Astrometry in Gaia's age
The astrometric core solution
Relativistic models for light propagation
A cross-check procedure
Conclusions

Time Transfer functions as tool to validate light propagation solutions for space astrometry

S. Bertone*, O. Minazzoli, M. Crosta, C. Le Poncin-Lafitte, A. Vecchiato, M.-C. Angonin

* SYRTE/Obspm - INAF/Università di Torino

June 07, 2013
Journées de la SF2A

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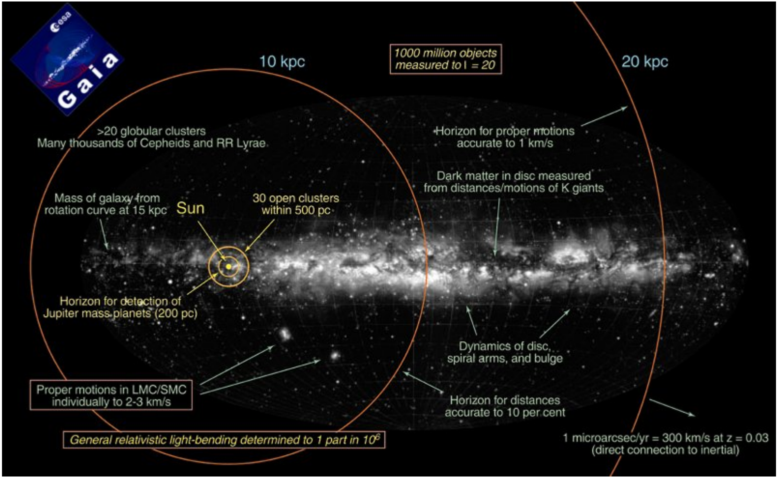
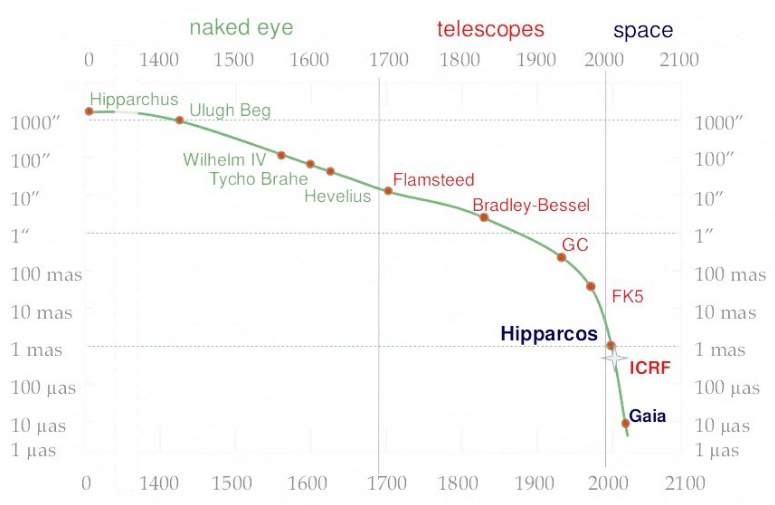
2. The astrometric core solution
3. Relativistic models for light propagation
4. A cross-check procedure
5. Conclusions



Schematic diagram showing the distances out to which Gaia will contribute to our knowledge of the Galaxy. Image: ESA



Accuracy of astrometric observations VS year. Image : S. Klioner, Porto 2011

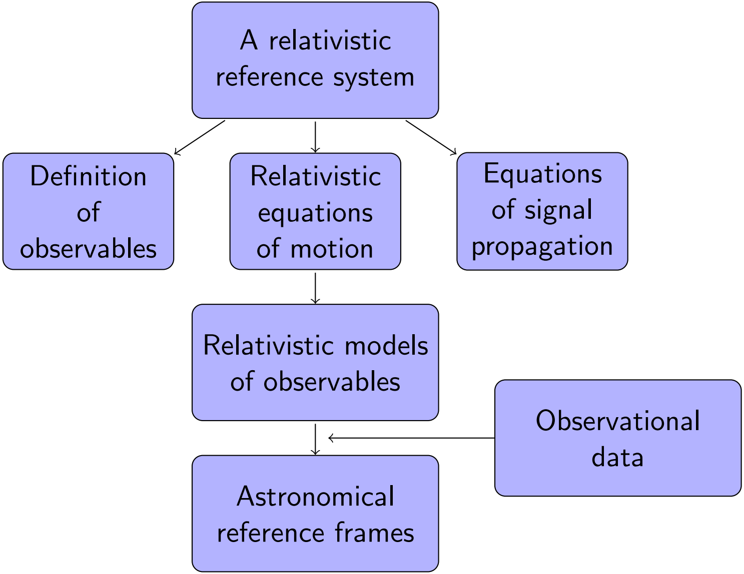
Relativistic deflection by Solar System planets :

