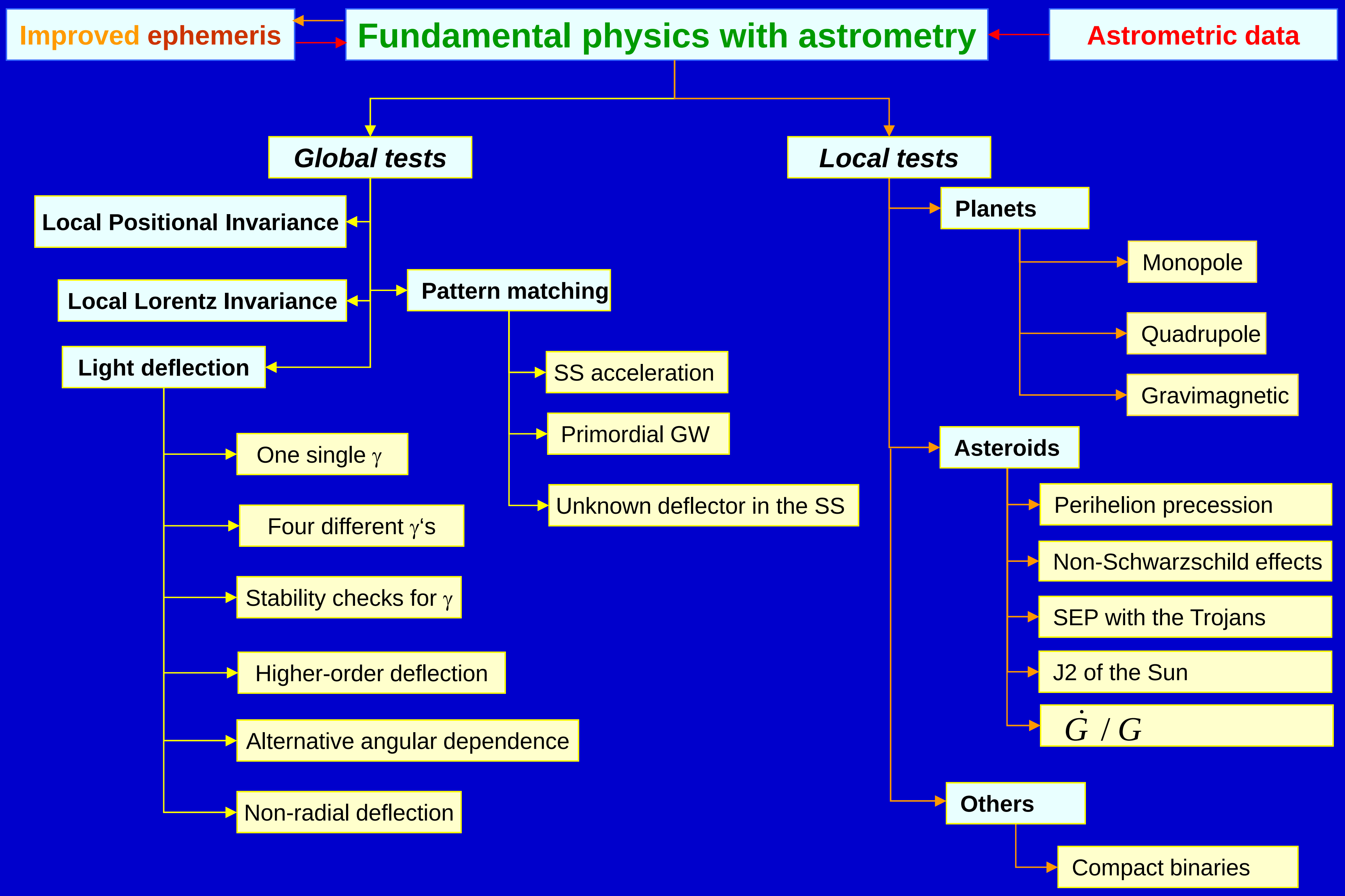
equivalence principle

Pulsars

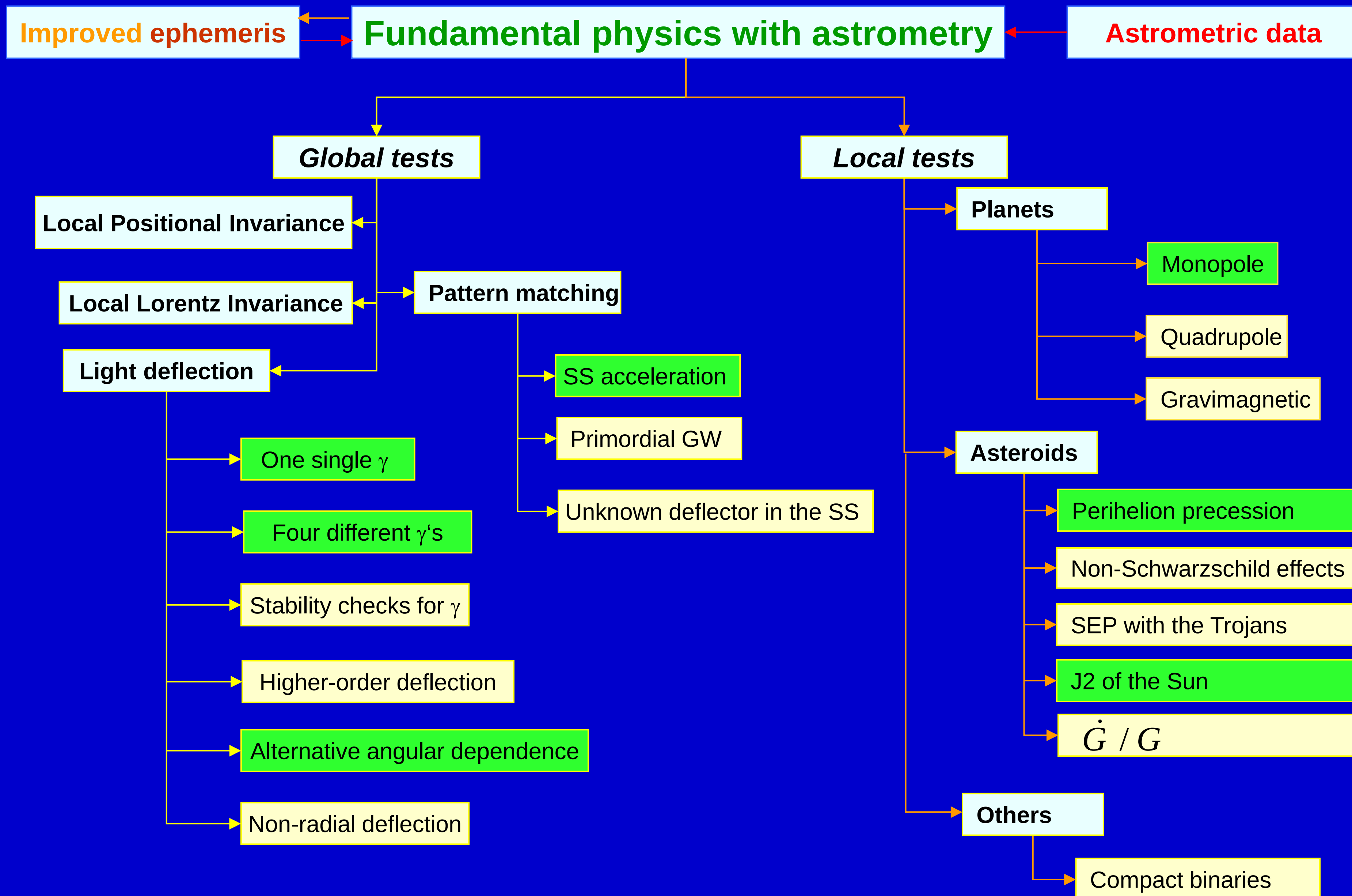
strong fields and gravitational waves

Solar Light deflection	$\sigma_{\gamma} < 1\times10^{-6}$
Orbits of minor planets	$\sigma_{\beta} < 5\times10^{-4}$
Orbits of minor planets	$\sigma_{\dot{G}/G} < 5\times10^{-13}\,\mathrm{yr}^{-1}$
Jupiter light deflection	$Q_{\mathrm{deflect}} > 3\sigma$

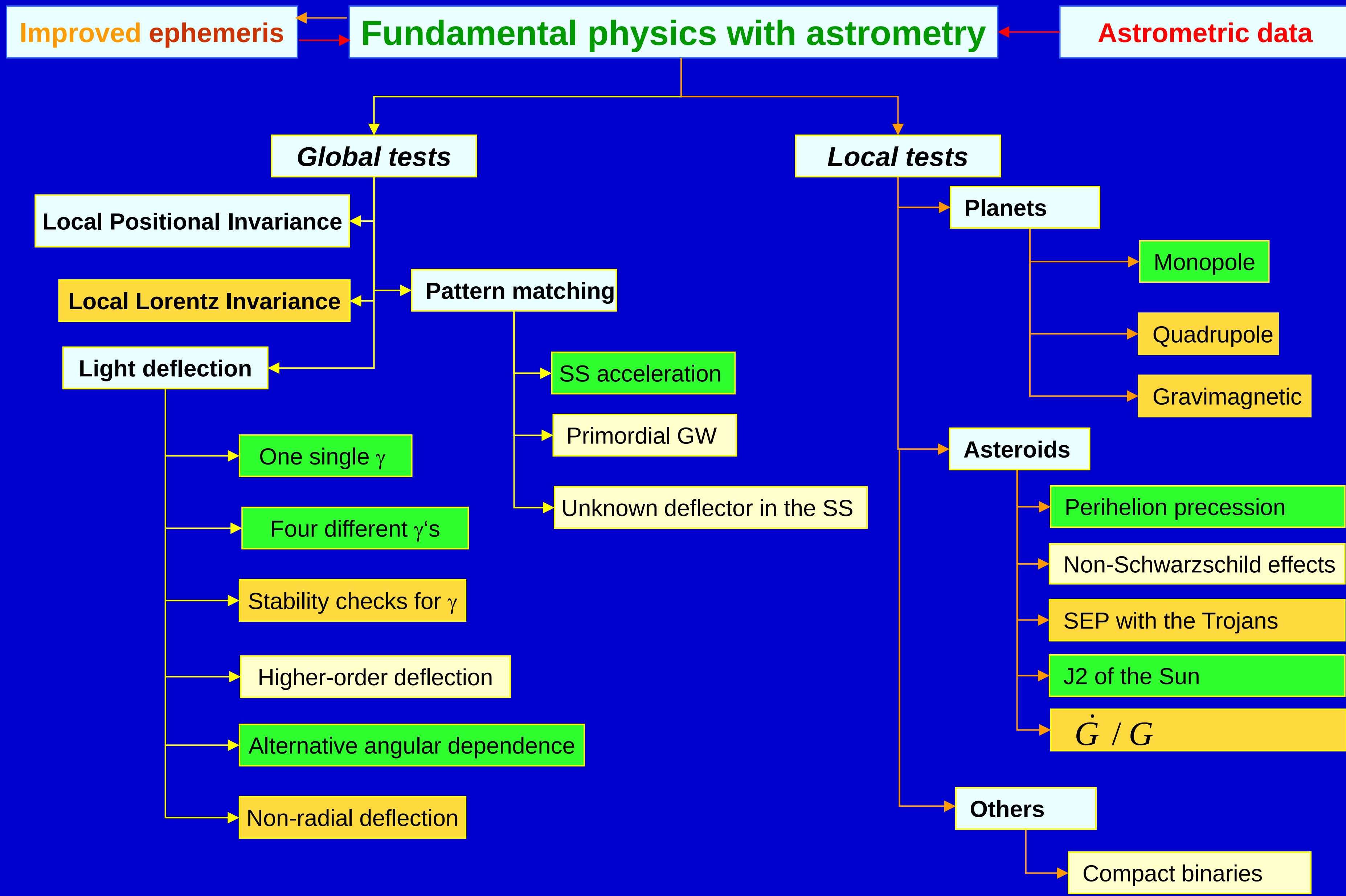
Relativity tests with accurate astrometry



