Resonances due to third body perturbations in the dynamics of MEOs

L. Stefanelli, G. Metris

Geoazur, Université de Nice Sophia-Antipolis,

CNRS (UMR 7329),

Observatoire de la Côte d'Azur

Introduction: long term dynamics of MEOs

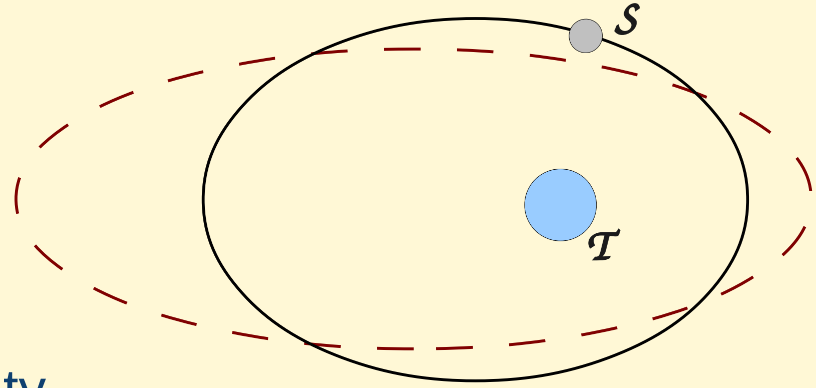
⋆ **Context:** space security

- orbits of positionning satellites constellations (GNSS): GPS, Galileo ...
- parking orbits for satellites at end of life

MEO: Medium Earth Orbits, altitude aboute 20 000 km.

Long term: at least 100 years (or more).

Main problem: growth of the eccentricity $\Rightarrow$ can leads dangerous orbits intersections



Resonances: potential source of instability

Lunisolar: the resonance source is a third body (Sun or Moon)

$$\Downarrow$$

⋆ **Aim:** study the interaction between resonances and MEO orbits $\Rightarrow$ analytical and numerical studies of resonances, mainly those close to Galileo orbits

Summary

Resonances:

- the resonance source is a third body gravity potential
- involve combinations of frequencies of the satellite and of the Sun/Moon:
  - the frequencies related to Sun/Moon are known
  - the frequencies of the satellite are determined by initial conditions and *perturbations*.

Main *perturbations* generating secular variations of the satellite frequencies:

- Earth gravitational field, in particular the quadrupole moment J_2
- Third-body perturbations: Sun and/or Moon
- The relative importance changes with the altitude: for MEOs the third body perturbations may become non negligible.

Outline:

- Identification of the resonances in presence of J_2 secular effects (cfr. Hughes 1980) and contribution of the third body secular effects (new).
- Analytical and numerical studies of resonances of the form $\alpha\dot{\omega} + \beta\dot{\Omega} \approx 0$, in particular $2\dot{\omega} + \dot{\Omega} \approx 0$ using a simplified model in three steps.
- Study of the eccentricity.

The Sun gravity potential as a source of resonances

$$\mathcal{P}_\odot = \mu_\odot \sum_{n=2}^{\infty} \frac{a^n}{a_\odot^{n+1}} \sum_{m=0}^{n} (2-\delta_{0m}) \frac{(n-m)!}{(n+m)!} \times \sum_{p=0}^{n} \bar{F}_{n,m,p}(i) \sum_{h=0}^{n} \bar{F}_{n,m,h}(\epsilon) \sum_{q=-\infty}^{\infty} G_{n,p,q}(e) \sum_{j=-\infty}^{\infty} H_{n,p,j}(e_\odot) \times \cos\psi$$

$$\psi = \alpha\omega + \beta\Omega + \zeta M + \eta\omega_\odot + \beta\Omega_\odot + \gamma M_\odot$$

$\alpha = n - 2p$, $\beta = m$, $\zeta = n - 2p + q$, $\eta = n - 2h$, $\gamma = n - 2h - j$;.

$a, e, i, \Omega, \omega, M$ orbital elements of the satellite

$\mu_\odot = Gm_\odot$, ϵ obliquity of the ecliptic

$a_\odot, e_\odot, \Omega_\odot, \omega_\odot, M_\odot$ orbital elements of the Sun

$\bar{F}_{n,m,p}$: inclination functions, $G_{n,p,q}$, $H_{n,p,q}$: Hansen coefficients (see Hughes 1980)

Resonance:

$$\dot{\psi} \approx 0$$

Here we study:

$$\alpha\dot{\omega} + \beta\dot{\Omega} \approx 0$$

Case of the Moon

$$\mathcal{P}_{☾} = \mu_{☾} \sum_{n=2}^{\infty} \frac{a^n}{a_{☾}^{n+1}} \sum_{m=0}^{n} \sum_{s=0}^{n} (-1)^m \frac{2-\delta_{0m}}{2} \frac{(n-s)!}{(n+s)!}$$

$$\times \sum_{p=0}^{n} \bar{F}_{n,m,p}(i) \sum_{h=0}^{n} \bar{F}_{n,m,h}(i_{☾}) \sum_{q=-\infty}^{\infty} G_{n,p,q}(e) \sum_{j=-\infty}^{\infty} H_{n,p,j}(e_{☾})$$

$$\times \left[(-1)^{n-s} U_{n,m,-s}(\epsilon) \cos\psi_+ + U_{n,m,s}(\epsilon) \cos\psi_-\right]$$

$$\psi_{\pm} = \alpha\omega + \beta\Omega + \zeta M \pm \eta\omega_{☾} \pm s\Omega_{☾} \pm \gamma M_{☾}$$

$\alpha = n - 2p$, $\beta = m$, $\zeta = n - 2p + q$, $\eta = n - 2h$, $\gamma = n - 2h - j$.

