



Software tools for the validation of the in-flight calibration performance of the MICROSCOPE space mission

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retour sur innovation

The Equivalence Principle

General Relativity

PE → **Universality of free fall** :
all bodies, independently of their
mass or intrinsic composition,
acquire the **same acceleration** in
the same uniform gravity field

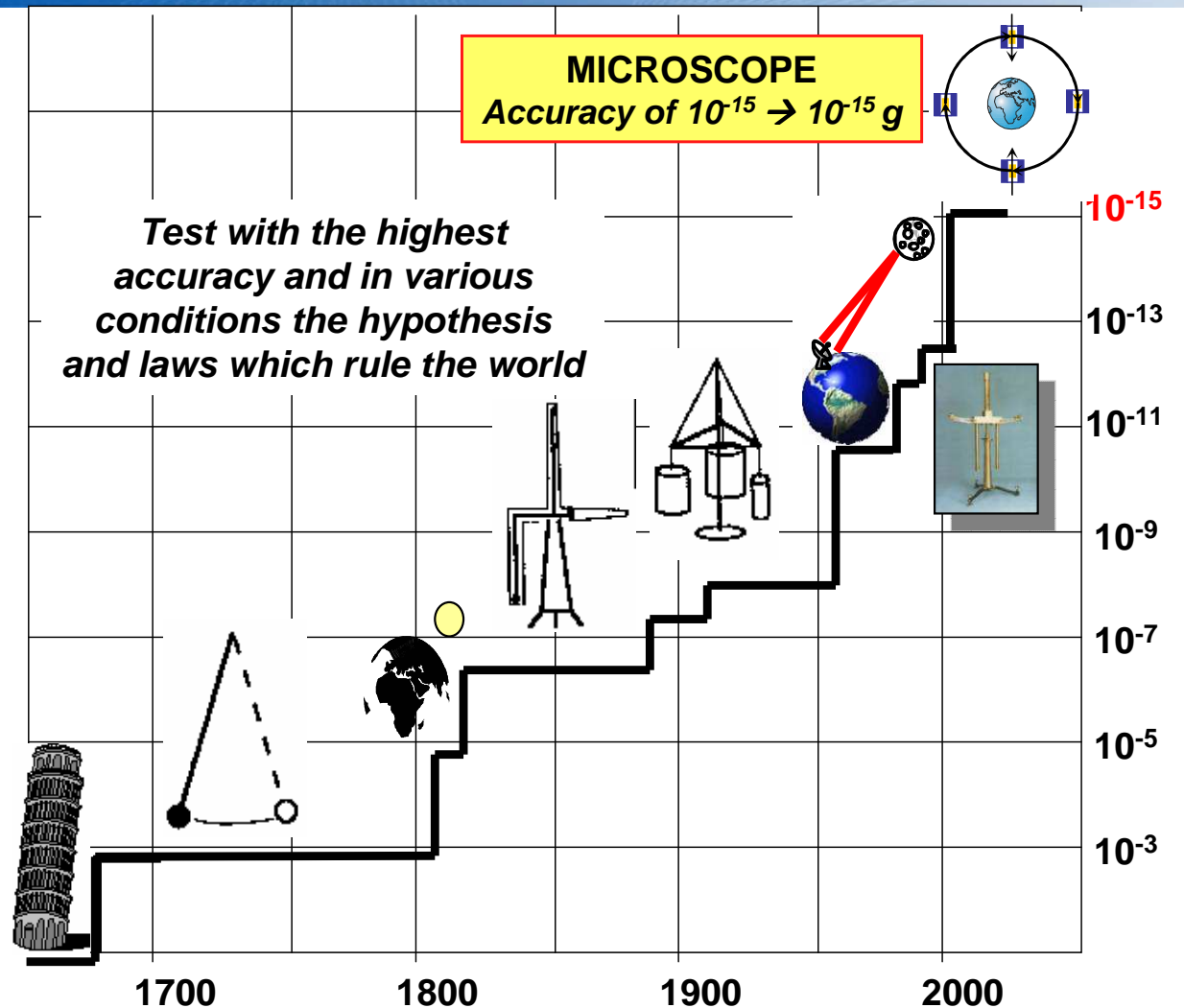
$$\frac{M_G}{M_I} = 1$$

Impossible to merge the
gravitation with the three other
fundamental interactions