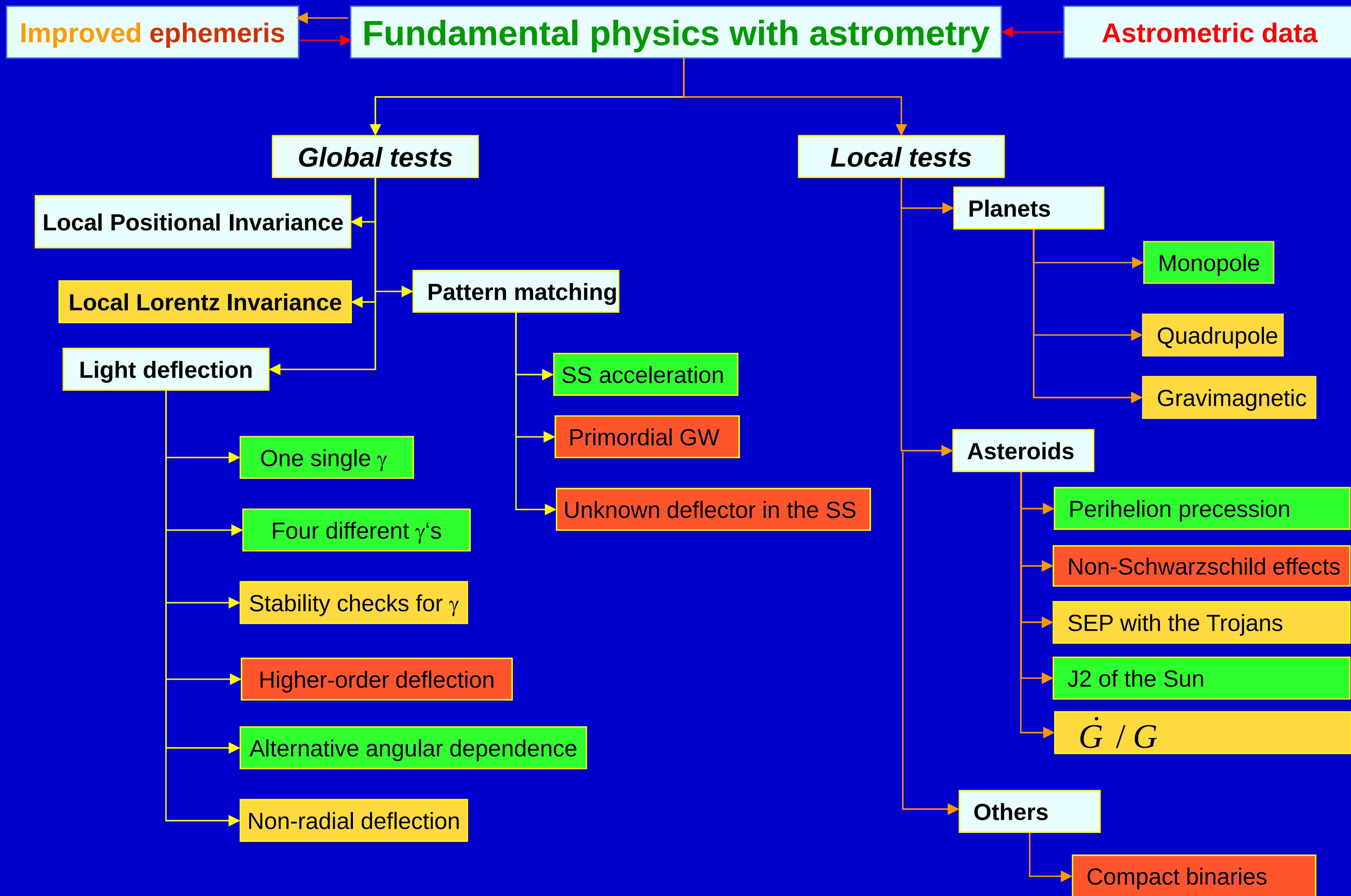
Gaia : Core tests – good results expected



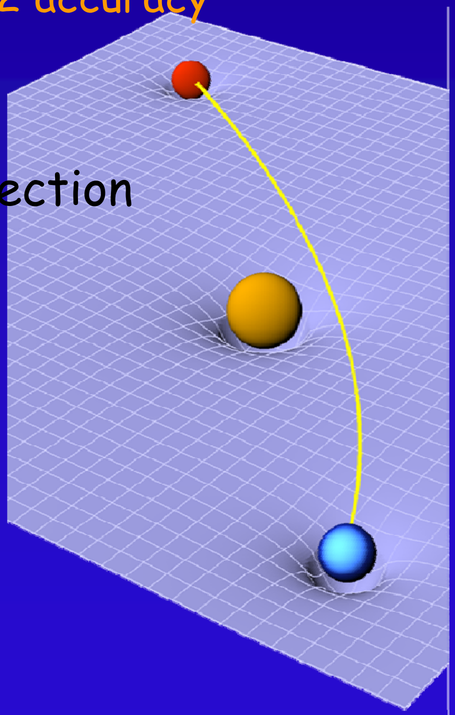
Gaia: complementary tests with parameter fitting



Gaia: tests on residuals



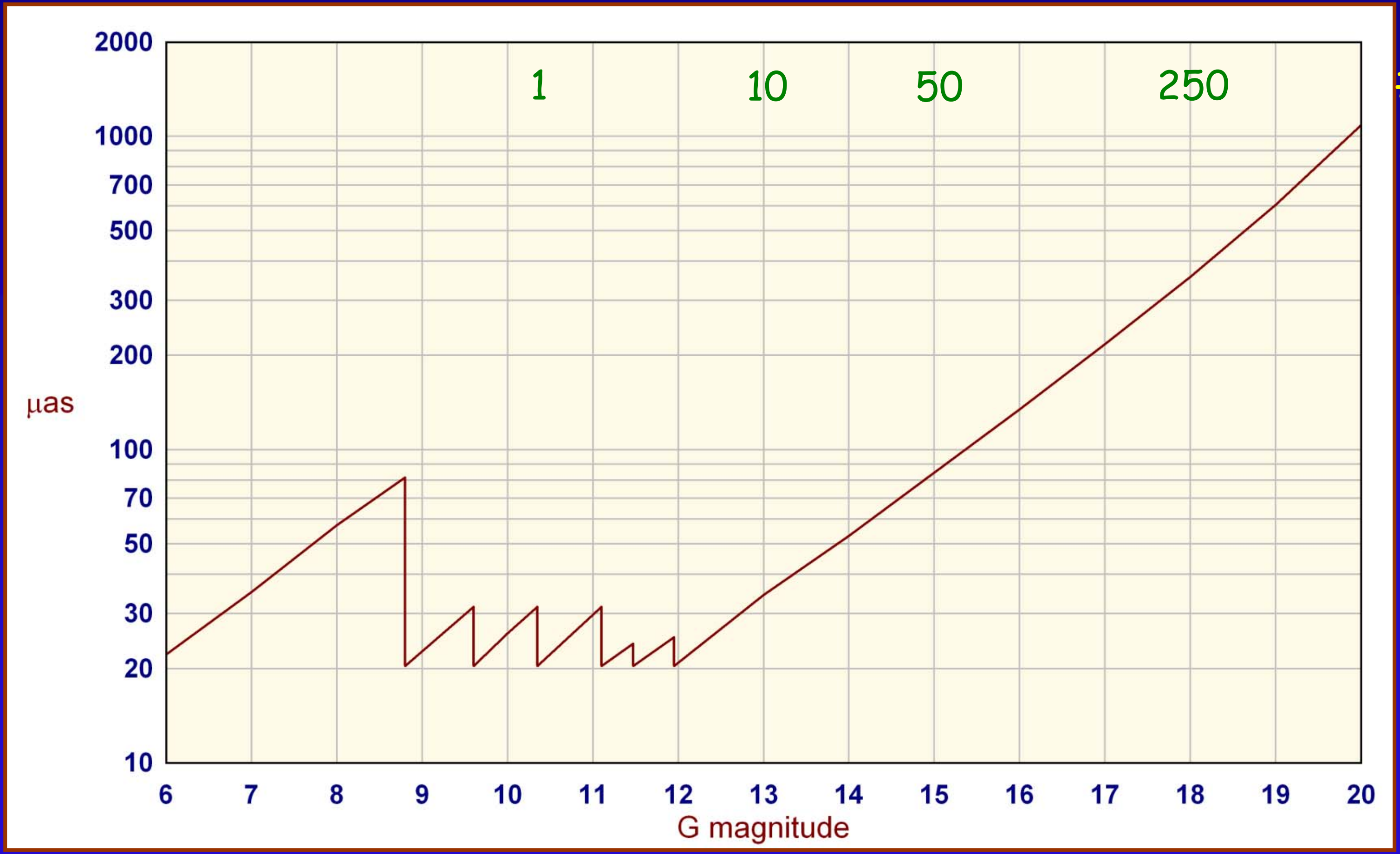
- Most precise test on γ with Gaia
  - Preliminary analysis (ESA, 2000, Mignard, 2001, Vecchiato et al., 2003)
- Advantages of Gaia experiment
  - Optical with accurate astrometry
    - One individual observation at 90° from the Sun ➔ γ to 0.02 accuracy
  - Deflection (not time delay involving nearly sun grazing)
  - Wide range of angular coverage ➔ mapping of the deflection
    - Test of alternate deflection law
  - No problem with solar corona
  - Full-scale simulation of the experiments
    - sensitivity analysis, systematic effects
  - Testing could be wider than PPN formulation



Gaia single observation accuracy

- One transit over the field-of-view
- Integration over 9 Astro CCDs

Solar deflection → 4mas @ 90°



stars 10^6

Photon path in a gravitational field

$$g_{00} = -1 + \frac{2}{c^2} w(x,t) - \frac{2}{c^4} \beta w^2(x,t)$$

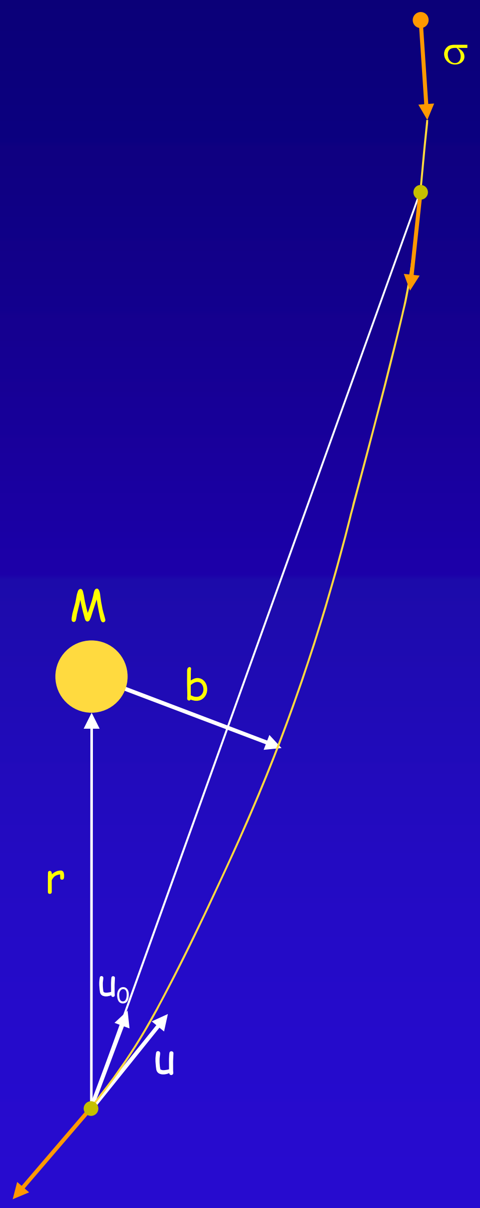
$$g_{0i} = -\frac{4}{c^3} w^i(x,t)$$

$$g_{ij} = \left(1 + \frac{2}{c^2} \gamma w(x,t)\right) \delta_{ij}$$

$$\mathbf{x}(t) = \mathbf{x}_0(t) + \boldsymbol{\sigma}(t - t_0) + \Delta\mathbf{x}(t)/c^2$$

