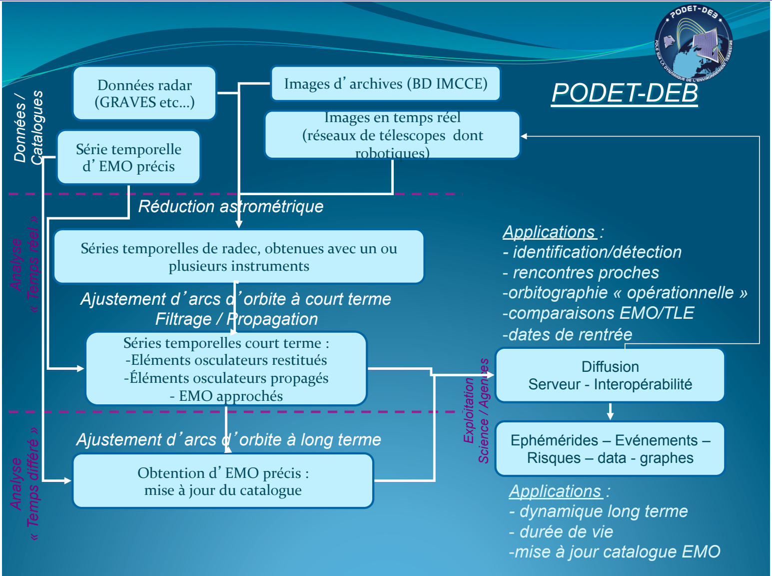
Orbit determination methods in view of the PODET project

Florent Deleflie[1], David Coulot[2], Romain Decosta[1], Basile van Cooten[3], Alain Vienne[1], Pascal Richard[4]

[1] Institut de Mécanique Céleste et de Calcul des Ephémérides, Paris et Université Lille 1
[2] IGN/LAREG, Université Paris Diderot, Paris
[3] ENPC Champs sur Marne
[4] Centre National d'Etudes Spatiales, Toulouse

7th June 2013

PODET and PODET-DEB projects



Overview and aim of the study

- Main goal: Fitting an orbit on tracking data
  - range: $\equiv$ SLR data
  - R.A., Decl.: $\equiv$ after astrometric reduction of images
- Usual methods and their main drawbacks
  - LS methods: "good enough" *a priori* values required
  - **not valid for uncatalogued objects !**
  - "Gauss, Laplace, Escobal...":
    - not valid in any geometrical configuration (singularities)
    - poor dynamical modelling (keplerian motion...)
    - very poor results for some cases, not helpful as *a priori*
- New approach based on a genetic algorithm
  - supposed to be valid in all dynamical configurations
  - can be used for **TSA**, or over a **couple of days**
  - without any *a priori* knowledge of the trajectory
  - 2 preliminary results shown: 1 SLR satellite, 1 GEO

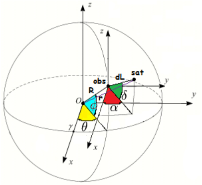
Orbital modelling and fit

- Equations of motion:

$$\frac{d^2\mathbf{r}}{dt^2} = \mathbf{F}(\mathbf{r}, \dot{\mathbf{r}}, t, \sigma)$$
$$\mathbf{r}(t_0) = \mathbf{r}_0 \qquad \dot{\mathbf{r}}(t_0) = \dot{\mathbf{r}}_0$$

- Estimation of initial conditions
  - Dedicated classical algorithm
  - Corrections to a "good-enough" *a priori*
  - Test of all possible configurations within a frame of dimension 6 (!)
  - Using of an algorithm selecting "good" initial conditions, and iterating

Laplace method



$$\mathbf{L}(t) \equiv \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix}$$

$$\mathbf{R}(t) \equiv -R_0 \begin{pmatrix} \cos\phi\cos\theta \\ \cos\phi\sin\theta \\ \sin\theta \end{pmatrix}$$

$$\mathbf{r} = d\mathbf{L} - \mathbf{R}$$

- Unknowns: $\mathbf{r}, \dot{\mathbf{r}}$ (and d, distance station-satellite)
- 3 observations: $\mathbf{L}(t_1), \mathbf{L}(t_2), \mathbf{L}(t_3)$
- Taylor expansion

$$\mathbf{L}(t_i) \simeq \mathbf{L}(t_0) + (t_i - t_0)\dot{\mathbf{L}}(t_0) + \frac{1}{2}(t_i - t_0)^2\ddot{\mathbf{L}}(t_0)$$

with $t_0 = \frac{1}{3}(t_1 + t_2 + t_3)$: $\mathbf{L}(t_0), \dot{\mathbf{L}}(t_0), \ddot{\mathbf{L}}(t_0)$ known