Alternative theories

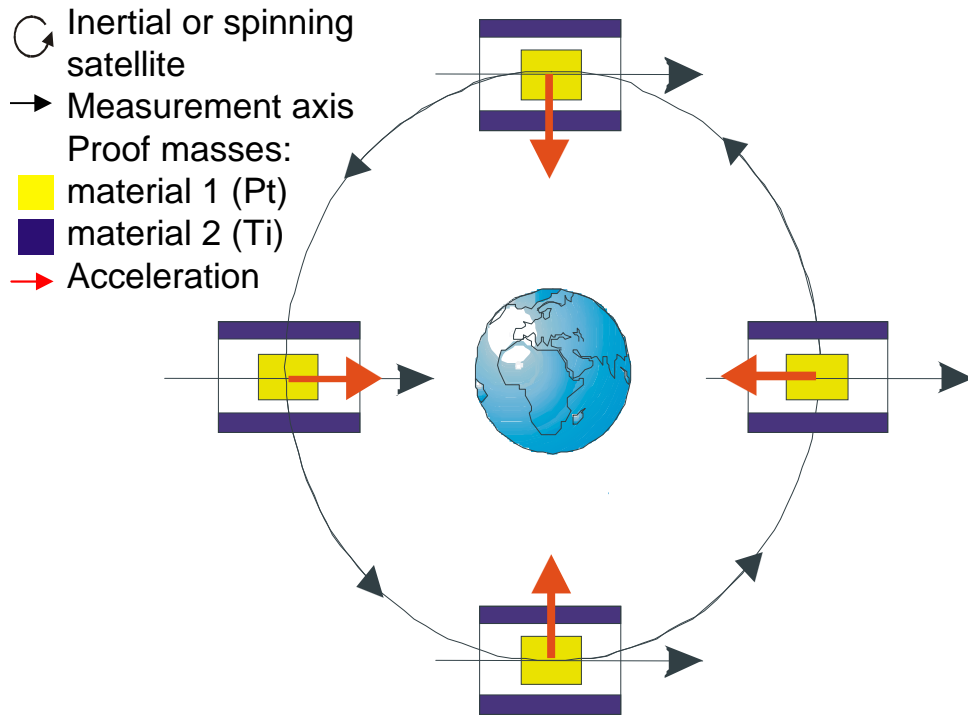
Possible unification
⇒ New interaction?
⇒ Violation of the Equivalence
principle?

$$\frac{M_G}{M_I} = 1 + \omega$$



**MICROSCOPE space experiment: test of the
Equivalence Principle with an accuracy of 10^{-15}**

The principle of the MICROSCOPE space mission



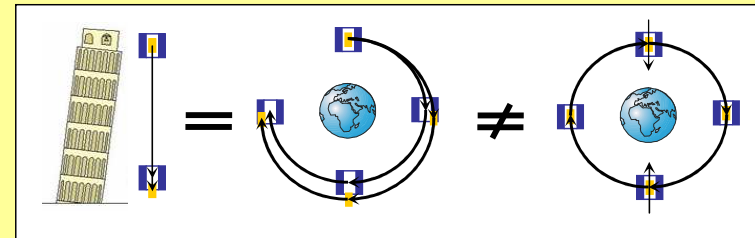
CNES MYRIADE Microsatellite

- Circular Orbit: 720 km, $e < 5 \cdot 10^{-3}$
- Inertial or Rotating: $7 \cdot 10^{-3}$ rd/s
- Mission duration: 12 months
- Mass of microsat: 200 kg
- Payload budgets: 35 kg, 40 Watts
- 2 differential electrostatic accelerometers (2 pairs of masses: Pt/Pt & Pt/Ti)

Gravitational source: **the Earth**

inertial acceleration: orbital motion

2 masses of **different composition**: controlled **on the same orbit** ($< 10^{-11}$ m) thanks to the measured electrostatic forces