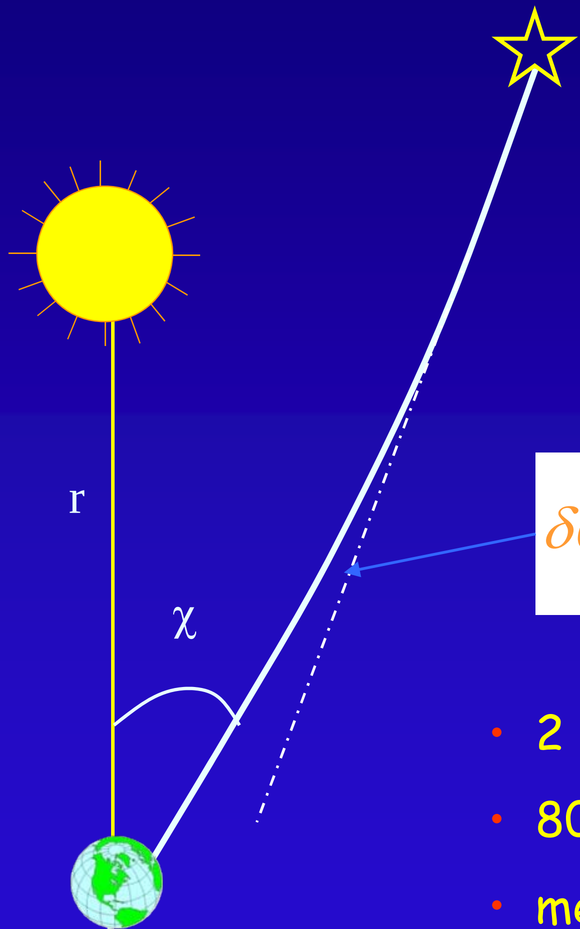
$$\mathbf{u} = \mathbf{u}_0 + \frac{(1+\gamma)GM}{c^2} \frac{\left[1 + (\mathbf{u}_0 . \mathbf{r})/r\right]\mathbf{b}}{b^2}$$

$$\delta\phi = \frac{(1+\gamma)GM}{c^2} \frac{1 + \cos\chi}{b}$$



Relativity Experiments

- Light Deflection : determination of γ



Light deflection in mas

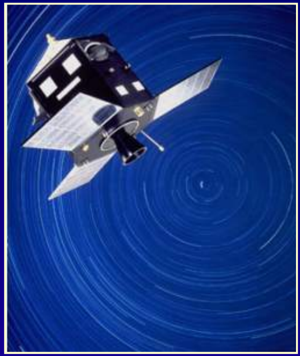
	min χ	χ = 45 deg
Sun	10 mas	10 mas
Jupiter	16 mas	2 μas
Saturn	6 mas	0.8 μas

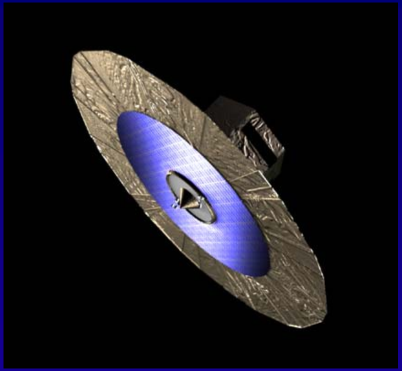
$$\delta\theta = \frac{2GM}{rc^2}\frac{1+\gamma}{2}\frac{\sin\chi}{1-\cos\chi}$$

Observable quantity

- 2×10^7 stars V < 14
- 80 observations per star
- measurable effect even at 135° from the Sun
- but large correlation with zero-point parallax (~ -0.85)

Expected Performance on γ





Hipparcos	GAIA	σ_H/σ_G
• 10^5 stars V < 10	• 8×10^6 stars V < 13	
• 2.5×10^6 abscissas	• 6×10^8 FOV transits	→ x 15
• σ ~ 3 to 8 mas	• σ ~ 40 μas	→ x 125
• χ > 47 degrees	• χ > 45 degrees	
	• + 10^9 fainter stars	
⬇	⬇	> 2000

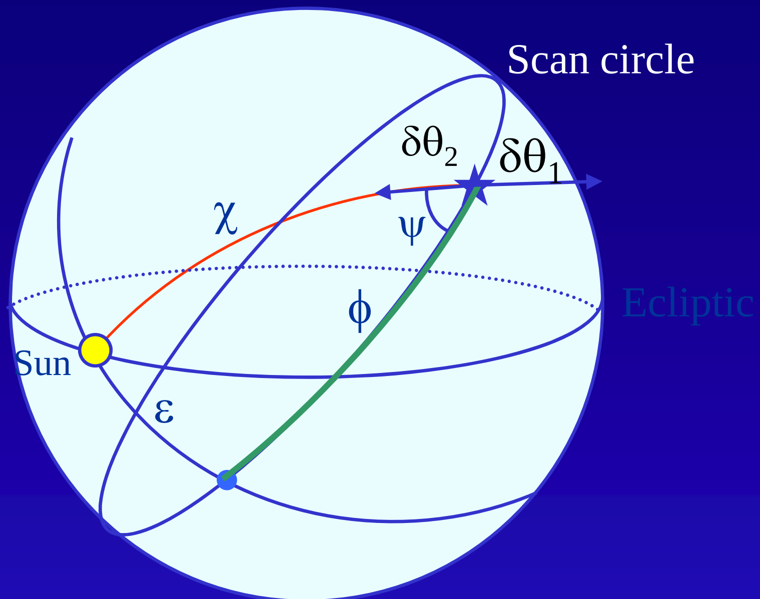
$$\gamma = 1 \pm 3\times10^{-3}$$

$$\sigma_\gamma \approx 2\times10^{-6} \text{ to } 6\times10^{-7}$$

Potential problems

- Special problems related to the procedure
  - many measurements are used and averaged out to get gamma
    - improvement in $1/n^{1/2}$ if no other unknown instrumental or physical effect is correlated with the deflection
    - very hard to establish at this level of accuracy
  - but these effects become significant only if constant over five years
- Known effects already identified
  - global parallax shift strongly correlated with γ
    - itself linked to instrument thermo-mechanical behaviour
  - relation with the velocity and aberration correction

Link with parallax



Deflection : $\delta\theta_1$

$$\delta\theta_1 = \frac{2GM}{ac^2}\frac{1+\gamma}{2}\frac{\sin\chi}{1-\cos\chi}$$

Parallax : $\delta\theta_2$

$$\delta\theta_2 = \pi\sin\chi$$

- Abscissa change : $\delta\phi_1 = \delta\theta_1\cos\psi = \dfrac{2GM}{ac^2}\dfrac{1+\gamma}{2}\dfrac{\cos\varepsilon\sin\phi}{1-\cos\varepsilon\cos\phi}$

$$\delta\phi_2 = -\delta\theta_2\cos\psi = -\pi\cos\varepsilon\sin\phi$$

- Correlation : 0.88 (0.92 with Hipparcos)

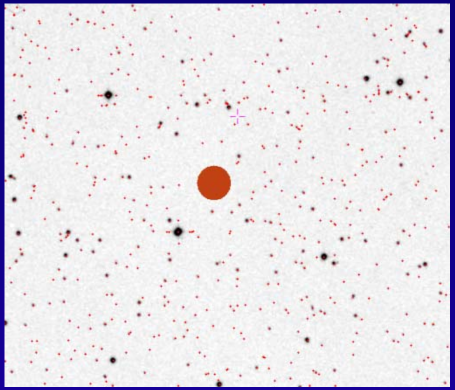
Beyond plain γ

- Observations over five years
  - processing over independent time intervals
  - check for systematic effects
- Repeated observations over many stars
  - Stability check: dependence of γ on various parameters
    - brightness, color, geometry
- Sampling of the angular distance to the Sun
  - mapping of the actual angular dependence
    - blind decomposition on spherical harmonics
- Higher order PPN terms could be included

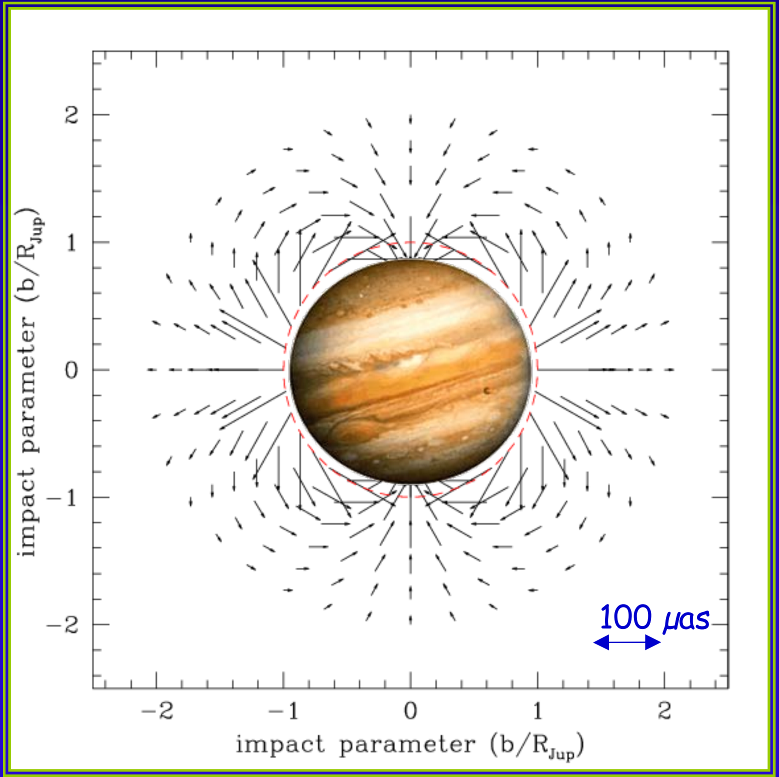
Light deflection by giant planets

	Monopole mas	Quadrupole μas
$1R_j$	16	240
$2R_j$	8	30
$5R_j$	3	2
$10R_j$	2	0.2
$1R_s$	6	95
$2R_s$	3	12
$5R_s$	1	0.8
$10R_s$	0.6	0.01

- Jupiter light deflection
  - Small field astrometry with Gaia
  - Relative measurements of star position around Jupiter
  - Same field observed earlier or later



Jupiter in 2013



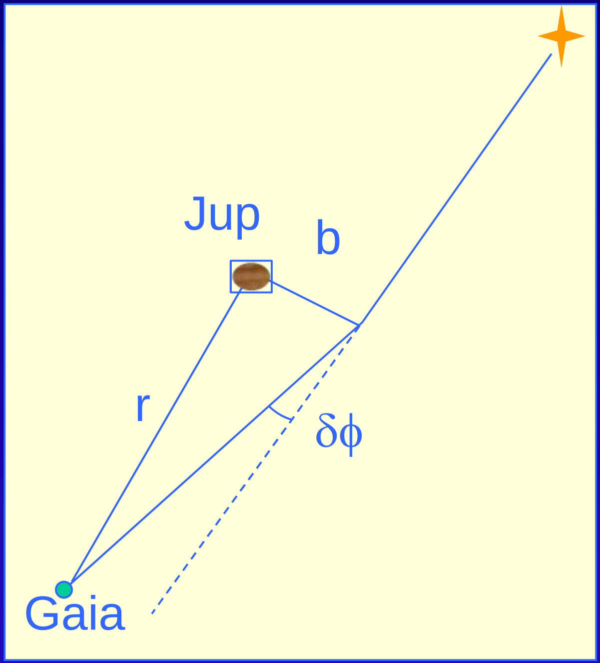
Deflection from Jupiter quadrupole

$$\delta\phi_Q = \frac{4GM_J}{c^2}\frac{J_2 R_J^2}{b^3}$$

- On-going work
  - optimize the mission parameters
  - search for observations close to bright stars
  - method of reduction

Crosta & Mignard, 2006, CQG

- Simplified formula for the quadrupole deflection



$$\delta\phi_M = \frac{4GM_J}{c^2 b}\frac{1+\gamma}{2}$$

$$\delta\phi_Q = \frac{4GM_J}{c^2}\frac{J_2 R_J^2}{b^3}$$

Exact expression with radial and non radial deflection

- Full derivation includes radial and non-radial deflection
  - Klioner, 2003 ; Crosta & Mignard, 2006 ; Leponcin-Lafitte & Teysandier.