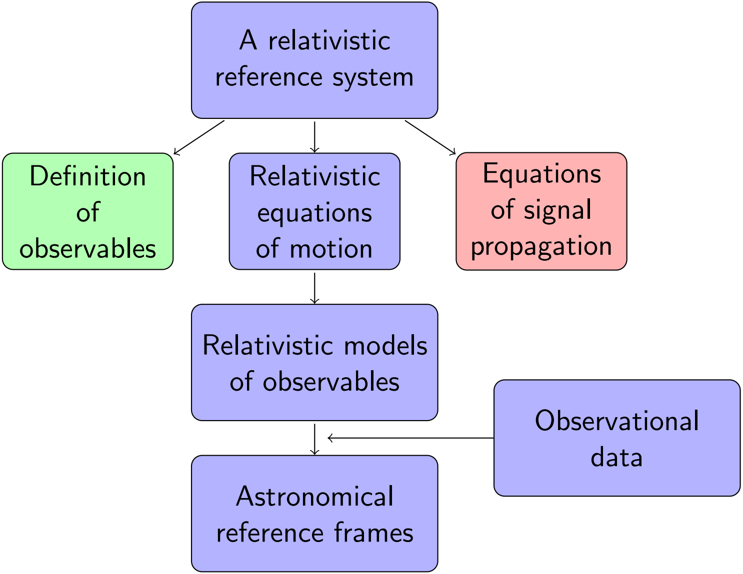
$10 - 10^4 \mu as$

Relativistic light deflections in the Solar System

Body	**Monopole**		**Quadrupole**	
	grazing	χ	grazing	χ
	mas	$\delta\theta = 1\ \mu$as	μas	$\delta\theta = 1\ \mu$as
Sun	17,000	180°		
Mercury	0.083	0.15°		
Venus	0.49	4.5°		
Mars	0.12	0.4°		
Jupiter	16.3	90°	240	8 R_J
Saturn	5.8	17°	95	4 R_S
Uranus	2.1	1.2°	8	2 R_U
Neptune	2.5	0.9°	10	2 R_N

Mignard,Klioner , Gaia : Relativistic modelling and testing, 2009


A relativistic reference system
Definition of observables
Relativistic equations of motion
Equations of signal propagation
Relativistic models of observables
Observational data
Astronomical reference frames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Astrometric Global Iterative Solution (AGIS)

Global Sphere Reconstruction (GSR)

Procedure

Definition of the observable

Light propagation

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Astrometric Global Iterative Solution (AGIS)		Global Sphere Reconstruction (GSR)
Iterative	Procedure	LSQR algorithm
	Definition of the observable	
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Iterative	Procedure	LSQR algorithm
Klioner 2004	Definition of the observable	Bini 2003
	Light propagation	

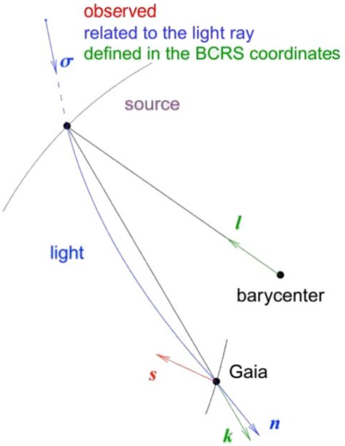
Astrometric Global Iterative Solution (AGIS)		Global Sphere Reconstruction (GSR)
Iterative	Procedure	LSQR algorithm
Klioner 2004	Definition of the observable	Bini 2003
GREM	Light propagation	RAMOD