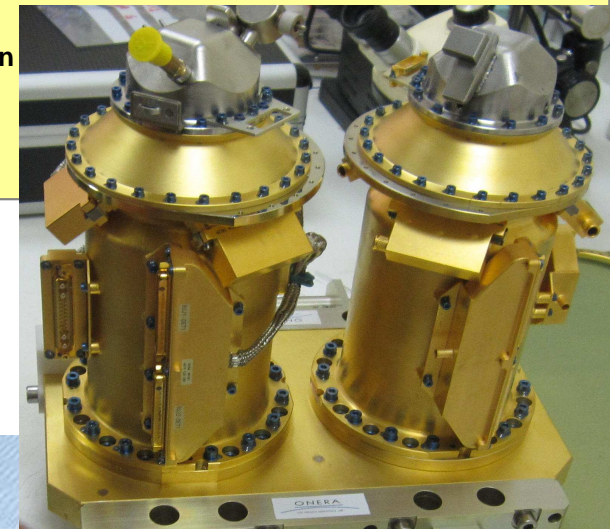
time span of the measurement: **non limited by the free fall** (> 20 orbits)

Environment: Very controlled or avoiding perturbations, **drag-free satellite**

Signal to be detected: phases & frequency are defined $f_{ep} =$

- **Inertial mode**: $f_{orb} = 1/\text{orbit}$

- **Spinning mode**: $f_{orb} + f_{spin}$



The measurement

Measurement of the accelerations applied to the test masses to keep them centered and concentric

d: differential mode (half différence)
→ contains the EP violation term

c: common mode (half sum)
→ command of the drag-free system

EP violation parameter : $\delta = \frac{m_{2g}}{m_{2I}} - \frac{m_{1g}}{m_{1I}}$

$$\Gamma_{mes,dx} = \frac{1}{2} (\Gamma_{mes,1} - \Gamma_{mes,2}) = \frac{1}{2} K_{1cx} \cdot \delta \cdot g_{x/sat} +$$

Gravity gradient Inertia gradient

$$\frac{1}{2} \begin{bmatrix} K_{1cx} \\ \eta_{cz} + \theta_{cz} \\ \eta_{cy} - \theta_{cy} \end{bmatrix}^t \cdot [T - In] \cdot \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix}$$

$$+ \begin{bmatrix} K_{1dx} \\ \eta_{dz} + \theta_{dz} \\ \eta_{dy} - \theta_{dy} \end{bmatrix}^t \cdot (\vec{\Gamma}_{res,df} + C_x) + 2 \cdot K_{2cxx} \cdot (\Gamma_{app,dx} + b_{1dx}) \cdot (\Gamma_{res,df,x} + C_x - b_{0cx})$$

$$+ K_{2dxx} \cdot \left((\Gamma_{res,df,x} + C_x - b_{0cx})^2 + (\Gamma_{app,dx} + b_{1dx})^2 \right)$$

b_0 : bias
 b_1 : parasitic forces
 $\Gamma_{res,df}$: drag-free residual
 C : drag-free command

Δ : décentrage
 K_1 : facteur d'échelle
 η : couplage

θ : mésalignement
 K_2 : terme quadratique

Budget before calibration

	Signal element	Parameter concerned	Contribution before calibration (m·s ⁻²)
Defects between the instrument and the satellite	$K_{1cx} \cdot T_{xx} \cdot \Delta_x$	$K_{1cx} \cdot \Delta_x < 20.2 \mu\text{m}$	8.4×10^{-14}
	$K_{1cx} \cdot T_{xz} \cdot \Delta_z$	$K_{1cx} \cdot \Delta_z < 20.2 \mu\text{m}$	8.6×10^{-14}
	$K_{1cx} \cdot T_{xy} \cdot \Delta_y$	$K_{1cx} \cdot \Delta_y < 20.2 \mu\text{m}$	6×10^{-16}
	$(\eta_{cz} + \theta_{cz}) \cdot T_{yy} \cdot \Delta_y$	$\eta_{cz} + \theta_{cz} < 2.6 \times 10^{-3} \text{ rad}$	8.6×10^{-16}
		$\Delta_y < 20 \mu\text{m}$	
	$(\eta_{cy} - \theta_{cy}) \cdot T_{zz} \cdot \Delta_z$	$\eta_{cy} - \theta_{cy} < 2.6 \times 10^{-3} \text{ rad}$	6.4×10^{-16}
$\Delta_z < 20 \mu\text{m}$			
Defects between the two sensors	$2 \cdot K_{1dx} \cdot \Gamma_{res_{df},x}$	$K_{1dx} < 10^{-2}$	2×10^{-14}
	$2 \cdot (\eta_{dz} + \theta_{dz}) \cdot \Gamma_{res_{df},y}$	$\eta_{dz} + \theta_{dz} < 1.6 \times 10^{-3} \text{ rad}$	3.0×10^{-15}
	$2 \cdot (\eta_{dy} - \theta_{dy}) \cdot \Gamma_{res_{df},z}$	$\eta_{dy} - \theta_{dy} < 1.6 \times 10^{-3} \text{ rad}$	3.0×10^{-15}
Quadratic non linearities	$4 \cdot K_{2,cxx} \cdot \Gamma_{app,dx} \cdot \Gamma_{res_{df},x}$	$K_{2cxx} < 20000 \text{ s}^2/\text{m}$	8.0×10^{-16}
	$2 \cdot K_{2,dxx} \cdot (\Gamma_{res_{df},x}^2 + \Gamma_{app,dx}^2)$	$K_{2dxx} < 20000 \text{ s}^2/\text{m}$	8.0×10^{-16}
	Total = $\sum $		2×10^{-13}