$a, e, i, \Omega, \omega, M$ orbital elements of the satellite

$\mu_{☾} = Gm_{☾}$, ϵ obliquity of the ecliptic

$a_{☾}$, $e_{☾}$, $i_{☾}$, $\Omega_{☾}$, $\omega_{☾}$, $M_{☾}$ orbital elements of the Moon

$\bar{F}_{n,m,p}$: inclination functions, $G_{n,p,q}$, $H_{n,p,q}$: Hansen coefficients (see Hughes 1980)

The J_2 perturbation - secular effets

$$\dot{\omega}_{J_2} = -c_\omega \left(\frac{R_\oplus}{a}\right)^{7/2} \frac{1-5\cos^2 i}{(1-e^2)^2}\,, \qquad \dot{\Omega}_{J_2} = -2c_\omega \left(\frac{R_\oplus}{a}\right)^{7/2} \frac{\cos i}{(1-e^2)^2}$$

$$c_\omega \equiv \frac{3}{4} J_2 \frac{\sqrt{Gm_\oplus}}{R_\oplus^{3/2}}\,,$$

Resonance: $\alpha\dot{\omega} + \beta\dot{\Omega} \approx 0 \;\Rightarrow\; c_\omega\, Y(a,e)\, P(\cos i) \approx 0 \;\Leftrightarrow\; P(\cos i) = 0$

$$Y(a,e) \equiv \left(\frac{R_\oplus}{a}\right)^{7/2} (1-e^2)^{-2}\,, \quad P(\cos i) \equiv \alpha\left(5\cos^2 i - 1\right) - 2\beta\cos i\,.$$

- The existence of resonances depends only on inclination: the zeros of $P(\cos i)$ “correspond” to the resonant inclinations.
- For each pair (α, β) there are two possible resonant inclinations.

Example: $\alpha = 2, \beta = 1 \;\rightarrow\; 2\dot{\omega} + \dot{\Omega} \approx 0$

$\cos i_1 = \;\; 0.5582 \qquad \Rightarrow\; i_1 \approx 56.1$ deg

$\cos i_2 = -0.3582 \qquad \Rightarrow\; i_2 \approx 110.1$ deg

The third body perturbation - secular effects

Secular effets due to the Sun/Moon perturbation:

$$\dot{\Omega}_{\rm p} = -c_{\rm p}\,(5-3\eta^2)\frac{\cos i}{n\eta}\,, \qquad \dot{\omega}_{\rm p} = c_{\rm p}\,\frac{5\cos^2 i - \eta^2}{n\eta}$$

with $\rm p = moon$ or $\rm p = sun$ and

$$c_{\rm sun} = \frac{3}{16}\frac{Gm_\odot}{a_\odot^3\eta_\odot^3}(3\cos^2\epsilon - 1)\,, \quad c_{\rm moon} = \frac{3}{16}\frac{Gm_☾}{a_☾^3\,\eta_☾^3}(3\cos^2 i_☾ - 1)(3\cos^2\epsilon - 1)$$

$\eta = (1-e^2)^{1/2},\ \eta_\odot = (1-e_\odot^2)^{1/2},\ \eta_☾ = (1-e_☾^2)^{1/2}$.

Including all the perturbations:

$$\dot{\Omega}_{\rm tot} = \dot{\Omega}_{J_2} + \dot{\Omega}_{\rm sun} + \dot{\Omega}_{\rm moon}\,, \quad \dot{\omega}_{\rm tot} = \dot{\omega}_{J_2} + \dot{\omega}_{\rm sun} + \dot{\omega}_{\rm moon}$$

$$\alpha\dot{\omega}_{\rm tot} + \beta\dot{\Omega}_{\rm tot} = -\frac{c_\omega}{\eta^4}\left(\frac{R_\oplus}{a}\right)^{7/2} P(\cos i, a, e)$$

Again

$$\alpha\dot{\omega}_{\rm tot} + \beta\dot{\Omega}_{\rm tot} \approx 0 \qquad \Leftrightarrow \qquad P(\cos i, a, e) = 0$$

The third body perturbation - secular effects

$$P(\cos i, a, e) = a_2 \cos^2 i + a_1 \cos i + a_0$$

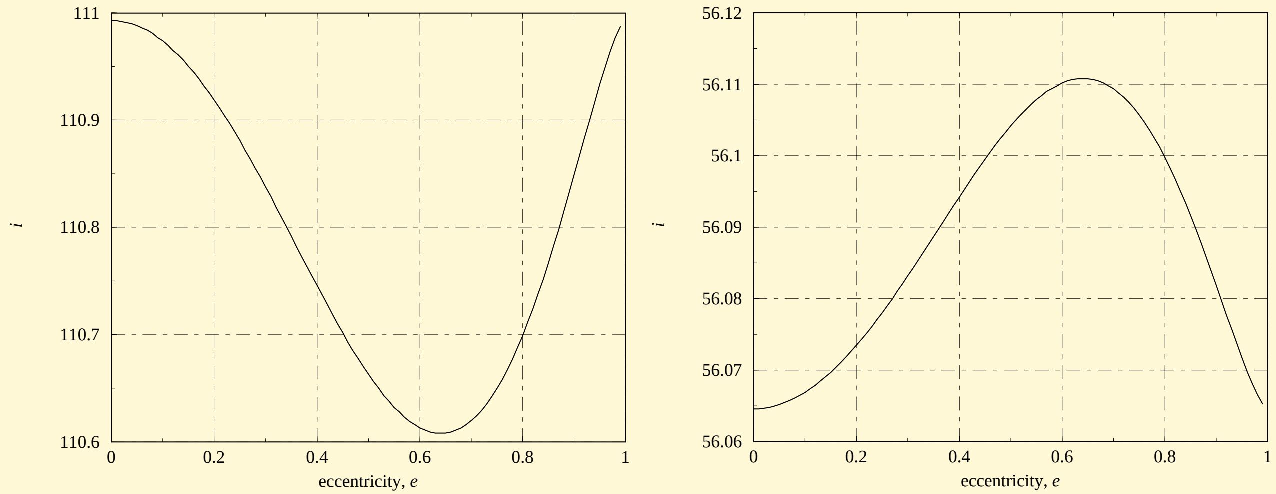
is a polynomial whose coefficients now depend on e and a by means of $\eta = (1-e^2)^{1/2}$ and the coefficient $c_{p,\omega} = -\frac{c_{\rm sun}+c_{\rm moon}}{n c_\omega (R_\oplus/a)^{7/2}}$:

$$a_2 = 5\alpha(\eta^3 c_{p,\omega} - 1) \; , \quad a_1 = \beta(2-\eta^3(5-3\eta^2)c_{p,\omega}) \; , \quad a_0 = \alpha(1-\eta^5 c_{p,\omega}) \; .$$

$\Rightarrow$ Main consequence: we are no more in the case of resonances depending only on inclination.

$\Rightarrow$ For each (α, β) the resonant inclinations change with eccentricity: fix $e \rightsquigarrow$ get i

Example: $\alpha = 2, \beta = 1$ and $a = 29\,600$ km. *Left*: i_2, *right*: i_1.



Study of the resonance $2\dot{\omega} + \dot{\Omega} \approx 0$

$\rightsquigarrow$ In the purely inclination-dependent case (just J_2 secular perturbation) the resonance

$$2\dot{\omega} + \dot{\Omega} \approx 0$$

- is associated to one of the largest amplitude terms of the third body perturbation
- is not far from Galileo inclination (about $56°$).

We introduce very long period terms : *resonant terms*.

- *Aim:* study of the stability properties of a resonance.
- *Method:* Hamiltonian formalism, stability analysis.
  Resonances are seen as *equilibrium points* of a simplified model.
- More precisely: the resonance $2\dot{\omega} + \dot{\Omega} \approx 0$:
  - stability analysis of the equilibrium points and of the motion in their neighbourhood
  - study of the evolution of the eccentricity

The Hamiltonian model

The Hamiltonian:

$$\mathcal{J} = \mathcal{J}_{\rm Kep} + \mathcal{J}_{J_2} + \mathcal{J}_{\rm 3b}$$

- $\mathcal{J}_{\rm Kep}$ Hamiltonian corresponding to the Keplerian motion

$$\mathcal{J}_{\rm Kep} = -\frac{\mu}{2a}$$

- $\mathcal{J}_{J_2}$ perturbation due the Earth's oblateness

$$\mathcal{J}_{J_2} = \frac{\mu}{a}\frac{J_2}{4}\left(\frac{R_\oplus}{a}\right)^2 \left(1 - 3\cos^2 i - 3\sin^2 i\ \cos(2f + 2\omega)\right)$$

$\mu = Gm_\oplus$, f = true anomaly.

- $\mathcal{J}_{\rm 3b}$ perturbation due to the third body (the Sun for the moment)

$$\mathcal{J}_{\rm 3b} = \mathcal{P}_\odot$$

Truncation of $\mathcal{P}_\odot$ at:

- $n = 2 \Leftarrow$ fast decrease of $(a/a_\odot)^n$
- $j = 0 \Leftarrow$ small value of $e_\odot$

Study of the resonance $\alpha\dot{\omega} + \beta\dot{\Omega} \approx 0$: averaging

1. First *averaging:* removing the mean anomalies $M, M_\odot$. [cfr. Brouwer 1959]

2. *Canonical change of variables* $(\omega, \Omega, G, H) \to (\sigma, \xi, \Sigma, \Xi)$ to introduce the resonant angle σ:

$$\sigma = \alpha\omega + \beta\Omega\,, \quad \Sigma = \frac{H}{\beta} = \frac{1}{\beta}\sqrt{\mu a(1-e^2)}\cos i\,,$$

$$\xi = \Omega\,, \quad \Xi = G - \frac{\alpha}{\beta}H = \left(1 - \frac{\alpha}{\beta}\cos i\right)\sqrt{\mu a(1-e^2)}\,.$$

3. Second *averaging:* removing the periodic non-resonant terms.

$$\Downarrow$$

New Hamiltonian:

$$\mathcal{K} = \mathcal{K}(\sigma, \xi, \Sigma, \Xi) = \mathcal{K}_{\rm Kep} + \mathcal{K}_{J_2,{\rm sec}} + \mathcal{K}_{\rm 3b,sec} + \mathcal{K}_{\rm 3b,res}$$

$\star$ Third body perturbation: we keep only secular and long period terms (= resonant terms associated to the resonance we want to study).

Study of the resonance $\alpha\dot{\omega} + \beta\dot{\Omega} \approx 0$: the reduced problem

Equations of motion - Hamilton equations:

$$\dot{\sigma} = \frac{\partial \mathcal{K}}{\partial \Sigma}\,, \quad \dot{\Sigma} = -\frac{\partial \mathcal{K}}{\partial \sigma}\,, \qquad (1)$$

$$\dot{\xi} = \frac{\partial \mathcal{K}}{\partial \Xi}\,, \quad \dot{\Xi} = -\frac{\partial \mathcal{K}}{\partial \xi}\,.$$

- $\mathcal{K}$ does not depend on $\xi \Rightarrow \Xi$ is constant $\Rightarrow$ we study only the two equations (1) with Ξ as a parameter
- we study the evolution of the resonant angle σ and the action Σ;
- Ξ is constant $\Rightarrow$ the evolution of Σ gives us the evolution of i and e.