General RElativistic Model (GREM)

Based on IAU reference systems

$s \longleftrightarrow n \longleftrightarrow \sigma \longleftrightarrow k \longleftrightarrow l, \pi$

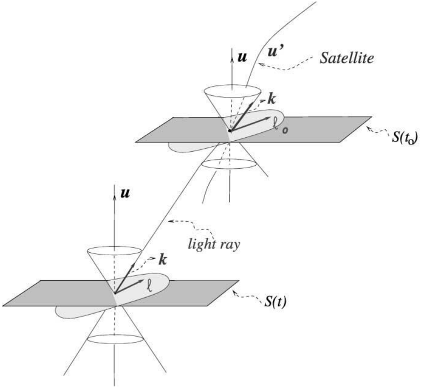
(1) aberration
(2) gravitational deflection
(3) coupling to finite distance
(4) parallax



General structure of the General RElativistic Model (GREM). Image: S. Klioner, PRD 2003

Relativistic Astrometric MODel (RAMOD)

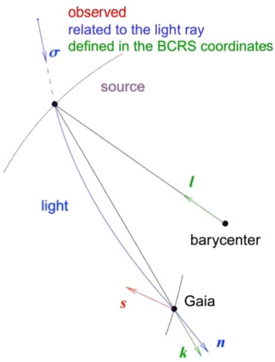
- Based on a measurement protocol
- local barycentric observer $\boldsymbol{u}$
- $\bar{\ell} =$ local line of sight of the fiducial observer $\boldsymbol{u}$



General structure of the Relativistic Astrometric MODel (RAMOD). Image: M. Crosta, Porto 2011

$$\frac{\mathrm{d}\bar{\ell}^k}{\mathrm{d}\zeta}+\bar{\ell}^i\bar{\ell}^j\left(\partial_i h_{kj}-\frac{1}{2}\partial_k h_{ij}\right)+\frac{1}{2}\bar{\ell}^k\bar{\ell}^i\partial_i h_{00}-\frac{1}{2}\partial_k h_{00}+\mathcal{O}\left(h^2\right)=0$$

How to relate their results?



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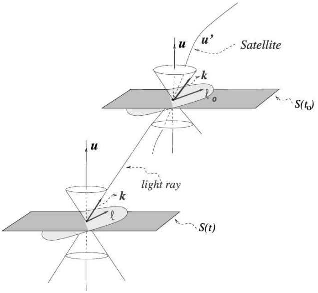

General structure of the Relativistic Astrometric MODel (RAMOD). Image: M. Crosta, Porto 2011

Previous studies : aberration (M. Crosta and A. Vecchiato 2010), geodesic equations (M. Crosta 2011)

Time Transfer Functions (TTF)

$$\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \frac{R_{AB}}{c} + \sum_{n=1}^{\infty} G^n \Delta_r^{(n)}(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B, g_{\mu\nu})$$

$$\hat{k}_i^{x_B} = \frac{k_i^{x_B}}{k_0^{x_B}} = -c\frac{\partial \mathcal{T}_r}{\partial x_B^i}\left[1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1}$$

Le Poncin-Lafitte et al. 2004, Teyssandier & Le Poncin-Lafitte 2008; T. 2012.

- Δ_r at any order in general static, spherically symmetric space-times
- without integrating the whole set of geodesic equations
- well adapted to a ray emitted and observed at points both at a finite distance x_A et x_B
- definition of the astrometric observable within the formalism (Bertone and Le Poncin-Lafitte 2012)

Time Transfer Functions (TTF)

$$\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \frac{R_{AB}}{c} + \sum_{n=1}^{\infty} G^n \Delta_r^{(n)}(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B, g_{\mu\nu})$$

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Le Poncin-Lafitte et al. 2004, Teyssandier & Le Poncin-Lafitte 2008; T. 2012.