Total contribution :
 $2 \cdot 10^{-13} \text{ m} \cdot \text{s}^{-2}$
 $> 8 \cdot 10^{-15}$ (mission accuracy objective)

→ an in-orbit calibration is necessary

Calibration procedures : off-centrings

$$\Gamma_{mes,dx} = \frac{1}{2} (\Gamma_{mes,1} - \Gamma_{mes,2}) = \frac{1}{2} K_{1cx} \cdot \delta \cdot g_{x/sat} + \frac{1}{2} \begin{bmatrix} K_{1cx} \\ \eta_{cz} + \theta_{cz} \\ \eta_{cy} - \theta_{cy} \end{bmatrix}^t \cdot [T - In] \cdot \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix}$$

$$+ \begin{bmatrix} K_{1dx} \\ \eta_{dz} + \theta_{dz} \\ \eta_{dy} - \theta_{dy} \end{bmatrix}^t \cdot (\vec{\Gamma}_{res_{df}} + C_x) + 2 \cdot K_{2cxx} \cdot (\Gamma_{app,dx} + b_{1dx}) \cdot (\Gamma_{res_{df},x} + C_x - b_{0cx})$$

$$+ K_{2dxx} \cdot \left((\Gamma_{res_{df},x} + C_x - b_{0cx})^2 + (\Gamma_{app,dx} + b_{1dx})^2 \right)$$

$K_{1cx} \Delta_x$ and $K_{1cx} \Delta_z$: use the important value of T_{xx} and T_{xz} at $2f_{orb}$

cosine part: $\Gamma_{mes,dx/cos}(2f_{orb}) = \frac{1}{2} K_{1cx} \cdot T_{xx}(2f_{orb}) \cdot \Delta_x$

sine part: $\Gamma_{mes,dx/sin}(2f_{orb}) = \frac{1}{2} K_{1cx} \cdot T_{xz}(2f_{orb}) \cdot \Delta_z$

$K_{1cx} \Delta_y$: T_{xy} too weak \rightarrow oscillate the satellite around Y_{sat}

$$\Gamma_{mes,dx/cos}(f_{cal/ang}) = \frac{1}{2} K_{1cx} \cdot (T_{xy}(f_{cal/ang}) - \alpha_0 \omega_{cal/ang}^2) \cdot \Delta_y$$

Other calibration procedures

$$\Gamma_{mes, dx} = \frac{1}{2} (\Gamma_{mes, 1} - \Gamma_{mes, 2}) = \frac{1}{2} K_{1cx} \cdot \delta \cdot g_{x/sat} + \frac{1}{2} \begin{bmatrix} K_{1cx} \\ \eta_{cz} + \theta_{cz} \\ \eta_{cy} - \theta_{cy} \end{bmatrix}^t \cdot [T - In] \cdot \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix}$$

$$+ \begin{bmatrix} K_{1dx} \\ \eta_{dz} + \theta_{dz} \\ \eta_{dy} - \theta_{dy} \end{bmatrix}^t \cdot (\vec{\Gamma}_{res_{df}} + C_x) + 2 \cdot K_{2cxx} \cdot (\Gamma_{app, dx} + b_{1dx}) \cdot (\Gamma_{res_{df}, x} + C_x - b_{0cx})$$

$$+ K_{2dxx} \cdot \left((\Gamma_{res_{df}, x} + C_x - b_{0cx})^2 + (\Gamma_{app, dx} + b_{1dx})^2 \right)$$