Resonances = equilibrium points $(\sigma^*,\, \Sigma^*)$ s.t.

$$\alpha\dot{\omega} + \beta\dot{\Omega} = 0 \quad \Leftrightarrow \quad \dot{\sigma} = 0$$

and

$$\dot{\Sigma} = 0$$

Remark: Σ^* gives us $\cos i^* \Rightarrow$ we identify the equilibrium points as $(\sigma^*,\, i^*)$.

Study of the resonance $\alpha\dot{\omega} + \beta\dot{\Omega} \approx 0$: the reduced problem

$$\begin{aligned} \dot{\sigma} &= \frac{\partial \mathcal{K}_{J_2}}{\partial \Sigma} + \frac{\partial \mathcal{K}_{3b,\mathrm{sec}}}{\partial \Sigma} + \frac{\partial \mathcal{K}_{3b,\mathrm{res}}}{\partial \Sigma} = A(\Sigma) + B(\Sigma)\cos\sigma \\ \dot{\Sigma} &= -\frac{\partial \mathcal{K}_{3b,\mathrm{res}}}{\partial \sigma} = e^2\, C(\Sigma)\sin\sigma \end{aligned}$$

In the following:

- linear stability of the equilibrium points of this system
- numerical integration of this system $\rightarrow$ motion near the equilibrium points

Outline: since $\frac{\partial \mathcal{K}_{3b,\mathrm{sec}}}{\partial \Sigma}, \frac{\partial \mathcal{K}_{3b,\mathrm{res}}}{\partial \Sigma} \ll \frac{\partial \mathcal{K}_{J_2}}{\partial \Sigma}$ we will proceed in 3 steps:

1. only J_2
2. J_2 + Sun secular perturbation
3. J_2 + Sun secular perturbation + Sun resonant perturbation

Step 1: only J_2 secular perturbation

$$\dot{\sigma} = \frac{\partial \mathcal{K}_{J_2}}{\partial \Sigma}\,, \qquad \dot{\Sigma} = -\frac{\partial \mathcal{K}_{3\mathrm{b,res}}}{\partial \sigma}$$

Recall: secular effects due to J_2 imply [cfr. Hughes 1980]:

$$\frac{\partial \mathcal{K}_{J_2}}{\partial \Sigma} = c_\omega Y(a,e) P(\cos i)\,,$$

$$Y(a,e) \equiv \left(\frac{R_\oplus}{a}\right)^{7/2} (1-e^2)^{-2}\,, \quad P(\cos i) \equiv \alpha\left(5\cos^2 i - 1\right) - 2\beta\cos i\,.$$

Moreover:

$$-\frac{\partial \mathcal{K}_{3\mathrm{b,res}}}{\partial \sigma} = e^2\, C(\Sigma)\sin\sigma$$

Resonance condition $\rightarrow$ equilibrium points $(\sigma^*,\, \Sigma^*)$ s.t.

$$0 = \dot{\sigma} = c_\omega Y\, P(\cos i^*)\,, \;\; 0 = \dot{\Sigma} = e^2 C(\Sigma^*)\sin\sigma^*\,.$$

Step 1: only J_2 secular perturbation

Looking for equilibrium points is equivalent to solve

- if $e \neq 0 \;\; P(\cos i) = 0\,, \quad \sin\sigma = 0$
- if $e = 0 \;\; P(\cos i) = 0\,, \quad \forall\sigma$

$\rightarrow$ the existence of resonances depends only on inclination $\rightarrow$ one or two well defined resonant inclinations i^* independent of a and e.

Case $2\dot{\omega} + \dot{\Omega} \approx 0$. Equilibrium points, $e \neq 0$:

$$\sigma^* = 0 \text{ or } \pi\,,\; \cos i^* = \frac{1 \pm \sqrt{21}}{10} \;\Leftrightarrow\; i_1^* \approx 56.046\,^\circ \text{ or } i_2^* \approx 110.993\,^\circ$$

$\rightarrow$ i^* correspond to the solution(s) of $P(\cos i) = 0$.

Linear stability:

- $\sigma^* = 0\,, i_1^*$ or i_2^* : two centers (stable);
- $\sigma^* = \pi\,, i_1^*$ or i_2^* : two saddles (unstable).

Step 2: J_2 + secular third body perturbation

$$\dot{\sigma} = \frac{\partial \mathcal{K}_{J_2}}{\partial \Sigma} + \frac{\partial \mathcal{K}_{3\mathrm{b,sec}}}{\partial \Sigma}\,, \qquad \dot{\Sigma} = -\frac{\partial \mathcal{K}_{3\mathrm{b,res}}}{\partial \sigma}\,.$$

Recall: secular effects due to the Sun imply:

$$\frac{\partial \mathcal{K}_{J_2}}{\partial \Sigma} + \frac{\partial \mathcal{K}_{3\mathrm{b,sec}}}{\partial \Sigma} = -\frac{c_\omega}{(1-e^2)^2}\left(\frac{R_\oplus}{a}\right)^{7/2} \tilde{P}(\cos i),$$

$$\tilde{P}(\cos i) = a_2 \cos^2 i + a_1 \cos i + a_0$$

Again: looking for equilibrium points is equivalent to solve

- if $e \neq 0$ $\tilde{P}(\cos i) = 0\,, \quad \sin\sigma = 0$
- if $e = 0$ $\tilde{P}(\cos i) = 0\,, \quad \forall\sigma$

but a_0, a_1, a_2 now depend on a and $e \Rightarrow$ the resonant inclinations now depend on a and $e \Rightarrow$ the equilibrium points are displaced into $i_1^{**}(a,e)$ and $i_1^{**}(a,e)$.