We propose to :

extract $\mathcal{T}$ and $\hat{k}_i$ from GREM and RAMOD

TTF formalism in closed form

(S. Bertone and C. Le Poncin Lafitte, Memorie SAI 2012)

$$\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \frac{R_{AB}}{c} + \frac{1}{c}\Delta_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) + \mathcal{O}(c^{-5})$$

$$\left(\widehat{k}_i\right)_B \approx N^i_{AB} + \frac{\partial \Delta_r}{\partial x^i_B} + N^i_{AB}\frac{\partial \Delta_r}{\partial x^0_B}$$

TTF formalism in closed form

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$$\Delta_r = \frac{1}{2} R_{AB} \int_0^1 \left[h^{(2)}_{00} + \frac{2}{c} N^i_{AB} h^{(3)}_{0i} + N^i_{AB} N^j_{AB} h^{(2)}_{ij} \right]_{z^{\alpha}_{-}(\lambda)} d\lambda$$

TTF for a time-dependent metric

(S. Bertone et al., 2013)

PPN metric : $h_{00} = \frac{2G}{c^2}\sum_P \frac{\mathcal{M}_P}{R_P(t,\boldsymbol{x})}$, $\quad h_{0i} = -(1+\gamma)h_{00}\beta^i_P(t)$, $\quad h_{ij} = \delta_{ij}\gamma h_{00}$

$$\boldsymbol{R}_P = \boldsymbol{R}^*_{\boldsymbol{PB}} - \lambda\, R_{AB}(\boldsymbol{N}_{AB} - \boldsymbol{\beta_P})$$

$$\begin{aligned}\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) \quad = \quad & \frac{R_{AB}}{c} + (\gamma+1)\frac{G}{c^2}\sum_P \mathcal{M}_P\Big[1 - \boldsymbol{\beta}_p(t_C)\cdot\boldsymbol{N}_{AB}\Big] \\ & \times \ln\left[\frac{R_{PA} - \boldsymbol{R_{PA}}\cdot\boldsymbol{N}_{AB} - \boldsymbol{\beta}_p(t_C)\cdot(\boldsymbol{R_{PA}} - \boldsymbol{N}_{AB}R_{PA})}{R_{PB} - \boldsymbol{R_{PB}}\cdot\boldsymbol{N}_{AB} - \boldsymbol{\beta}_p(t_C)\cdot(\boldsymbol{R_{PB}} - \boldsymbol{N}_{AB}R_{PB})}\right]\end{aligned}$$

$$\begin{aligned}\left(\widehat{k}_i\right)_B \quad = \quad & -N^i_{AB} + (\gamma+1)\frac{G}{c^2}\sum_P \frac{\mathcal{M}_P}{R_{AB}R_{PB}\Big[R^2_{PB}g^2_P - (\boldsymbol{R}_{PB}\cdot\boldsymbol{g}_P)^2\Big]} \\ & \times\Big\{g_P N^i_{AB}\Big[\Big(\boldsymbol{R}_{PB}\cdot\boldsymbol{N}_{AB}\Big)\Big(R^2_{PB} - R_{PA}R_{PB} - R_{AB}\boldsymbol{R}_{PB}\cdot\boldsymbol{\beta}_P(t_C)\Big) \\ & \qquad - R^2_{PB}R_{AB}g^2\Big] + R^i_{PB}g^2_P\Big[R_{PB}R_{PA} - R^2_{PB} + R_{AB}\boldsymbol{R}_{PB}\cdot\boldsymbol{g}_P\Big] \\ & + \beta^i_P(t_C)R_{PB}\Big[(R_{PA} - R_{PB})(\boldsymbol{R}_{PB}\cdot\boldsymbol{N}_{AB}) + R_{PB}R_{AB}\Big]\Big\} \\ & + (\gamma+1)\frac{G}{c^2}\sum_P \mathcal{M}_P \frac{\beta^i_P(t_C) - N^i_{AB}\boldsymbol{\beta}_P(t_C)\cdot\boldsymbol{N}_{AB}}{R_{AB}g_P}\ln\frac{g_P R_{PB} + \boldsymbol{R}_{PB}\cdot\boldsymbol{g}_P}{g_P R_{PA} + \boldsymbol{R}_{PA}\cdot\boldsymbol{g}_P} \\ & + \mathcal{O}(c^{-4})\,.\end{aligned}$$