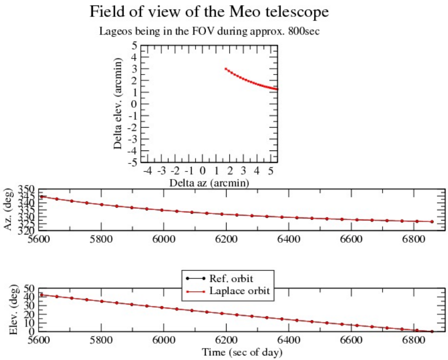
- at t_0: $\ddot{\mathbf{r}} = \ddot{d}\mathbf{L} + d\ddot{\mathbf{L}} + 2\dot{d}\dot{\mathbf{L}} - \ddot{\mathbf{R}}$ where $\ddot{\mathbf{r}} = -\mu\frac{\mathbf{r}}{r^3}$

  Shape of the solution for d: $d = \frac{A}{r^3} + B$

  Shape of the solution for $\dot{d}$: $\dot{d} = \mu\frac{A_2}{r^3} + \frac{B_2}{r^3}$
- after algebraic steps:
  - r obtained from $r^2 = d^2 + R^2 - 2\mathbf{L}.\mathbf{R}$
    Polynom of deg. 8 and then $\mathbf{r}(t_0)$, $\dot{\mathbf{r}}(t_0)$
  - but some singularities exist

Improvement of the Laplace method

$$\ddot{\vec{r}} = \mu \left(-1 + \frac{3}{2} J_2 \left(\frac{R_\oplus}{r} \right)^2 (3 \sin^2 \phi - 1) \right) \frac{\vec{r}}{r^3} \quad \text{(Laas-Bourez, et al., 2012)}$$



Multi-Objective Genetic Algorithms (MOGA)
How do genetic algorithms work ?

- Find the **set of initial conditions** (keplerian elements) that minimizes criteria at hand. These criteria are defined as functions of the initial conditions.
- evaluation for a **set of vectors** of possible initial conditions (implicit parallelism).
- Between two successive iterations, some vectors are replaced by others and the best are archived. The evolution of the set of initial conditions is governed by **mutations** (random small changes in vectors of possible initial conditions) and **crossover** (mix two vectors of possible initial conditions).
- At the end of the iteration procedure, a set of solutions is supplied.
- MOGA used here: ϵ-MOEA [Deb et al., 2003] Deb, K., M. Mohan, S. Mishra (2003) A Fast Multi-objective Evolutionary Algorithm for Finding Well-Spread Pareto-Optimal Solutions. KanGAL Report Number 2003002.

Orbital modeling

- An analytical approach
  - Huge number of different cases to be tested: very quick computations **(required)**
  - Main perturbations accounted for (required), at least J_2
  - Valid for **all dynamical configurations**: written in a set of equinoctial elements

$$\mathbf{E} \equiv \left(a, \Omega+\omega+M, e\cos(\Omega+\omega), e\sin(\Omega+\omega), \sin\frac{i}{2}\cos\Omega, \sin\frac{i}{2}\sin\Omega\right)$$

- Shape of the solution

$$\mathbf{E}(t) = \bar{\mathbf{E}}(t) + \mathcal{L}(\bar{\mathbf{E}})\frac{\partial W}{\partial \bar{\mathbf{E}}}(\bar{\mathbf{E}}(t))$$

- Initial conditions
  - Mean initial condition: $\bar{\mathbf{E}}(t_0)$ (6 elements adjusted)
  - Osculating initial condition: $\mathbf{E}(t_0) = \bar{\mathbf{E}}(t_0) + \mathcal{L}(\bar{\mathbf{E}})\frac{\partial W}{\partial \bar{\mathbf{E}}}(\bar{\mathbf{E}}(t_0))$