Step 2: phase portrait around equilibrium points

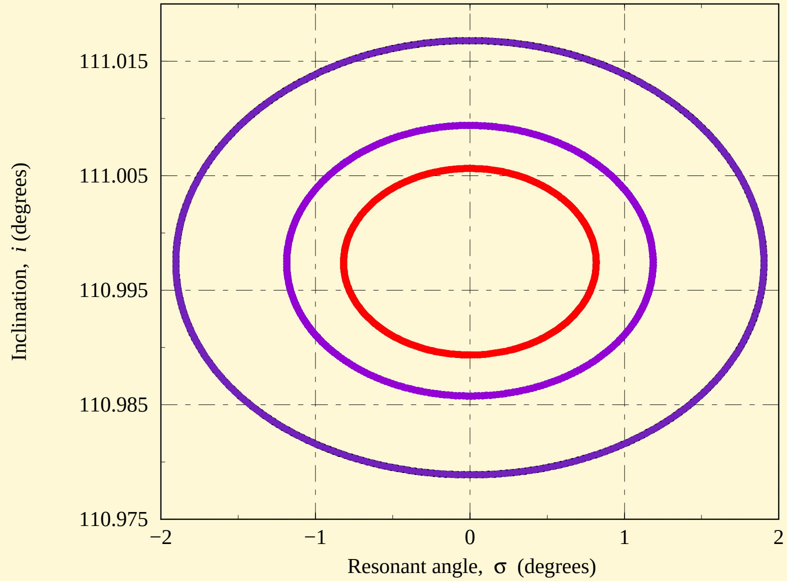
Case $2\dot{\omega} + \dot{\Omega} \approx 0$. Equilibrium points, $e = 0.1$:

$$\sigma^{**} = 0 \text{ or } \pi\,, \quad i_1^{**} = 56.065 \text{ or } i_2^{**} = 110.098$$

Question: does this (small) displacement change the stability?

Answer: numerically: integration of motion.

$\rightarrow$ The point $(\sigma^* = 0, i_2^{**} = 110.989)$ remain a center (stable).



Phase portrait around $(0, 110.989)$, and initial ecccentricity $e_0 = 0.1$.

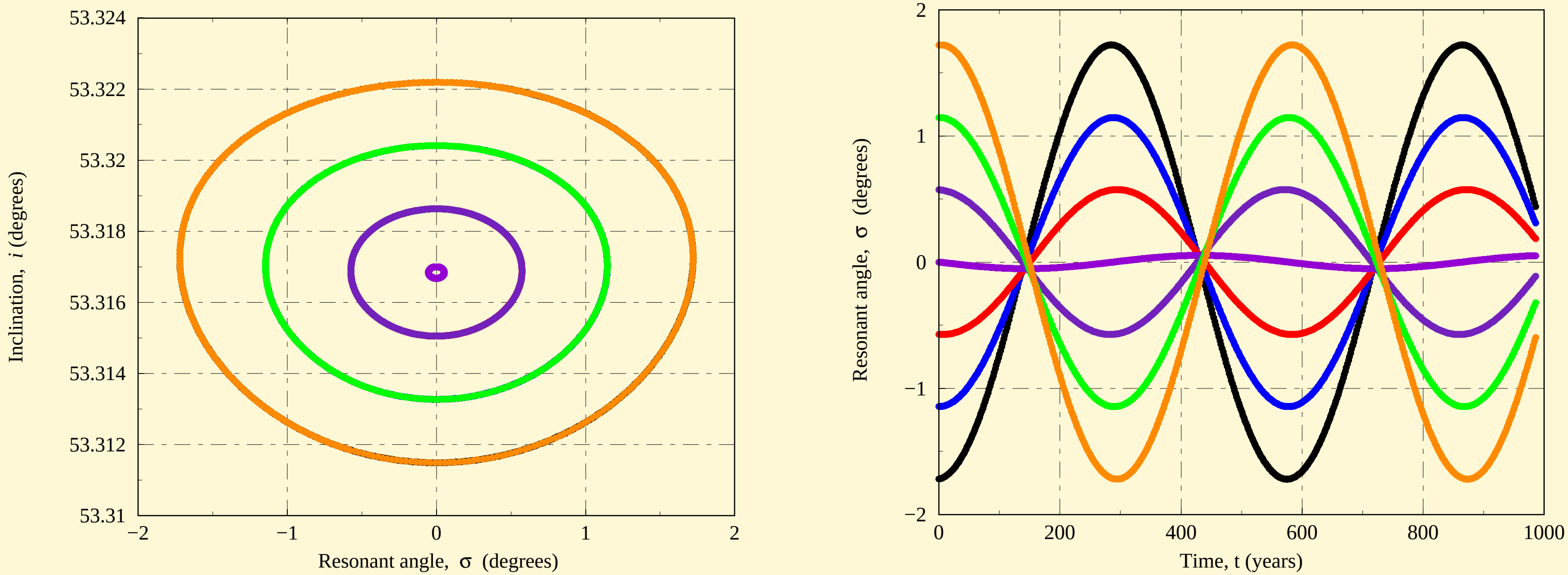
Step 3: phase portrait around the equilibrium point

$$\dot{\sigma} = \frac{\partial \mathcal{K}_{J_2}}{\partial \Sigma} + \frac{\partial \mathcal{K}_{3b,sec}}{\partial \Sigma} + \frac{\partial \mathcal{K}_{3b,res}}{\partial \Sigma}, \quad \dot{\Sigma} = -\frac{\partial \mathcal{K}_{3b,res}}{\partial \sigma}$$