TTF for a static metric

Static metric : $h_{00} = \frac{2G}{c^2} \sum_P \frac{\mathcal{M}_P}{R_P(t, \boldsymbol{x})}\ , \qquad h_{0i} = 0\ , \qquad h_{ij} = \delta_{ij}\, \gamma\, h_{00}$

$$\boldsymbol{R}_P = \boldsymbol{x}_\gamma - \boldsymbol{x}_P = \boldsymbol{R_{PB}} - \lambda\, R_{AB} \boldsymbol{N}_{AB}$$

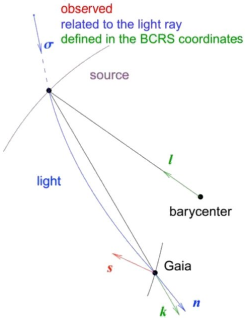
$$\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) \quad = \quad \frac{R_{AB}}{c} + (\gamma+1)\frac{G}{c^2} \sum_P \mathcal{M}_P \ln\left[\frac{R_{PA} - \boldsymbol{R_{PA}} \cdot \boldsymbol{N}_{AB}}{R_{PB} - \boldsymbol{R_{PB}} \cdot \boldsymbol{N}_{AB}}\right]$$

$$\begin{aligned}\left(\widehat{k}_i\right)_B \quad = \quad & -N^i_{AB} + (\gamma+1)\frac{G}{c^2} \sum_P \frac{\mathcal{M}_P}{R_{AB} R_{PB}\left[R^2_{PB} g^2_P - (\boldsymbol{R}_{PB} \cdot \boldsymbol{N}_{AB})^2\right]} \\ & \times \Big\{ N^i_{AB} \Big[\Big(\boldsymbol{R}_{PB} \cdot \boldsymbol{N}_{AB}\Big)\Big(R^2_{PB} - R_{PA} R_{PB}\Big) \\ & \qquad - R^2_{PB} R_{AB}\Big] + R^i_{PB}\Big[R_{PB} R_{PA} - R^2_{PB} + R_{AB} \boldsymbol{R}_{PB} \cdot \boldsymbol{N}_{AB}\Big] \\ & + \mathcal{O}(c^{-4})\ .\end{aligned}$$

GREM < − > TTF

$$\mathcal{T} = t_B - t_A = \Delta t_{AB}$$

$$\hat{k}_i = \frac{k_i}{k_0} = \frac{g_{ij}k^j + g_{0i}k^0}{g_{00}k^0 + g_{0i}k^i}$$
$$\approx -\frac{\dot{x}^i}{c} - 2h_{00}\sigma^i - (\delta_{ij} + \sigma^i\sigma^j)h_{0j}$$



S. Klioner and S. Kopeikin 1992, S. Klioner 2003

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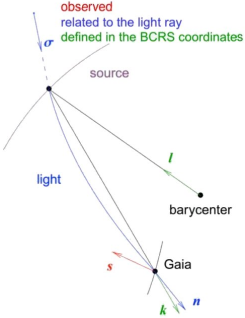

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$$x^i(t) = x^i(t_B) + c\sigma^i(t - t_B) + \Delta x^i(t, x^i, t_B, x^i_B)$$

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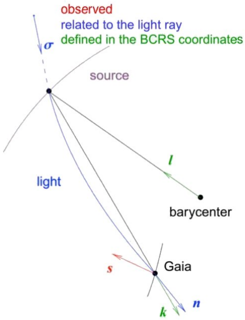

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$$\frac{\dot{x}^i(t, \boldsymbol{x})}{c} = \sigma^i + \frac{\Delta\dot{x}^i(t, \boldsymbol{x})}{c}$$

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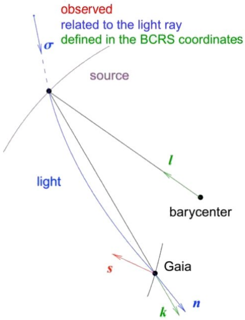
$$\boldsymbol{x}(\boldsymbol{x}_B, \boldsymbol{\sigma}, \Delta t) = \boldsymbol{x}_A$$