Equations of Motion for a test body

- EIH equations with $M_s \gg M_p$, $V_s \ll V_p$
  - Heliocentric form
  - good for gravitation on asteroids and comets

$$\frac{d^2\mathrm{r}}{dt^2} = -\frac{k^2\mathrm{r}}{r^3}$$

Newton Sun

$$-k^2\sum_p M_p\left[\frac{\mathrm{r}-\mathrm{r}_p}{\left|\mathrm{r}-\mathrm{r}_p\right|^3}+\frac{\mathrm{r}_p}{r_p^3}\right]$$

Newton planets

$$+\frac{k^2}{c^2r^3}\left[2(\gamma+\beta)\frac{k^2\mathrm{r}}{r}-\gamma(\dot{\mathrm{r}}\cdot\dot{\mathrm{r}})\mathrm{r}+2(1+\gamma)(\dot{\mathrm{r}}\cdot\mathrm{r})\dot{\mathrm{r}}\right]$$

Einstein + PPN

$$\Delta\varpi = \frac{2\pi GM}{a(1-e^2)c^2}\left[-(\gamma+\beta) \qquad +\gamma \qquad +2(1+\gamma)\right]$$

Precession per orbit

Determination of β : Orbits of minor planets

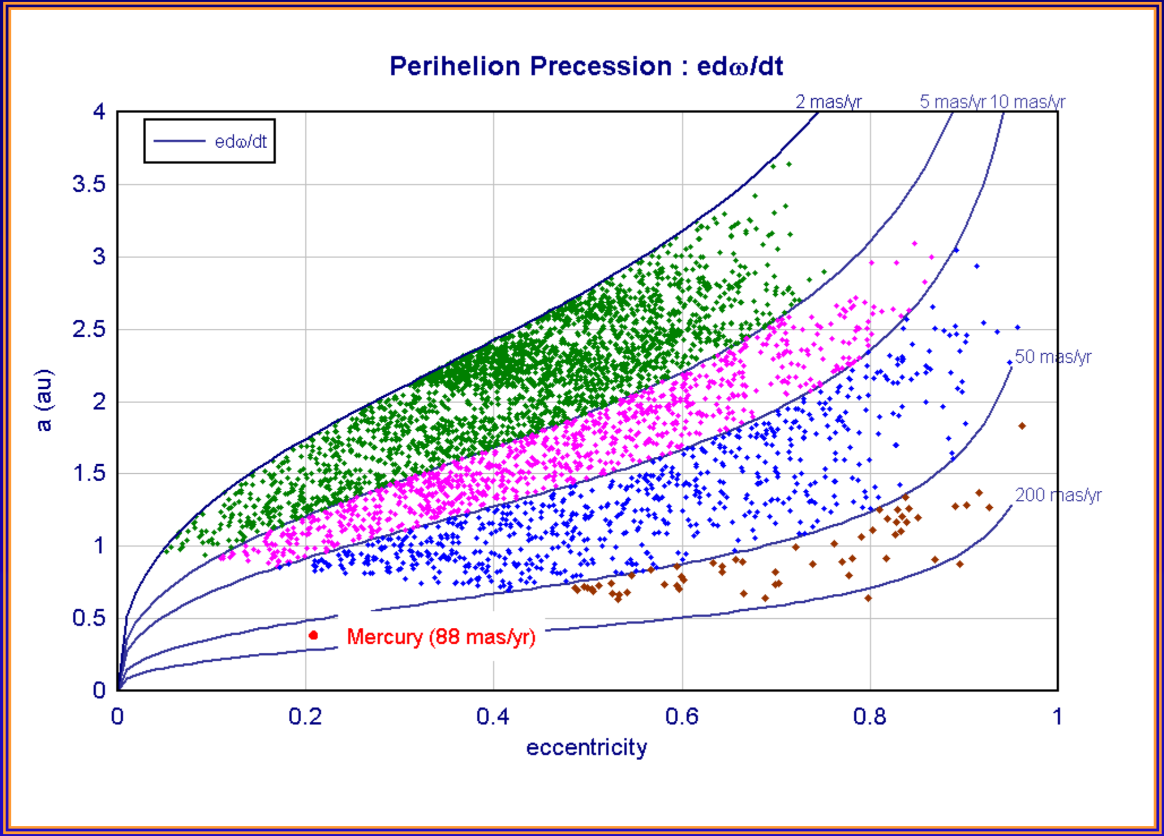
- About 300,000 planets observable with Gaia
- Accurate astrometry corrected for phase effect
- ~ 60 observations each over 5 years
- Accurate orbits determined with Gaia data
- Perihelion precession included in the dynamical model

$$\Delta\varpi = \frac{6\pi\lambda\, GM}{a(1-e^2)c^2} + \frac{3\pi\, J_2 R^2}{a^2(1-e^2)^2}$$

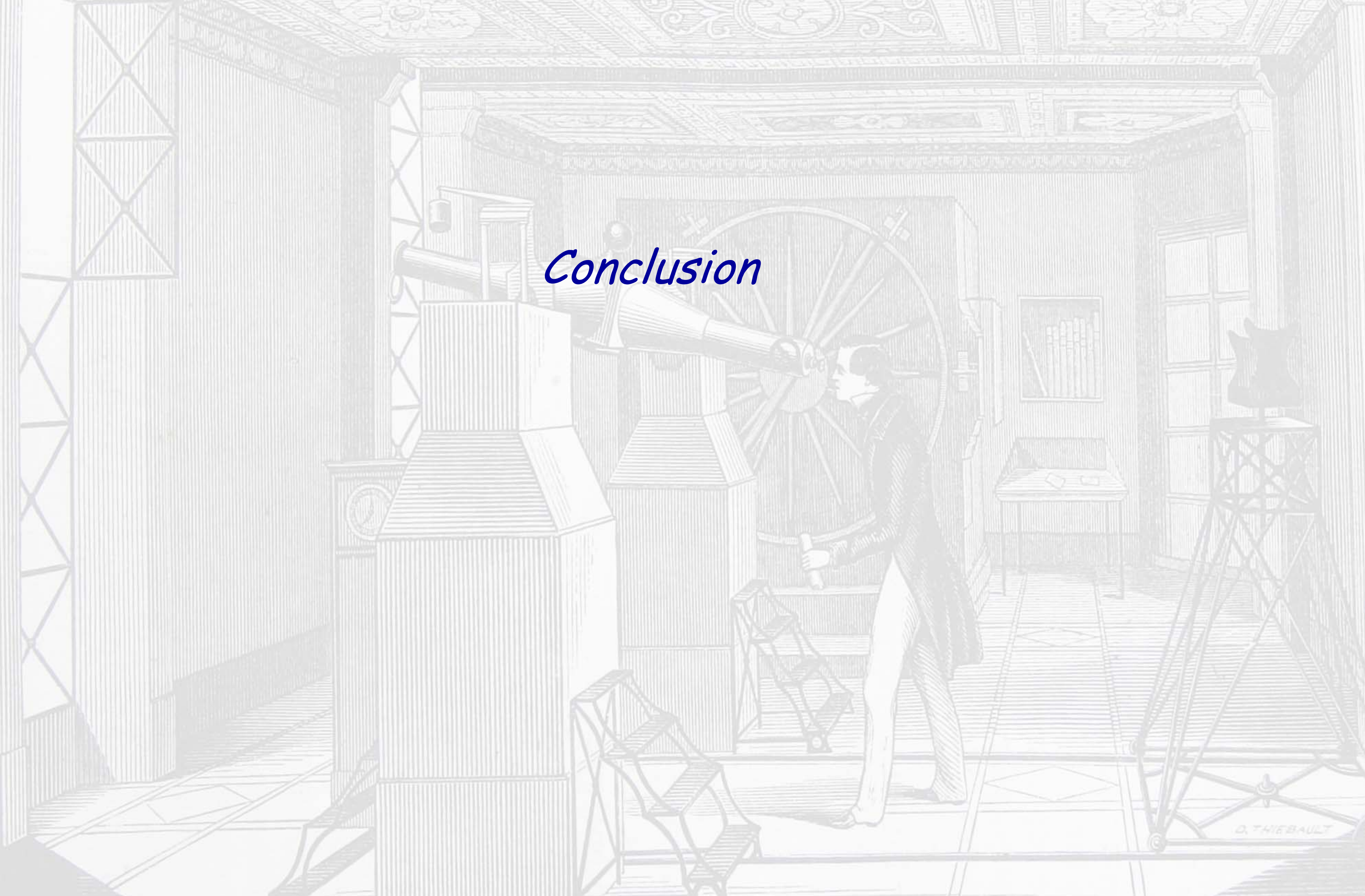
$$\lambda = (2\gamma - \beta + 2)/3$$

$$\dot{\omega} = \frac{38\lambda}{a^{5/2}(1-e^2)} + \frac{0.04(J_2/10^{-6})}{a^{7/2}(1-e^2)^2} \quad \text{mas/yr} \ \ (a \text{ in AU})$$

Perihelion precession : $e d\varpi/dt$



- Parameters fitted with Gaia
  - PPN β , Solar J2, $\dot{G}/G$
- Expected precision $\sigma(\beta) \sim 2 \times 10^{-4}$



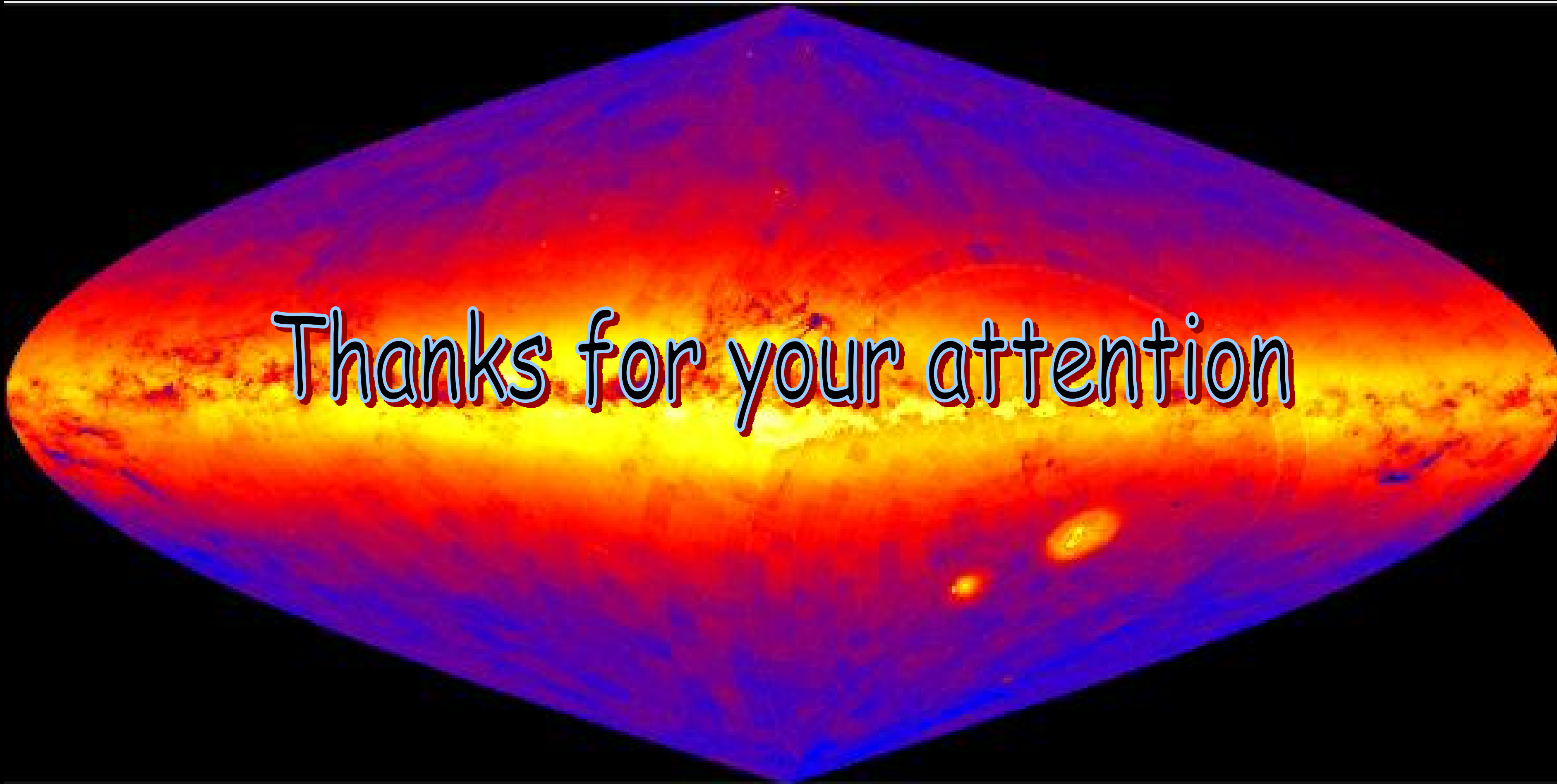
Conclusion

- Gaia launch is scheduled for September 2013
  - Observing mission to start 3 months after
  - Continuous scanning of the sky for 5 yrs