- Parameters of the common sensitivity matrix ($\eta_{cz} + \theta_{cz}$, $\eta_{cy} - \theta_{cy}$):** oscillation of the test masses along Y and Z at f_{TM} + modulation of the Earth gravity gradient at $2f_{orb}$ → calibration signal at $f_{TM} + 2f_{orb}$
- Parameters of the differential sensitivity matrix (K_{1dx} , $\eta_{dz} + \theta_{dz}$, $\eta_{dy} - \theta_{dy}$):** oscillation of the satellite along X, Y or Z through the drag-free command C
- Differential quadratic factor K_{2dxx} :** oscillation of the satellite along X through the drag-free command C → calibration signal at $2f_{cal/lin}$
- Common quadratic factor K_{2cxx} :**
 - K_{21xx} : oscillation of the test mass 1 along X, drag-free locked on the sensor 2 → calibration signal at $2f_{TM}$
 - K_{22xx} : oscillation of the test mass 2 along X, drag-free locked on the sensor 1 → calibration signal at $2f_{TM}$

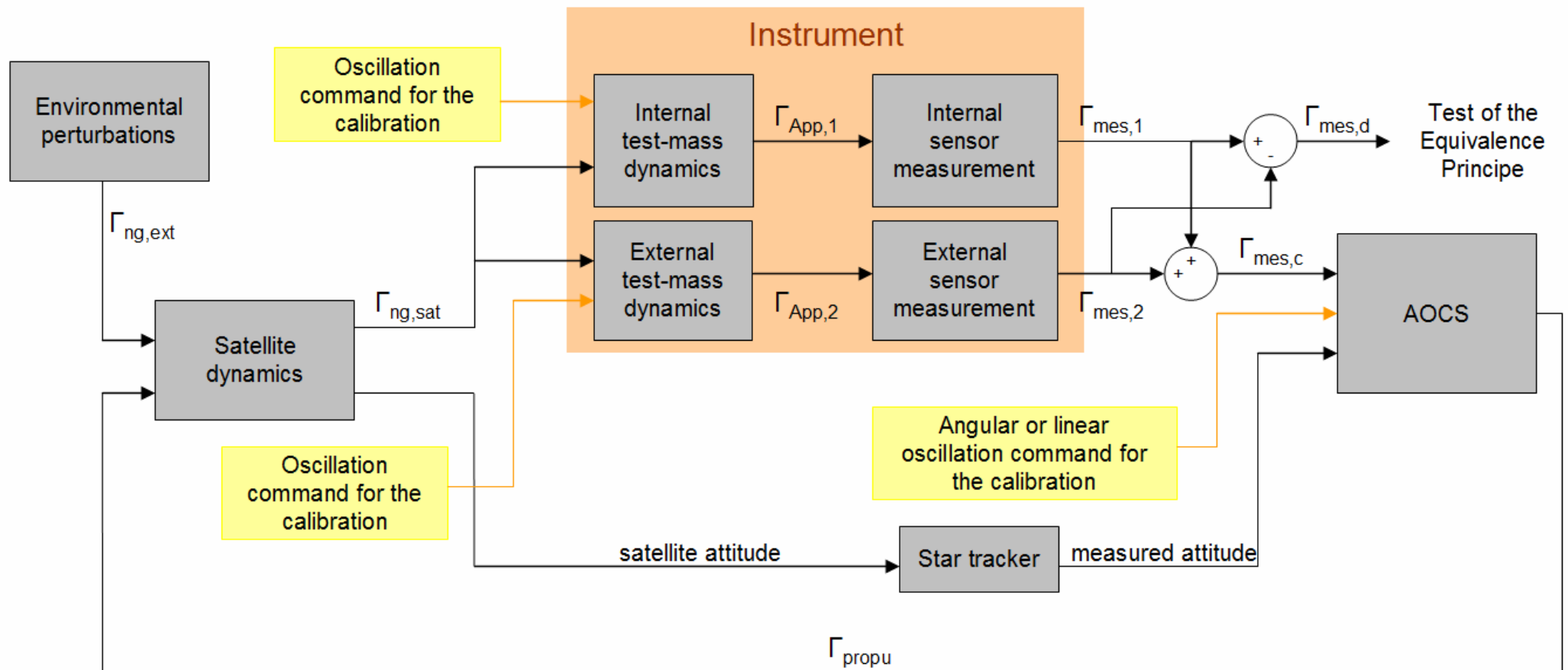
Evaluated calibration budget

$T_{\text{cal}} = 10$ orbits

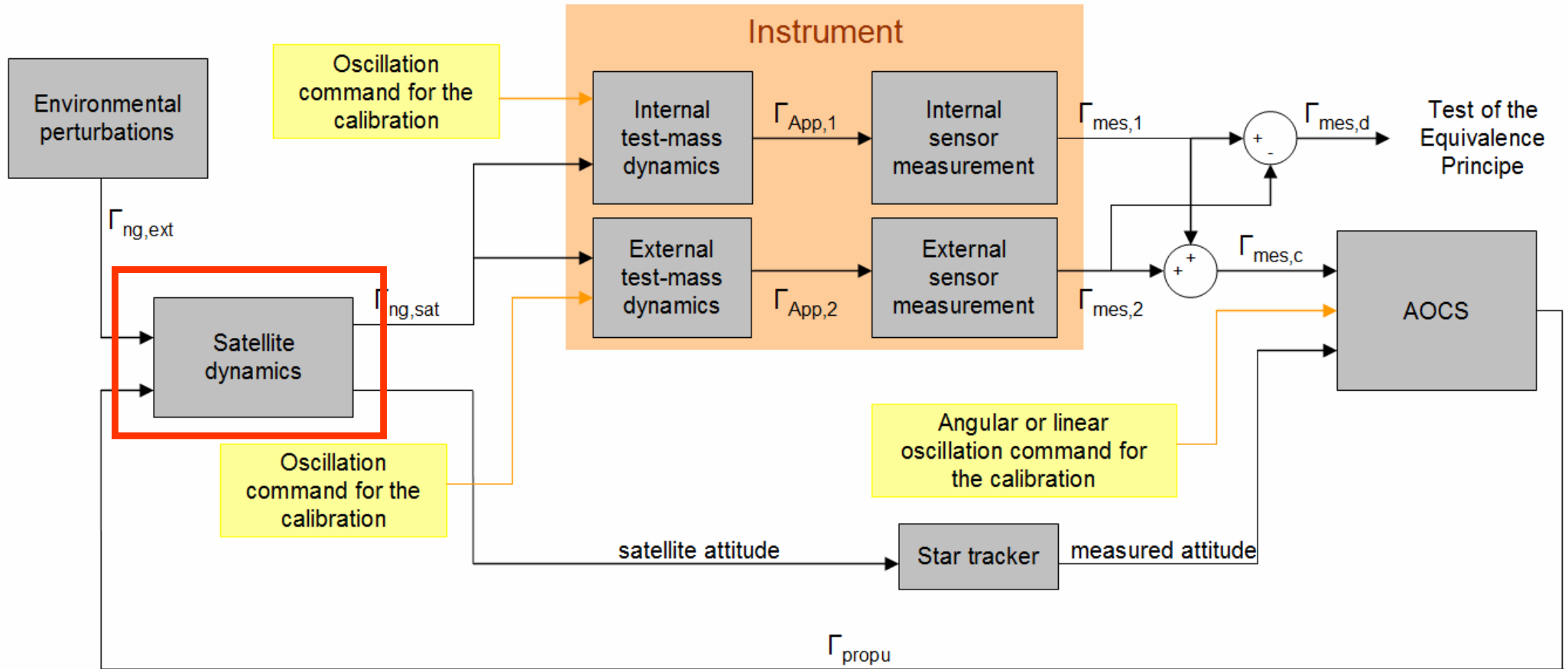
Parameter to be calibrated	Perfo. after calibration	Specification
$K_{1cx} \cdot \Delta_x$	0.10 μm	0.1 μm
$K_{1cx} \cdot \Delta_z$	0.11 μm	0.1 μm
$K_{1cx} \cdot \Delta_y$	1.2 μm	2 μm
$(\eta_{cz} + \theta_{cz})$	1.0×10^{-3} rad	9.0×10^{-4} rad
$(\eta_{cy} - \theta_{cy})$	9.5×10^{-4} rad	9.0×10^{-4} rad
(K_{1dx} / K_{1cx})	3.1×10^{-5}	$1.5 \cdot 10^{-4}$
Θ_{dz}	2.3×10^{-6} rad	$5 \cdot 10^{-5}$ rad
Θ_{dy}	2.3×10^{-6} rad	$5 \cdot 10^{-5}$ rad
K_{2dxx} / K_{1cx}^2	50.2 s^2/m	250 s^2/m
K_{2cxx} / K_{1cx}^2	581.9 s^2/m	1000 s^2/m