Equilibrium points: $\sigma^* = 0 \text{ or } \pi\,,\ i^{***} = i_1^{***}(a,e) \text{ or } i_2^{***}(a,e)$

Example: $e = 0.1 \Rightarrow i_1^{***} \approx 53.317°,\ i_2^{**}(a,e) = 112.089°$

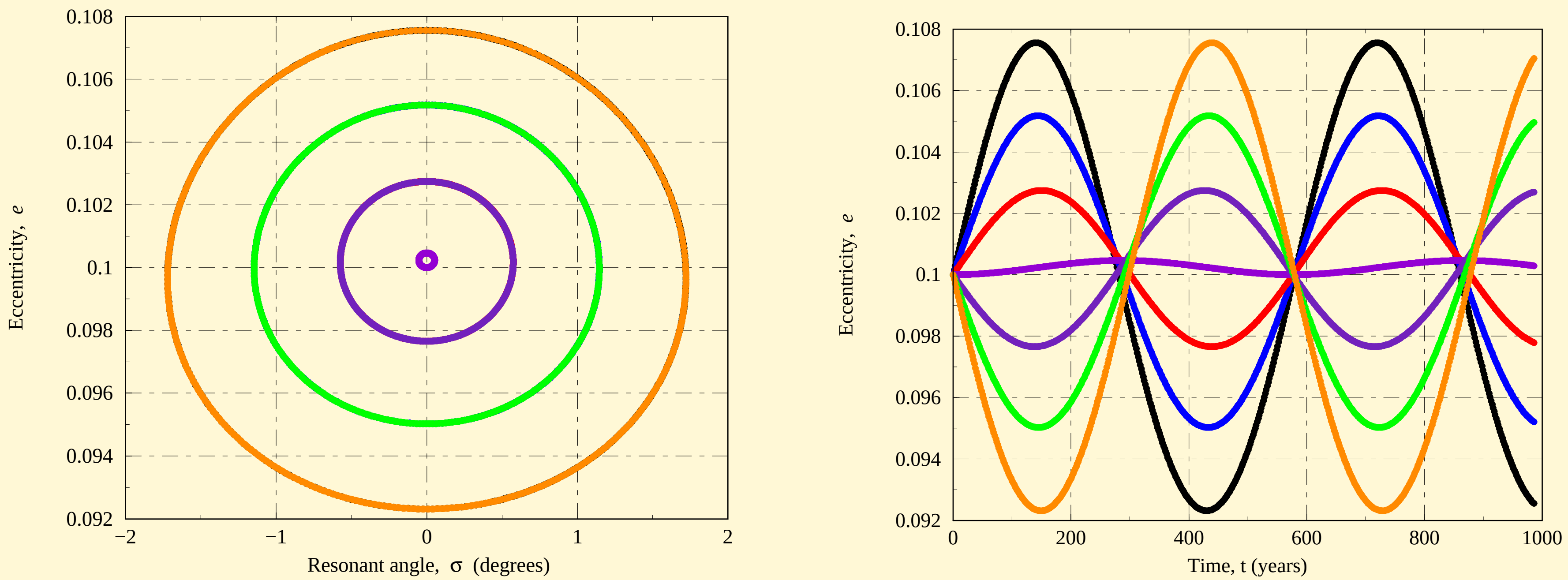
$\longrightarrow$ The point $(0, i_2^{***} = 53.317°)$ is still a center.



Left: Phase portrait (σ, i) around the equilibrium point $(0, i^{***} = 53.317°)$.
Right: time evolution of the resonant angle.
Initial eccentricity $e_0 = 0.1$, integration time 1000 years.

Step 3: phase portrait around the equilibrium point

$\rightarrow$ The point $(0, i_2^{***} = 53.317°)$ is still a center.



Left: Phase portrait (σ, e) around the equilibrium point $(0, i^{***} = 53.317°)$.

Right: time evolution of the eccentricity.

Initial eccentricity $e_0 = 0.1$, integration time 1000 years.

Step 3: J_2 + sec + res third body perturbation

$$\dot\sigma = \frac{\partial \mathcal{K}_{J_2}}{\partial \Sigma} + \frac{\partial \mathcal{K}_{3b,sec}}{\partial \Sigma} + \frac{\partial \mathcal{K}_{3b,res}}{\partial \Sigma} = A(\Sigma) + B(\Sigma)\cos\sigma\ ,$$

$$\dot\Sigma = -\frac{\partial \mathcal{K}_{3b,res}}{\partial \sigma} = e^2\ C(\Sigma)\sin\sigma$$

⋆ Case $e = 0$. Equilibrium points (analytically):

$$\sigma^* = \pm\arccos(-A/B)\,,\quad \forall\, i \text{ s.t. } |B| > |A|\ (A(\Sigma) \neq 0)$$

in particular:

$$i = i^{**} \Rightarrow A(\Sigma) = 0\,,\quad \sigma^* = \pm\pi/2$$

⋆ $e = 0 \Rightarrow A, B$ constant $\Rightarrow$ if $|B| > |A|$ and $A \neq B$, the general form of the solution for the equation $\dot\sigma = A + B\cos\sigma$ is:

$$\sigma(t) = -2\arctan\left[\sqrt{\frac{A+B}{B-A}}\tanh\left(\frac{\sqrt{B^2-A^2}}{2}t + \phi\right)\right]$$

where the constant ϕ depends on the initial conditions of the problem.