Analytical approach

- mean elements $\bar{\mathbf{E}}$: long periodic and secular effects
  - induced mainly by zonal coefficients on the angles
  - main effect: J_2

$$\Delta\dot{\Omega} = -\frac{3}{2}\left(\frac{R_e}{a}\right)^2 nJ_2 \frac{\cos i}{(1-e^2)^2} \quad \Delta\dot{\omega} = -\frac{3}{4}\left(\frac{R_e}{a}\right)^2 nJ_2 \frac{1-5(\cos i)^2}{(1-e^2)^2}$$

$$\Delta\dot{M} = -\frac{3}{4}\left(\frac{R_e}{a}\right)^2 nJ_2 \frac{1-3(\cos i)^2}{(1-e^2)^{3/2}}$$

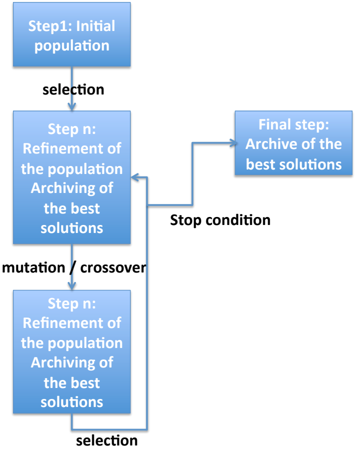
- short periodic part described through the so-called "generator" W:

$$\begin{aligned} W_2 &= -\mu J_2 \frac{1}{\bar{n}a}\left(\frac{R_0}{a}\right)^2 \frac{1}{\eta^3}\Bigg(\left(\frac{1}{2}-\frac{3}{4}\sin^2 i\right)(v-u+e\sin u+e\sin v) \\ &+ 4\cos^2\frac{i}{2}\sin\frac{i}{2}\cos(\omega+v)\sin\frac{i}{2}\sin(\omega+v)e\cos v \\ &+ 3\cos^2\frac{i}{2}\sin\frac{i}{2}\cos(\omega+v)\sin\frac{i}{2}\sin(\omega+v) \\ &- \cos^2\frac{i}{2}\left(\left(\sin\frac{i}{2}\cos(\omega+v)\right)^2-\left(\sin\frac{i}{2}\sin(\omega+v)\right)^2\right)e\sin v\Bigg) \end{aligned}$$

Example: $a(t) = \bar{a} + \frac{3}{2} J_2 \frac{R_e}{\bar{a}} \sin^2 \bar{i} \cos 2(\bar{\omega} + \bar{M})$

An iteration of the MOGA

- The MOGA provides a vector of initial conditions.
- Initial conditions used to compute an analytical orbit.
- Analytical orbit used to compute predicted measurements.
  - range
  - RADEC
  - ...
- Predicted measurements compared to the true data
  - Norm of the differences used as a criterion to minimize.
  - LS cost function



Preliminary step: Parameterization of the MOGA

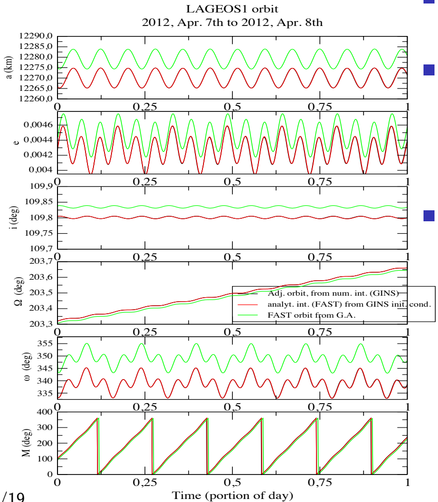
- GA
  - Initialization: population of 400 chromosomes
    - sma $a \in [12200 \;\; 15600]$ km for Lageos, $a \in [40000 \;\; 45000]$ km for Telecom-2D
    - eccentricity $\in [0 \;\; 0.1]$ (to save CPU time)
    - inclination $\in [0 \;\; 180^\circ[$
    - angles Ω, ω $M \in [0 \; 360^\circ[$
  - Mutation. $p = 0.9$
  - Crossover. $p = 0.16667 = 1/6$
  - Stop condition: end after 500 000 iterations (total CPU: 30h)
- Boundaring the intervals in a realistic way
  - Example: Choice of an *a priori* s.m.a. (based on the **observed period**)
    1. Circular orbit hypothesis
    2. and then possible changes to evaluate r instead of a during a pass (large eccentricities accounted for)
  - Use of admissible regions

Test case1: orbit determination of the Lageos SLR sat.