→ Simulator to test the validity of the planned calibration procedures

Simulateur d'étalonnage : structure

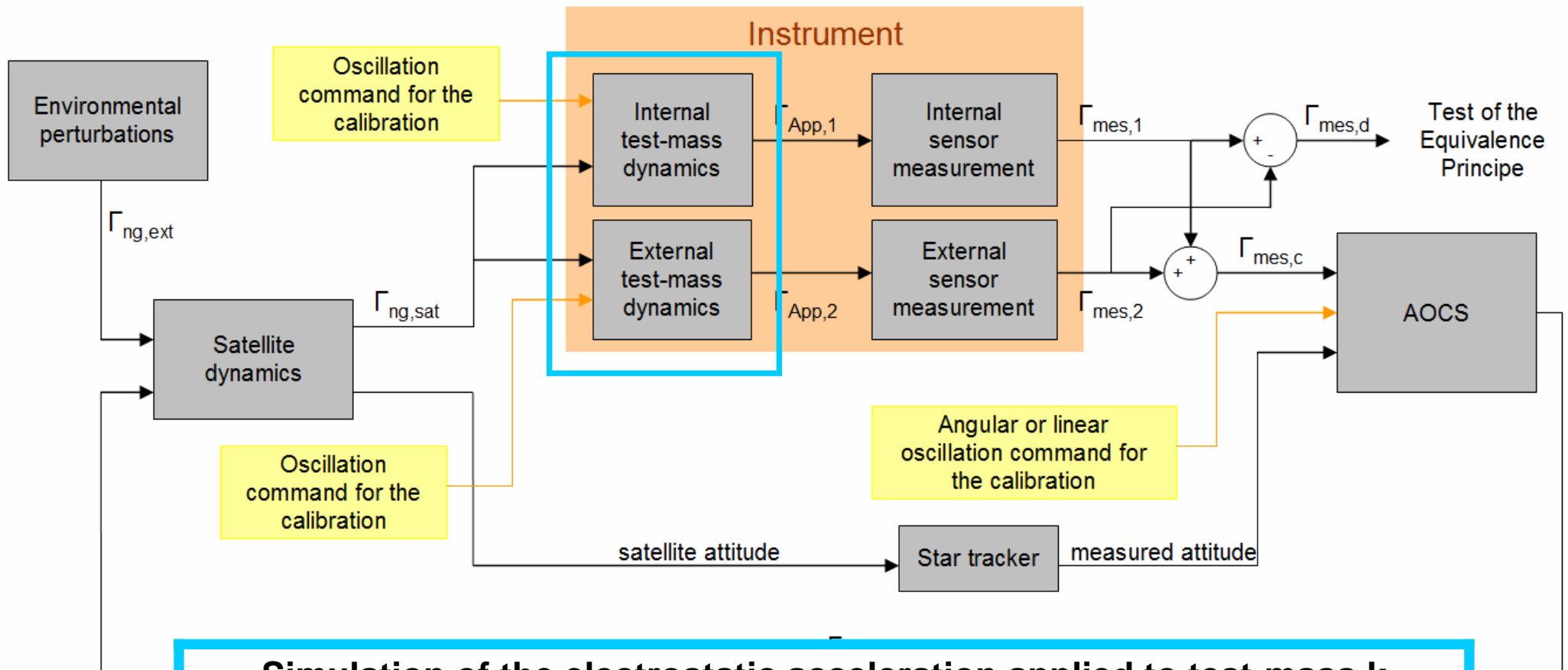


Dynamique du satellite dans un environnement perturbé



Satellite dynamics	
Input	Output
Thrust accélération Γ_{propu}	- Non-gravitationnal acceleration of the satellite $\Gamma_{ng,sat}$ - Attitude of the satellite for the star tracker
Influence of the environmental perturbations	
<ul style="list-style-type: none"> - solar radiation pressure - residual atmospheric drag $\Gamma_{ng,sat} = \Gamma_{propu} + \Gamma_{ext,sat}$	