Step 3: J_2 + sec + res third body perturbation

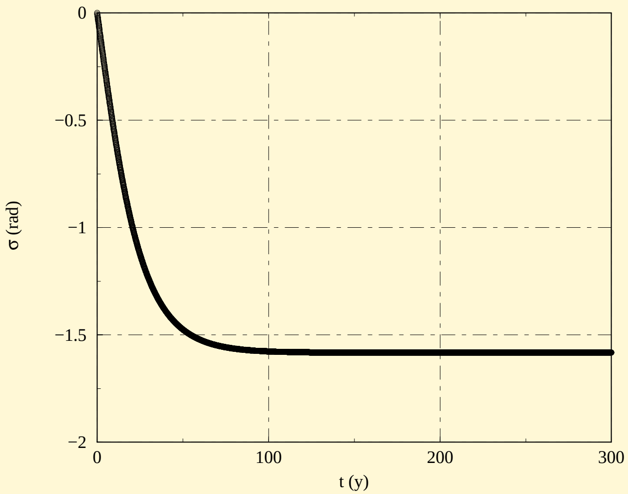
We have:

$$\lim_{t\to\infty} \sigma(t) = -\arccos\left(-\frac{A}{B}\right) = \sigma^*$$

$\Rightarrow \forall$ initial value σ_0, σ tends to the equilibrium point $e = 0$, $i = i_0$, $\sigma = \sigma^*$.

$\star$ Case $2\dot{\omega} + \dot{\Omega} \approx 0$, $e = 0$:

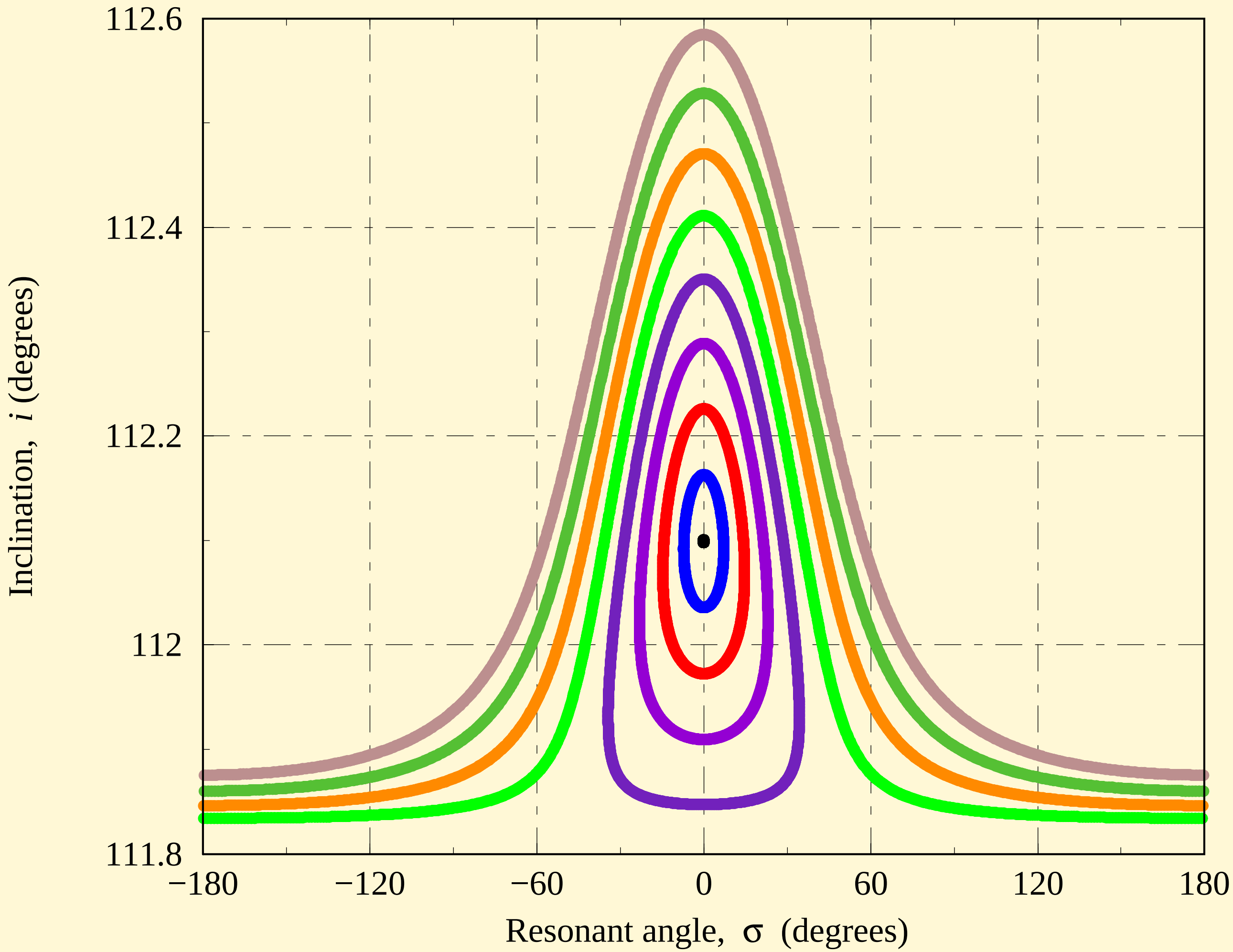
- $-\arccos\left(-\frac{A}{B}\right) = -90.69\,^\circ$ (cfr figure).
- numerically, resonant angle converges to the asymptotic value $-90.69\,^\circ$



$e = 0\,,\ i = 56.1\,^\circ$. Evolution of the resonant angle over a time of 300 years, starting with the initial value $\sigma_0 = 0$. It converges to the value $-1.582\,\mathrm{rad} = -90.69\,^\circ$.