- Some intermediate releases to begin L + 2yrs

- Early results on γ at mid-mission

- Accurate reference frame at mission completion



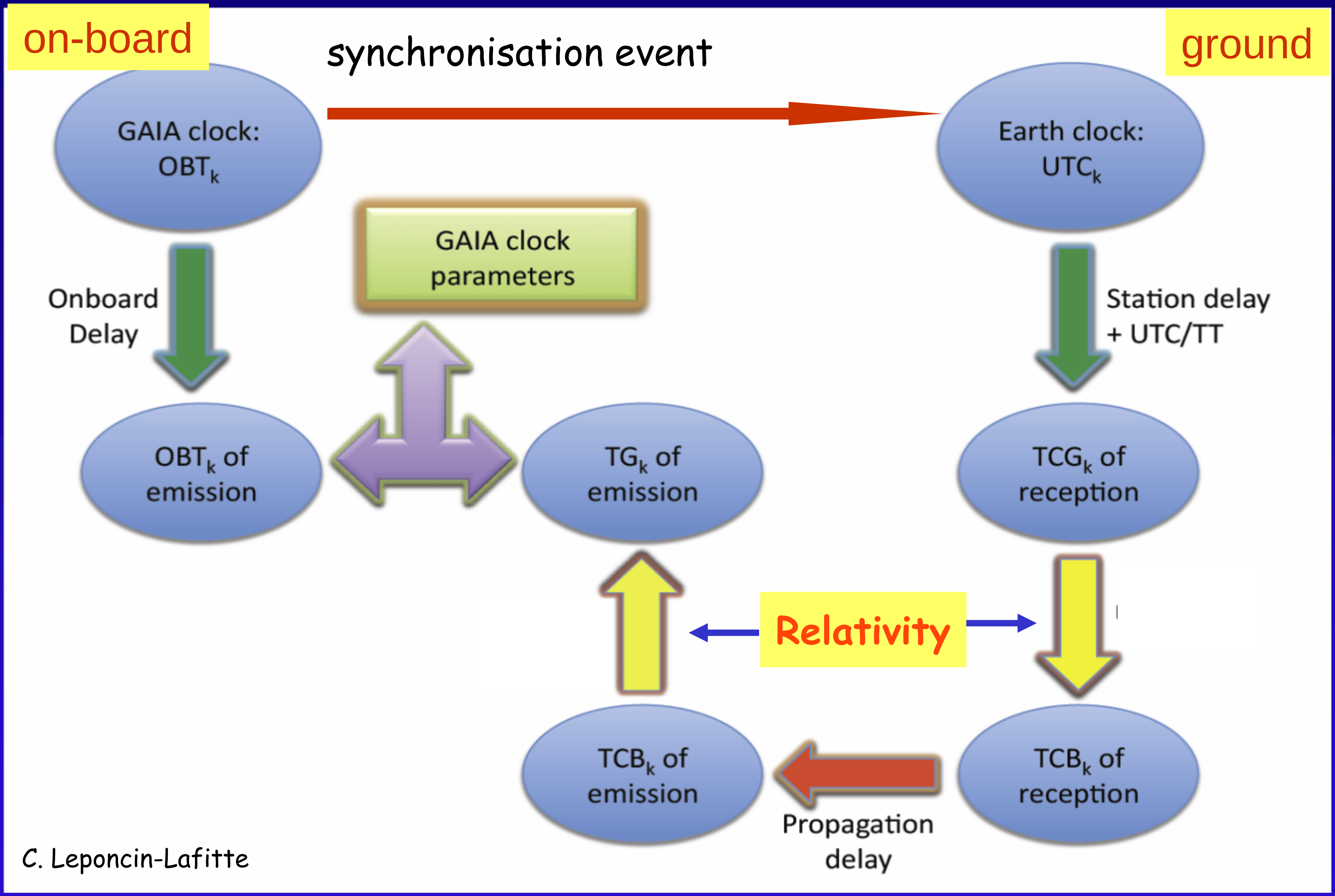
USNO A2

Time Metrology

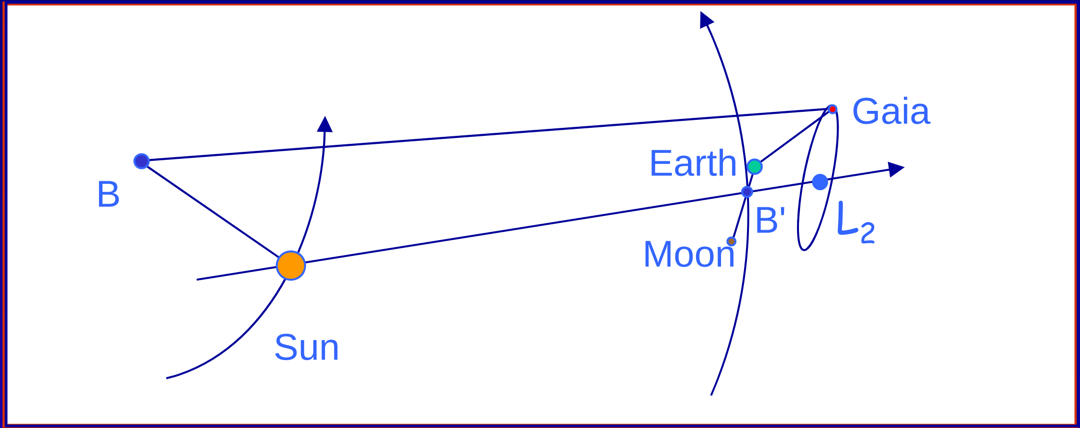
Accurate time metrology for Gaia

- Time stamping accuracy is high for Gaia
  - The requirements in the timing of on-board event to 1 μs
  - Clock stability over ~ 1 day of 10^{-12}
    - daily link with ground stations over ~8 h
  - One Rb clock on-board
- Objective: link between on-board time and astronomical time to 0.1 μs
  - Clock model and clock monitoring
    - relationship between OBT (clock delivered time) and TG (Gaia proper time)
  - Relativistic modeling of the time metrology chain
    - events timed in UTC, TT, TCG, TCB, TG
  - Details depends on Gaia position and velocity
- Synchronization sessions every day during visibility period
  - Synchronisation event triggered on-board every ~ s
  - real time downlink in current TM frame

Overall synchronisation scheme



- Orbit of Gaia around L2



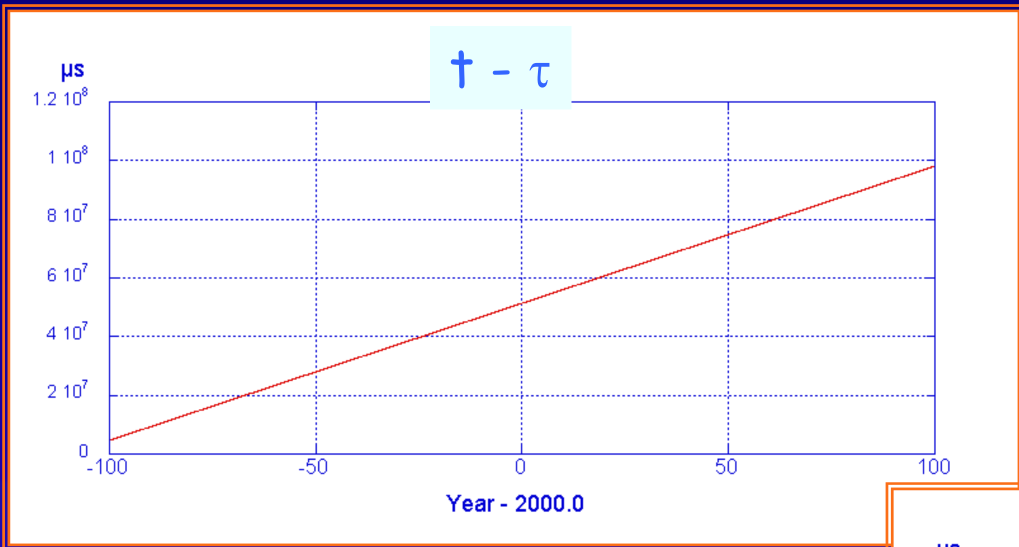
$$\frac{d\tau}{dt} \approx 1 - \frac{1}{c^2}\left[\frac{V^2}{2} + U\right] + \frac{1}{c^4}\left[-\frac{V^4}{8} - \frac{3}{2}V^2U + \frac{U^2}{2} + 4\mathbf{V.W}\right]$$

$$t - \tau = \int\left(\frac{V^2}{2c^2} + \frac{U}{c^2}\right)dt + \int\left(\frac{1}{8}\frac{V^4}{c^4} + \frac{3}{2}\frac{V^2U}{c^4} - \frac{U^2}{2c^4} - 4\mathbf{V.W}\right)dt$$

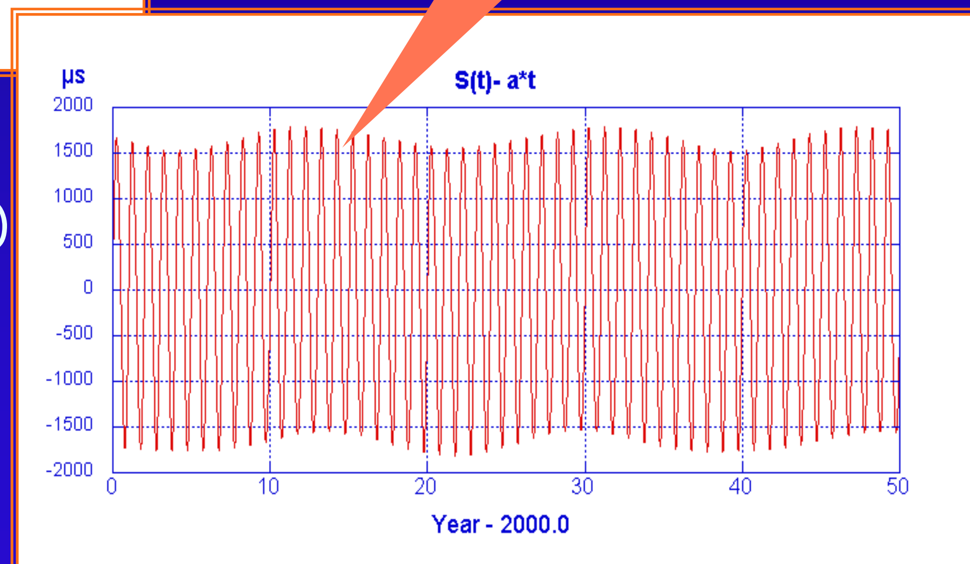
- Numerical quadrature + solar system ephemerides