- Very precise orbit from the ILRS network available at the level of $\approx 1cm$
- This case:
  - Eight days of SLR data (MJD 56 024 56 031 included, April 2012)
  - 29 tracking stations (2 034 measurements).
- Particularities of the test
  - Search not only for the best vector of initial conditions,
  - Additionally search for an optimal sub-network of SLR stations
  - Two objectives are considered:
    - the RMS of differences between predicted measurements and the real data (**to be minimized**)
    - the number of SLR stations involved in the computation (**to be maximized**).

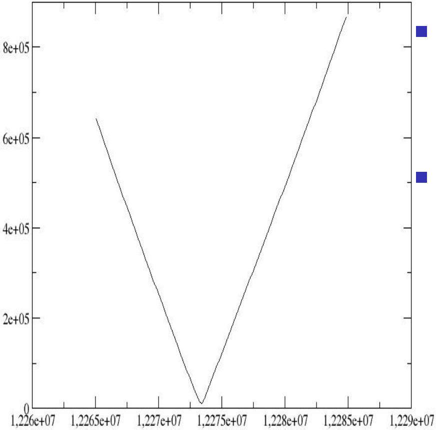
      Without it, the MOGA would probably tend to use a minimal set of stations to get better results regarding the initial conditions.

Test case1: results



- Reference orbit (gins s/w):
  RMS of differences is 2.15cm
- Adjusted orbit
  - $a_0 = 12274.840$ (12270.009) km: $\Delta a = 4.831$km
  - $e_0 = 0.004408$ (0.004261): $\Delta e = 0.000147$
  - $I_0 = 109.839(109.801)^{\circ}$: $\Delta I = 0.038^{\circ}$
  - $\Omega_0 = 203.306\ (203.323)^{\circ}$:$\Delta\Omega = 0.017^{\circ}$
  - $\omega_0 + M_0 = 76.538\ (76.616)^{\circ}$: $\Delta(\omega + M) = 0.078^{\circ}$
- Interpretations
  - Analytical model suitable for the dynamics
  - G.A. have a good capability over the global scale. For better results:
    - Change of the GA parameters
    - LS adjustment

Test case1: validation of the dynamical model



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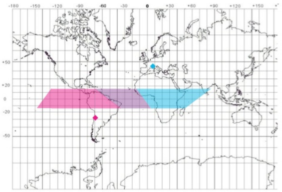
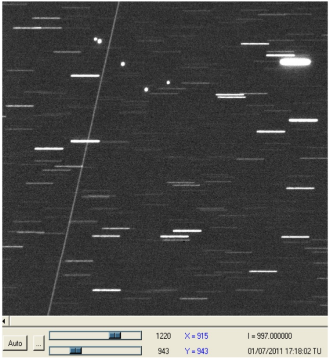
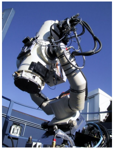
- Sensitivity analysis
  - $a_0^{\text{diff min}} = 12273.440$ km *(cf Figure: a (x-axis, m), rms (y-axis, m))*
  - $I_0^{\text{diff min}} = 110.639^\circ$
  - $\Omega_0^{\text{diff min}} = 201.261^\circ$

Test case2: Telecom2D

- **The TAROT network**
- Calern (Grasse,F) ; la Silla (Chile)
- large FOV (1.86° × 1.86°)
- aperture: 250 mm
- automatic
- Upper magnitude: 15 (GEO)
- Measur. accuracy: 700 m (GEO)