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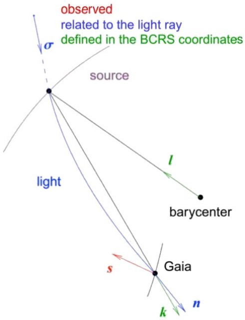

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$$x^i(t_A) = x^i(t_B) + c\sigma^i\Delta t_{AB} + \Delta x^i(\Delta t_{AB}, R^i_{AB})$$

$$\frac{\dot{x}^i(t, \boldsymbol{x})}{c} = \sigma^i + \frac{\Delta\dot{x}^i(t, \boldsymbol{x})}{c}$$

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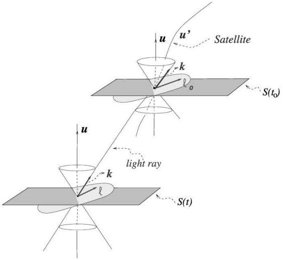
$$\mathcal{T} = t_B - t_A = \Delta t_{AB}$$

$$\bar{\ell}^i = -\frac{k^i}{u^0 k_0}$$



F. de Felice et al. 2004,
M. Crosta 2011

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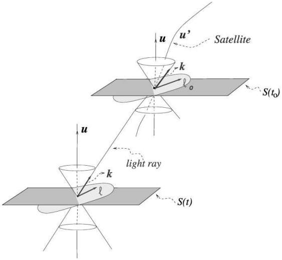

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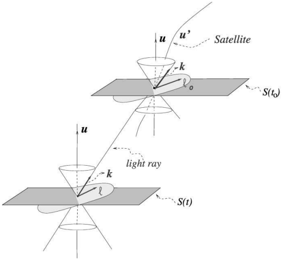
$$u^0 \equiv \frac{cdt}{d\zeta} = \frac{1}{\sqrt{-g_{00}}}$$

RAMOD $< - >$ TTF

$$\mathcal{T} = t_B - t_A = \Delta t_{AB}$$

$$\hat{k}_i^B = -\bar{\ell}_B^i \sqrt{g_{00}(x_B)}\, g_{ij}(x_B)$$



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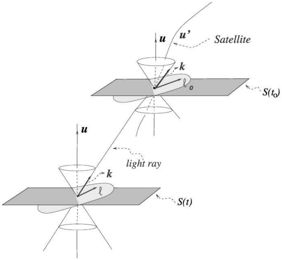

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$$\bar{\ell}_B^i = \frac{x_B^i - x_A^i}{\Delta\zeta_{AB}} + \frac{G}{c^2}\Delta\bar{\ell}(\boldsymbol{x}_A, \boldsymbol{x}_B, \Delta\zeta_{AB}, \boldsymbol{x}_P) + O\left(Gc^{-3}\right)$$

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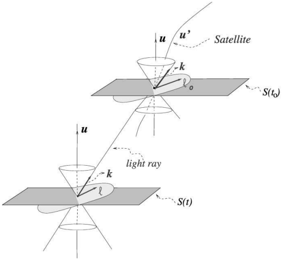

F. de Felice et al. 2004,
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$$u^0 \equiv \frac{cdt}{d\zeta} = \frac{1}{\sqrt{-g_{00}}}$$

$$=> c\Delta t_{AB} \approx \Delta\zeta_{AB} + \frac{G}{c^2}\Delta t_{AB}^{(2)}(\boldsymbol{x}_A, \boldsymbol{x}_B, \Delta\zeta_{AB}, \boldsymbol{x}_P)$$

$$\bar{\ell}_B^i = \frac{x_B^i - x_A^i}{\Delta\zeta_{AB}} + \frac{G}{c^2}\Delta\bar{\ell}(\boldsymbol{x}_A, \boldsymbol{x}_B, \Delta\zeta_{AB}, \boldsymbol{x}_P) + O\left(Gc^{-3}\right)$$

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- Absolute high-precision astrometry needs indipendent verifications
- More than one model of relativistic light propagation to interpret experimental data
- We propose a procedure to relate and understand them using the TTF formalism

Perspectives

- Development of a GSR-TTF

Phd financed by : Vinci Fellowship - UIF
Partner establishments : OBSPM - Università di Torino