Raw difference TCB – TG

- Makes sense over a long term

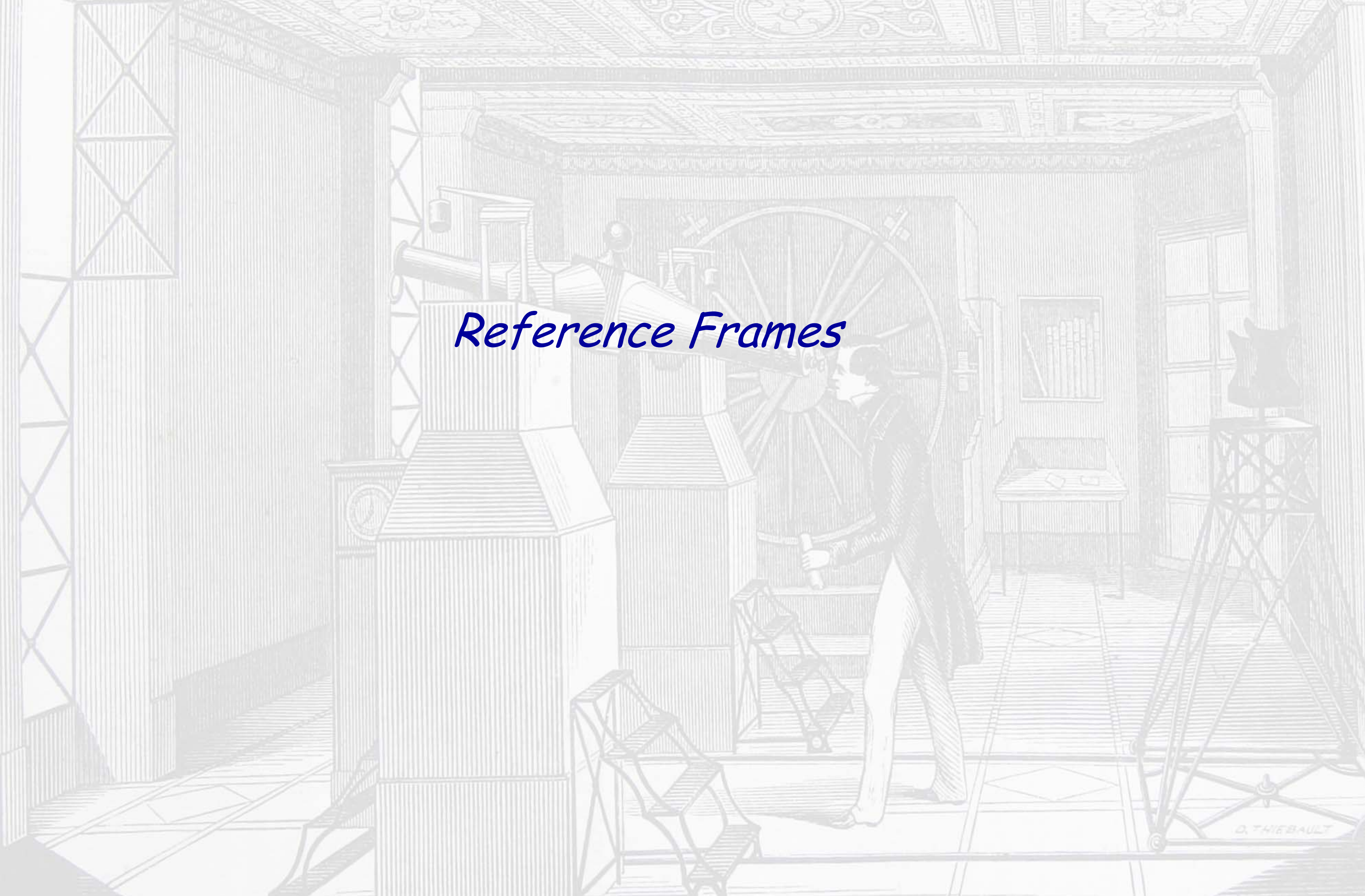


amplitude : 1.66 ms + modulation



Secular term :
TCB - τ = 1.481259949x10^{-8} (JD-J2010)

Mignard et al., 2006

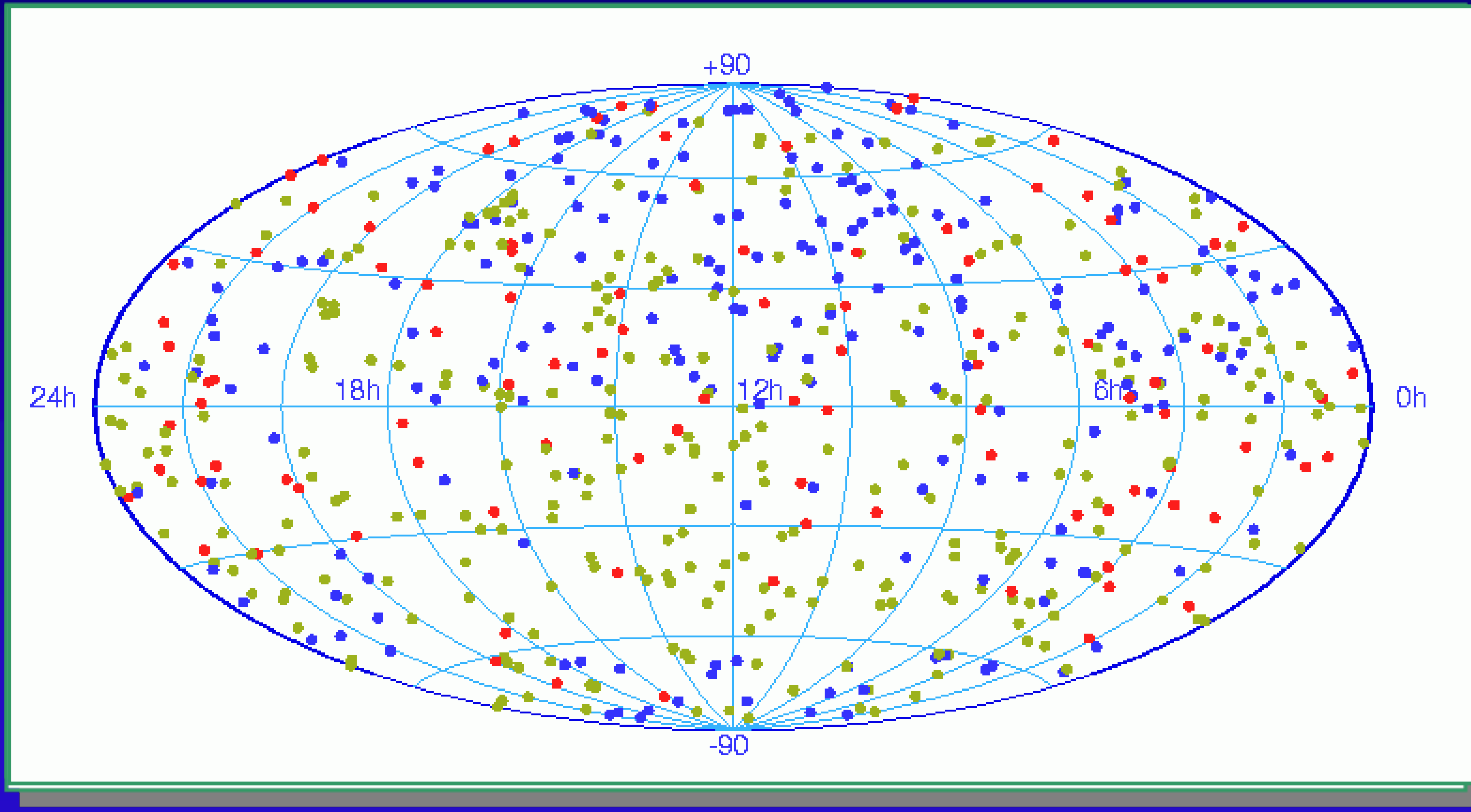


Reference Frames

Reference frame with GAIA

- Different from Hipparcos :
  - no link, primary frame determined
- Observe extragalactic sources in the visible
- There are plenty brighter than V =20
  - about 500,000 observable with Gaia
  - 20,000 $V < 18$
- Look for the anomalous proper motions to clean the sample
- The remaining set will display an overall spin
- Find ω and apply $-\omega$ everywhere
- The results will be referred to the best non-rotating frame
  - paradigm of the ICRS

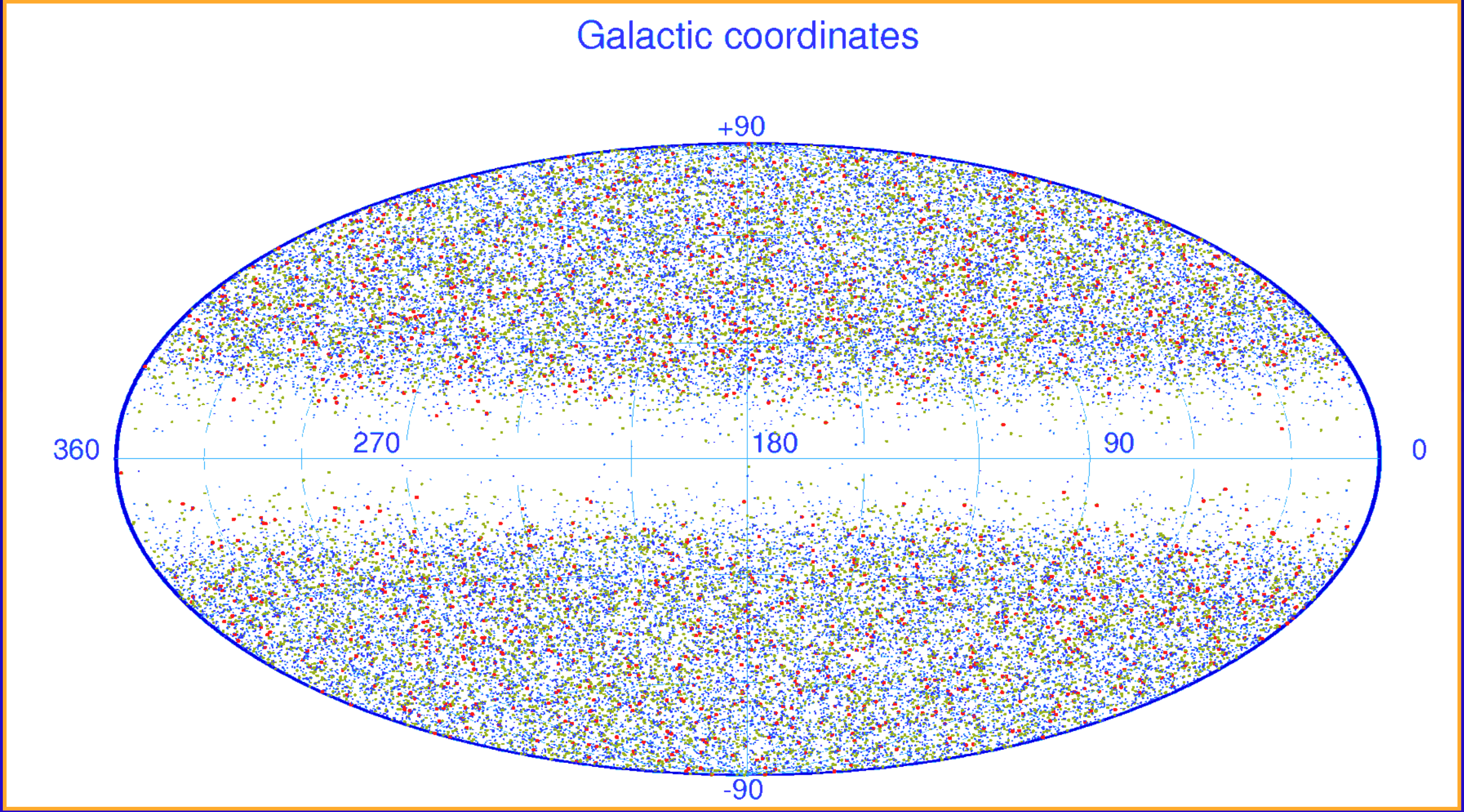
ICRF 1998



- Definition sources (212)
- Candidate sources (294)
- Other sources (102)

QSOs distribution with Gaia

- Based on the simulation used in the DPAC Universe model



● G < 18 ● 18< G < 19 ● 19< G < 20

Slezak & Mignard, 2007

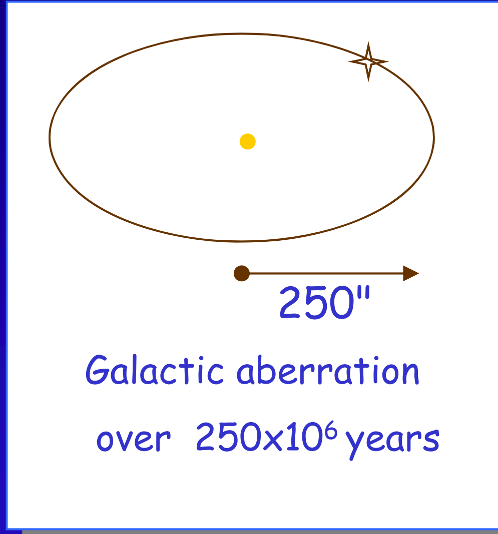
Transverse motion

- So far no systematic transverse motion detected
  - QSOs have fixed comoving coordinates
- If $V_t \sim H_0 D$ ===> $\mu \sim 10$ μas/yr
  - VLBI in 20 yrs with $s_{pos} \sim 1$ mas ===> $\mu < 50$ μas
  - but sub-mas structure instabilities
- Other sources :
  - microlensing $P = 10^{-6}$ (Belokurov) → only a handful
  - matter ejection, superluminous motion
  - Macrolensing $P = 10^{-2}$ (Mignard, 2003)
  - Variable galactic aberration (Kovalevsky, 2003)
  - Accelerated motion in the local group ?