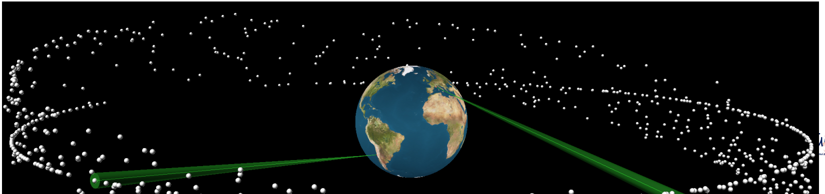




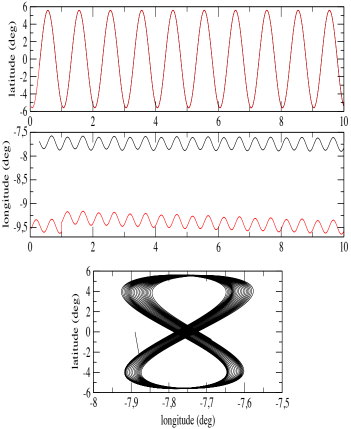


Test case2: Telecom2D, the data

- Data set:
  - nine days of angular data (MJD 56 147 56 156 included, Apr. 2012) from the two TAROT-telescopes
  - 86 measurements (27 for Chile and 59 for France).
- The MOGA searches for the best vector of initial conditions
- Two objectives are considered (both **to be minimized**), the RMS of differences between predicted measurements and the real data for:
  - elevation
  - azimut



Test case2, Telecom2D: results



- Reference orbit (Romance s/w)
- The final archive provided by the MOGA. For the best solution regarding both RMS of differences, the RMS values of differences are:
  - 0.0485° for elevation
  - 0.0742° for azimut
- Sensitivity analysis: Same concl. as for LAG1
- Adjusted orbit (orbital elements):
  - $a_0 = 42171.560$ (42165.980) km: $\Delta a = 5.580$km
  - $e_0 = 0.0000923$ (0.0001906): $\Delta e = 0.0000983$
  - $I_0 = 5.578(5.583)^{\circ}$: $\Delta I = 0.005^{\circ}$
  - $\Omega_0 = 62.897$ $(61.480)^{\circ}$:$\Delta\Omega = 1.417^{\circ}$
  - $\omega_0 + M_0 = 257.180$ $(256.934)^{\circ}$: $\Delta(\omega + M) = 0.246^{\circ}$
- Interpretations
  - Analytical model suitable
  - but to be improved (zonal+tesseral parameters)

Next step: admissible regions

$$\mathcal{E}_E(\rho, \dot{\rho}) = \frac{1}{2}\|\dot{P}\|^2 - \frac{\mu_E}{\|P\|},$$

(Tommei et al., 2007)

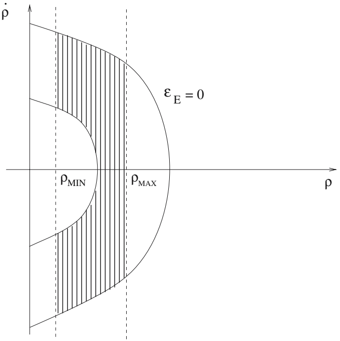


Fig. 4 Admissible region for a space debris $\mathcal{D}$ taking into account the condition (A′) instead of (A)

Example of admissible region from optical data

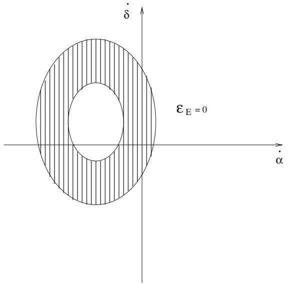


Fig. 6 Admissible region for a space debris $\mathcal{D}$ from radar data when condition (11) is satisfied a
taking into account the observation: $\mathcal{E}_E = 0$ is the curve of zero geocentric energy and it is an ellips

Example of admissible region from radar/range data

Conclusions and prospects

- Already done
  - Combination of G.A. and a modelling of the orbital motion
  - Different kinds of data (that can be combined)
  - Goal reached: determining from scratch the order of the initial values of an orbital arc
- G.A. refinements
  - Optimization on the choice of parameters
  - Implementing a better stop condition (to reduce CPU time)
- Analytical modelling enhancements
  - Tesseral parameters
  - Atmospheric drag for longer arcs ?
  - ... but impact on the total CPU time
- Multiplying the tests
  - Really testing the capabilities for TSA...
  - .. and in downgraded conditions (data sparse in time)