$\star$ Note that starting from a different value i_0 (and thus Σ_0) we observe the same behavior but the asymptotic value changes.

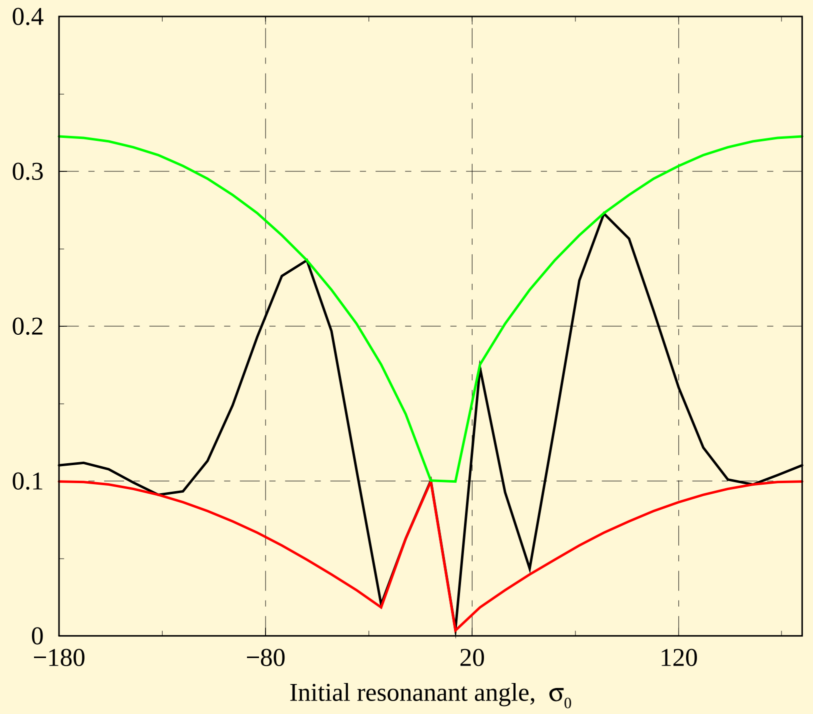
Step 3: global phase portrait

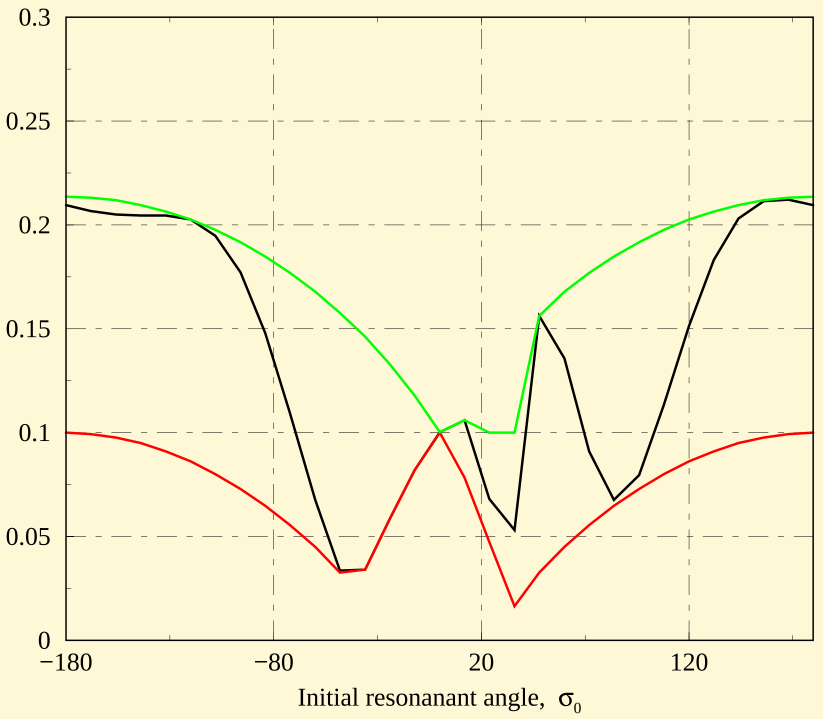


Global phase portrait (σ, i), centered on $(0, i_2^{***} = 112.098°)$.
Initial eccentricity $e_0 = 0.1$, integration time 1000 years.

Study of the eccentricity - complete case - numerical

Dependence of the evolution of the eccentricity on the initial resonant angle





Maximum (green), minimum (red) and final (black) eccentricity, after 300 years. Initial eccentricity $e_0 = 0.1$.

Left: $i_0 = i_1^{***} \approx 53.317°$, *right:* $i_0 = i_2^{***} \approx 112.098°$

Conclusions and perspectives

- Characterization and stability of the equilibrium points for the resonance $2\dot{\omega} + \dot{\Omega} \approx 0$, for Sun perturbation $\rightarrow$ resonance does not means necessarily danger!
- Existence of asymptotic equilibrium(a) for the circular case.
- Study of the eccentricity as a function of the resonant angle.
- Analytical and numerical study of the eccentricity.
- Extension of the theory to the Moon perturbation and to other selected resonances.
- Formulation in orbital elements if useful.
- Numerical study of the complete (not averaged) system (like in Rossi,2008).