- GAIA has the opportunity to test the ICRS paradigm
- QSO survey will be the largest available

More on QSOs and reference frame

- The solar system is in motion in the Galaxy, $V \sim 220$ km s^{-1}
  - constant aberration of ~ 250" for the QSO wrt to comoving frame
  - not detectable (principle of relativity)
  - $\delta\mathbf{u} = v/c$

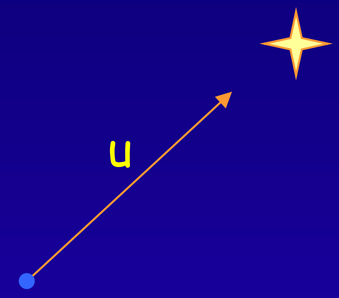


- But the solar motion is not uniform
  - ~ circular motion of radius $R \sim 8.5$ kpc and period 250×10^6 yrs
  - the aberration is then variable

$$\delta\mu = \frac{d(\delta\mathbf{u})}{dt} = \frac{\mathbf{\Gamma}}{c} = \frac{V^2}{cR} \approx 4\mu as / yr$$

- For any acceleration Γ of the SS wrt Quasars :

$$\frac{d\mathbf{u}}{dt} = \frac{\mathbf{\Gamma}}{c} - (\frac{\mathbf{\Gamma}}{c} \cdot \mathbf{u})\mathbf{u}$$



Observations

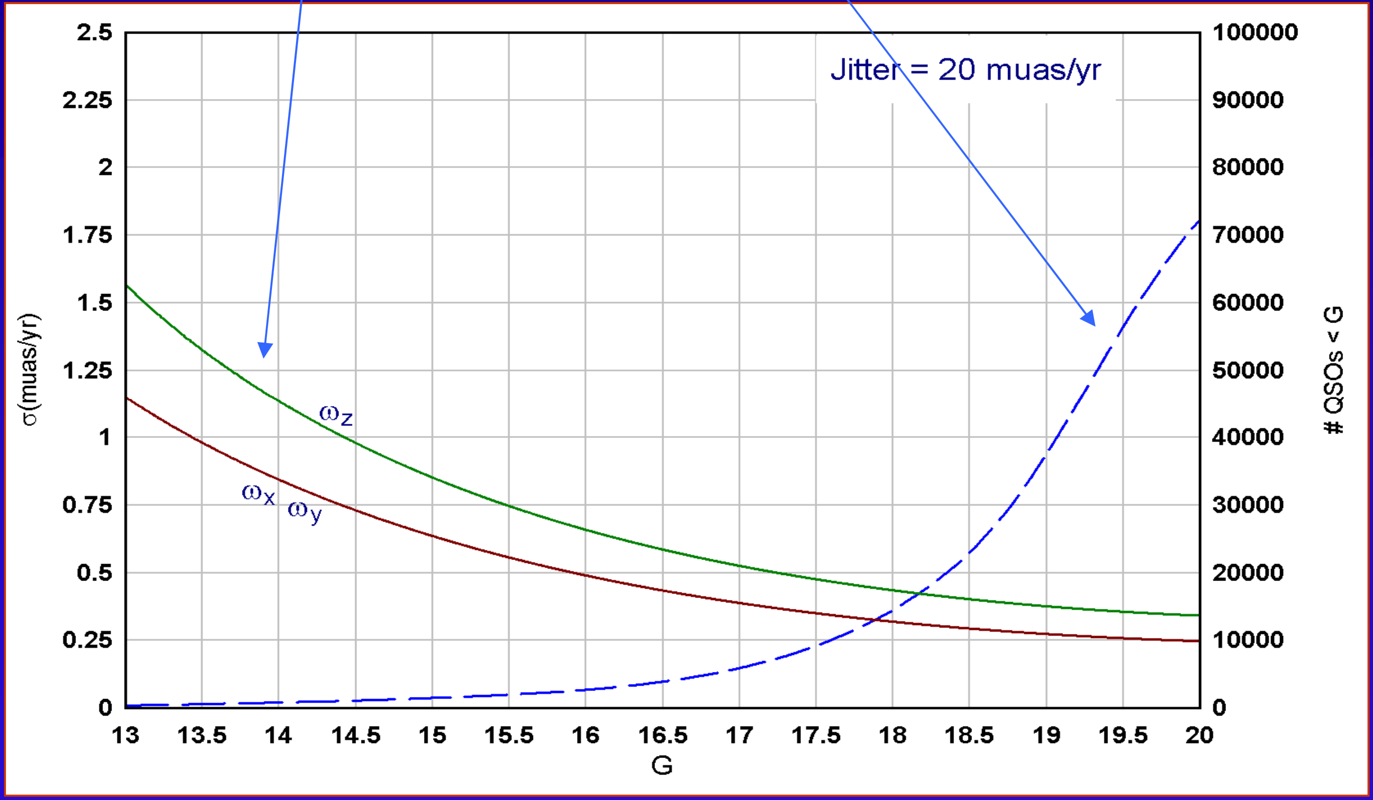
$$\mu_\alpha \cos\delta = -\frac{\Gamma_x}{c}\sin\alpha + \frac{\Gamma_y}{c}\cos\alpha$$

$$\mu_\delta = \frac{\Gamma_x}{c}\sin\delta\cos\alpha + \frac{\Gamma_y}{c}\sin\delta\sin\alpha - \frac{\Gamma_z}{c}\cos\delta$$

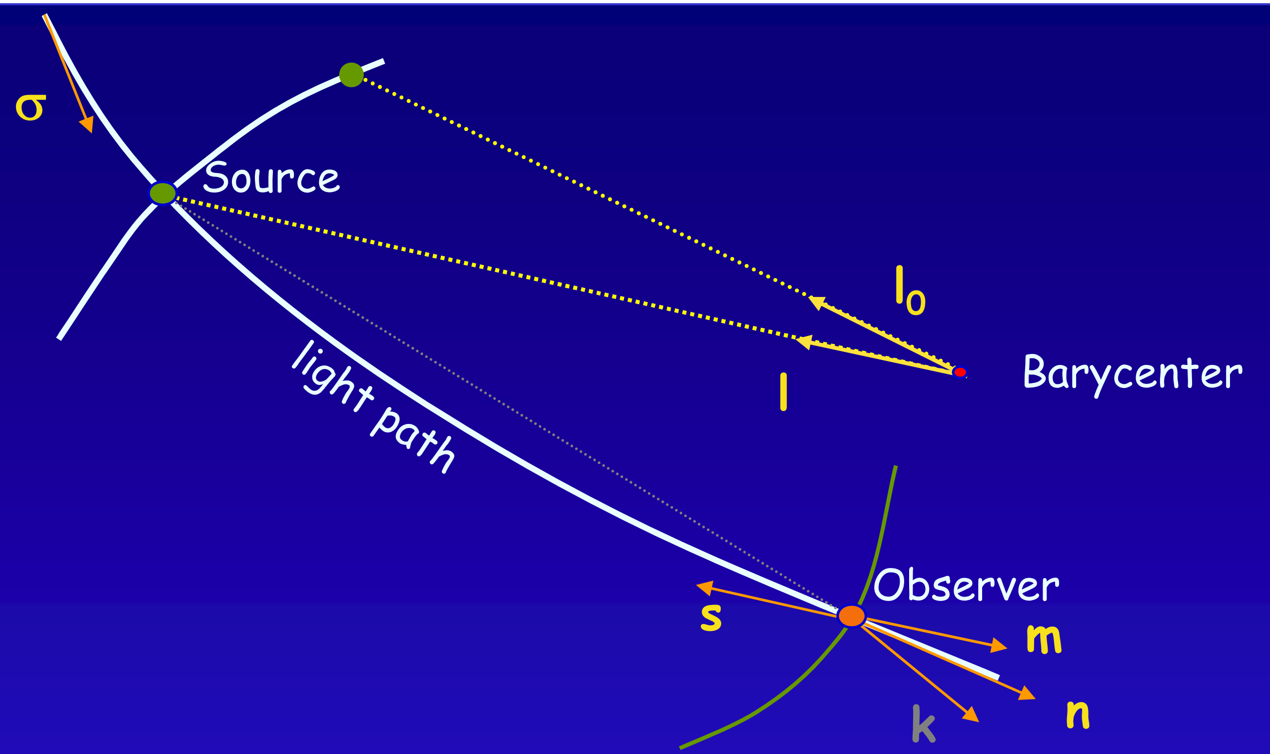
- Equations similar to global rotation.
- Precision of ~ 0.4 µas/an (2 prad/yr) on Γ/c
  = 0.2×10^{-10} m s^{-2} (γ Pionner/40)
- Galactic rotation (µ ~ 4 µas/yr)
- Acceleration of the Local Group→ CDM ?

Residual rotation of the Gaia frame

	today
• ICRF directly in the visible	radio
• Between 20,000 et 50,000 primary sources	212
• Inertiality < 0.3 muas/yr	50 muas/yr



Standard astrometric model: stars (from S. Klioner)



s : observed direction by an observer at **r** and at t_r

m : direction of the incoming photon as seen by the observer **m** = - **s**

n : same as **m** relative to the BCRS at **r** *aberration* ➔ velocity

σ : initial direction of the light path at t = - ∞ *light-deflection* ➔ position

k : allowing for the finite distance of the source

l : Direction of the source from the origin of the BCRS at t_r *parallax* ➔ position

$\mathbf{l}_0$: Same at some reference time $t_0 \sim t_r$ *proper motion*

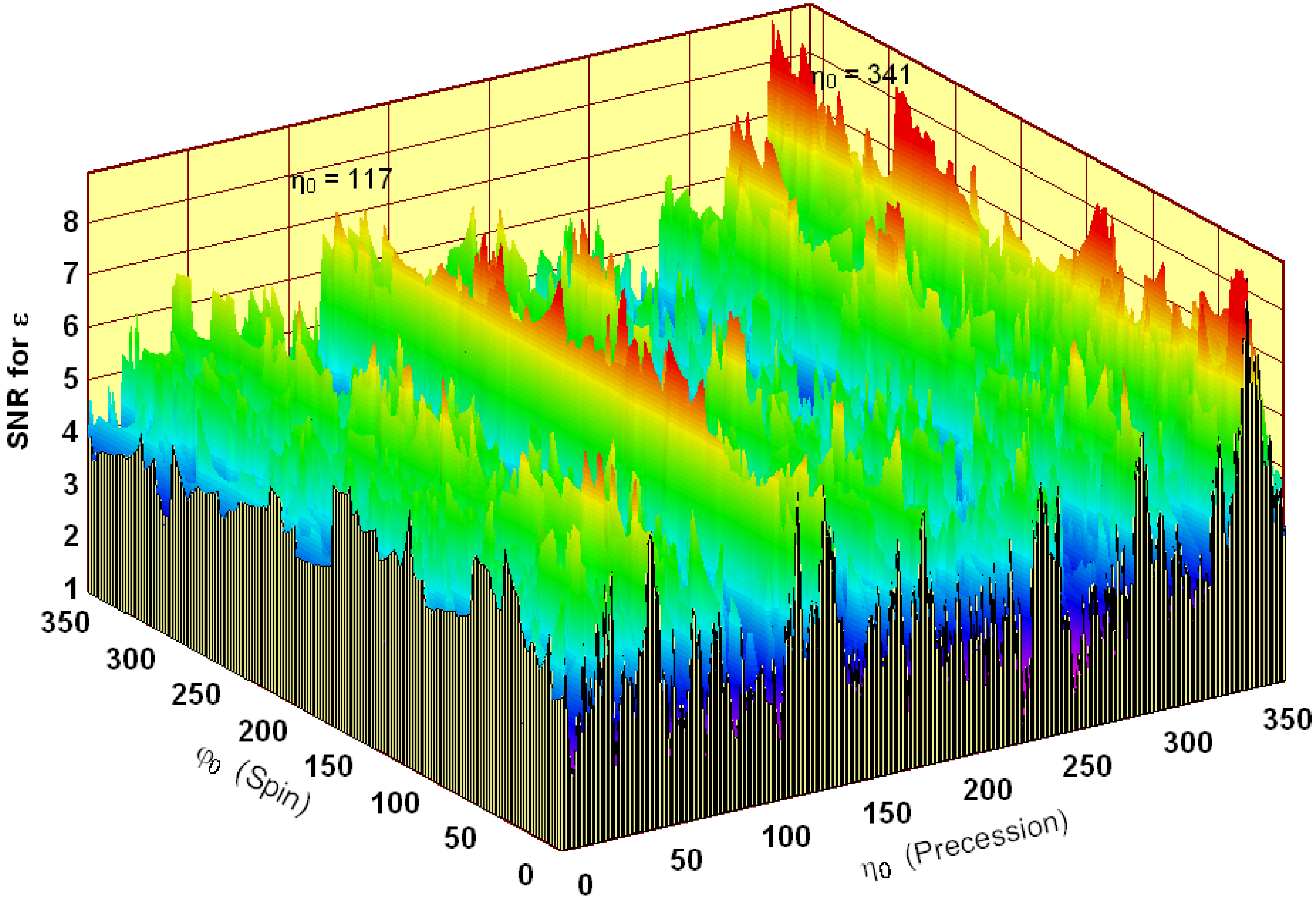
Solar light deflection with Gaia: order of magnitude

- Gaia will observe between 45° to 135° from the Sun
  - very large excursion
  - for all angles light deflection is very large compared to single observation accuracy
- One astrometric observation at 90° from the Sun gives:
  - deflection of 4 mas
  - astrometric measurement to 30 μas with Gaia on bright stars
    - 10^7 stars brighter than V = 13
  - potentially one single measurement tells something on γ to 0.01
  - Gaia will collect 10^8 – 10^9 such measurements
- Gaia is potentially sensitive to γ-1 deviations as small as 10^{-6}

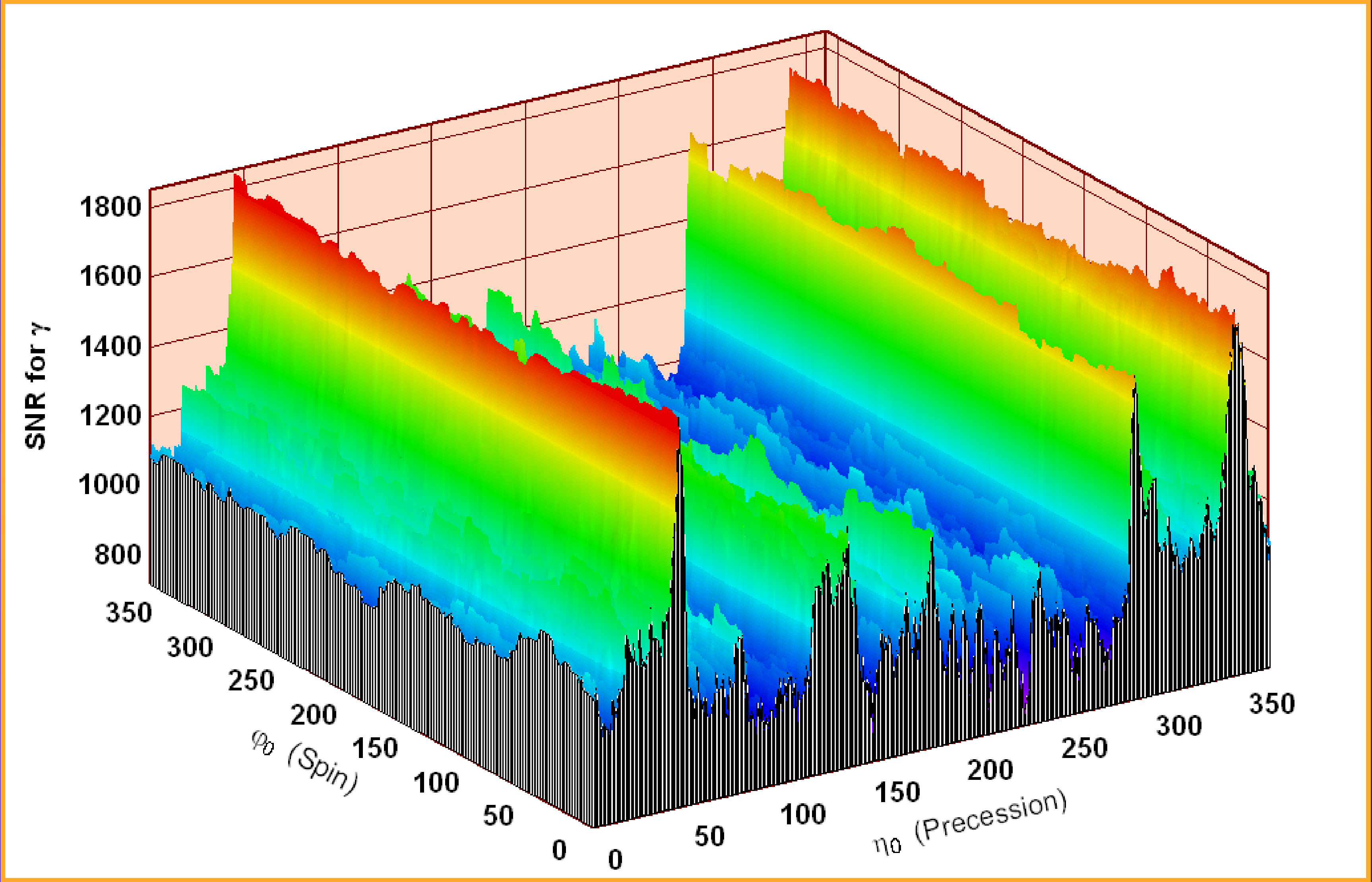
Optimisation of the scanning law

- There are two free parameters
  - Initial precession and spin phases
- Initially several conflicting mission constraints
  - but today Jupiter experiment has the lead
- Search for the conditions that yield observations of Jupiter with bright stars at very small distance
  - Full simulation of Jupiter transits
  - performance assessment cumulated over the transits
- Implementation
  - A standalone S/W will be used immediately after launch
  - Gaia orbit will be known for the whole mission
  - Optimal Scanning parameters will be selected

Performances on ε (quadrupole)



Performances on γ with Jupiter



Observations from Earth vicinity

Body	Monopole mas	χ $\delta\theta$ = 1 μas	Quadrupole μas	χ $\delta\theta$ = 1 μas
Sun	17,000	180°		
Mercury	0.083	0.15°		
Venus	0.49	4.5°		
Mars	0.12	0.4°		
Jupiter	16.3	90°	240	8 R_J
Saturn	5.8	17°	95	4 R_S
Uranus	2.1	1.2°	8	2 R_U
Neptune	2.5	0.9°	10	2 R_N

Other deflecting bodies

- Main experiment to be conducted with the Sun
- But observations will also take place close to the planets
  - Jupiter and Saturn to provide large signatures
  - deflection not correlated with parallax
  - independent measurements with each planet
- Much smaller number of observations
  - 70 observations with Jupiter in the FOV
  - 1.6 mas deflection at 10 Jupiter radii
  - only 100 to 1000 stars involved at each visibility period
  - $1/n^{1/2}$ statistics will not crash into systematic effect limit

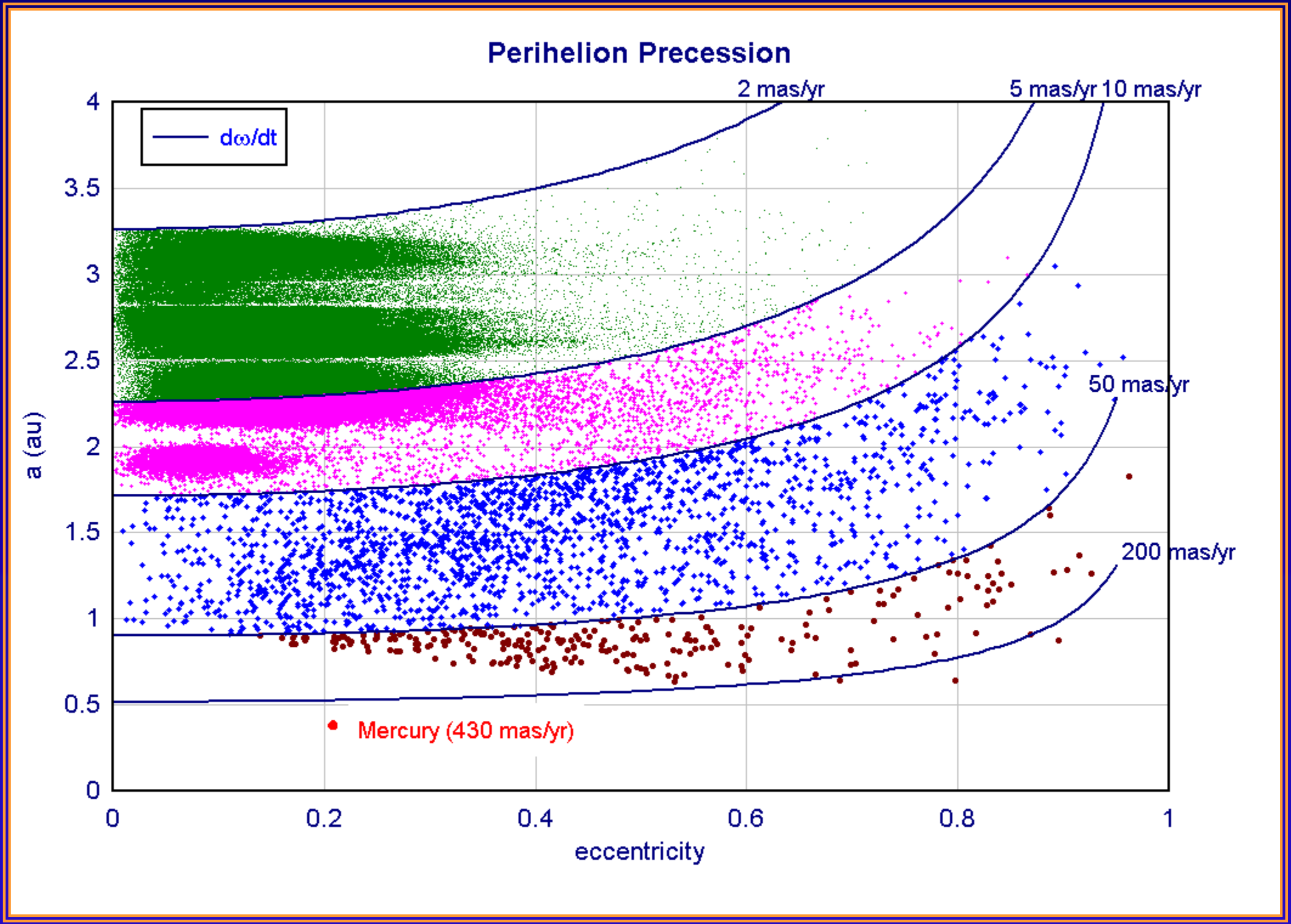
Jupiter Experiment

- Goal : detection of the light deflection by Jupiter
  - Monopole for a grazing ray ~ 16 mas – falls off as 1/r
  - Quadrupole for a grazing ray ~ 250 μas – falls off as $1/r^3$

- This is the only experiment planned with Gaia
  - i.e. some orbital or scanning parameters can be optimised
  - signal strength depends on very few favourable observations
  - Gaia orbit won't be known before few weeks days launch
- Principles and performances presented independently by
  - Crosta & Mignard (2006), Anglada & Klioner (2006)

- On-going activity and questions:
  - optimisation of the scanning law to increase the signal
  - how close can we observe from Jupiter ?
  - reduction principles (two methods investigated)

Perihelion precession : $d\varpi/dt$



- 20,000 minor planets in the plot
  - includes the largest NEOs
- Range of *a* and *e*

Summary

- Astrometry & Relativity

- Relativity relevance with Gaia
  - modeling
  - testing
  - accurate time metrology
  - frame building