The instrument

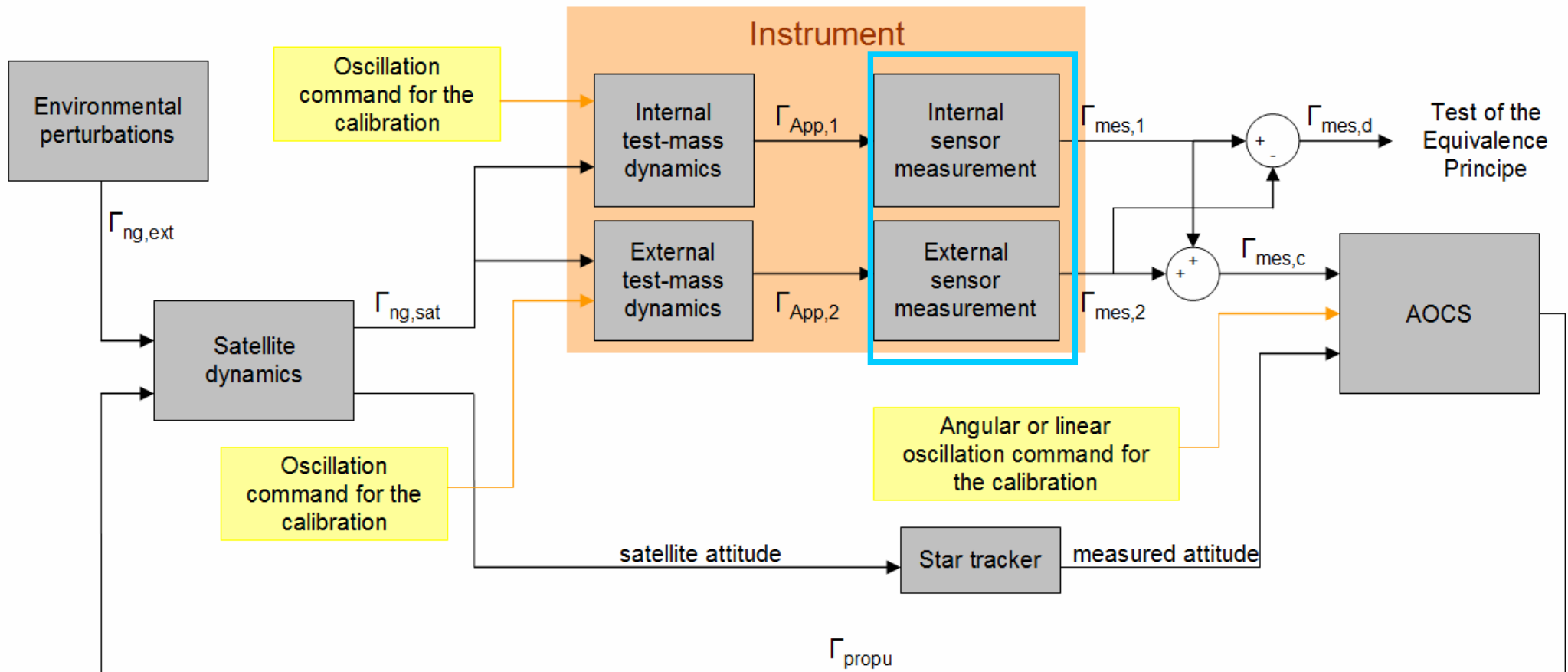


Simulation of the electrostatic acceleration applied to test-mass k

Input	Output
<ul style="list-style-type: none"> - Non gravitationnal acceleration of the satellite $\Gamma_{ng,sat}$ - Position of the test-mass relatively to the center of the electrostatic cage $O_c O_k$ 	Electrostatic acceleration applied to the test-mass k in order to keep it centered

$$\vec{\Gamma}_{App,k} = \frac{\vec{F}_{NGsat}}{M_{Isat}} + ([T] - [In]) \cdot \vec{O}_k O_c - [Cor] \dot{\vec{O}}_k O_c - \ddot{\vec{O}}_k O_c$$

The instrument

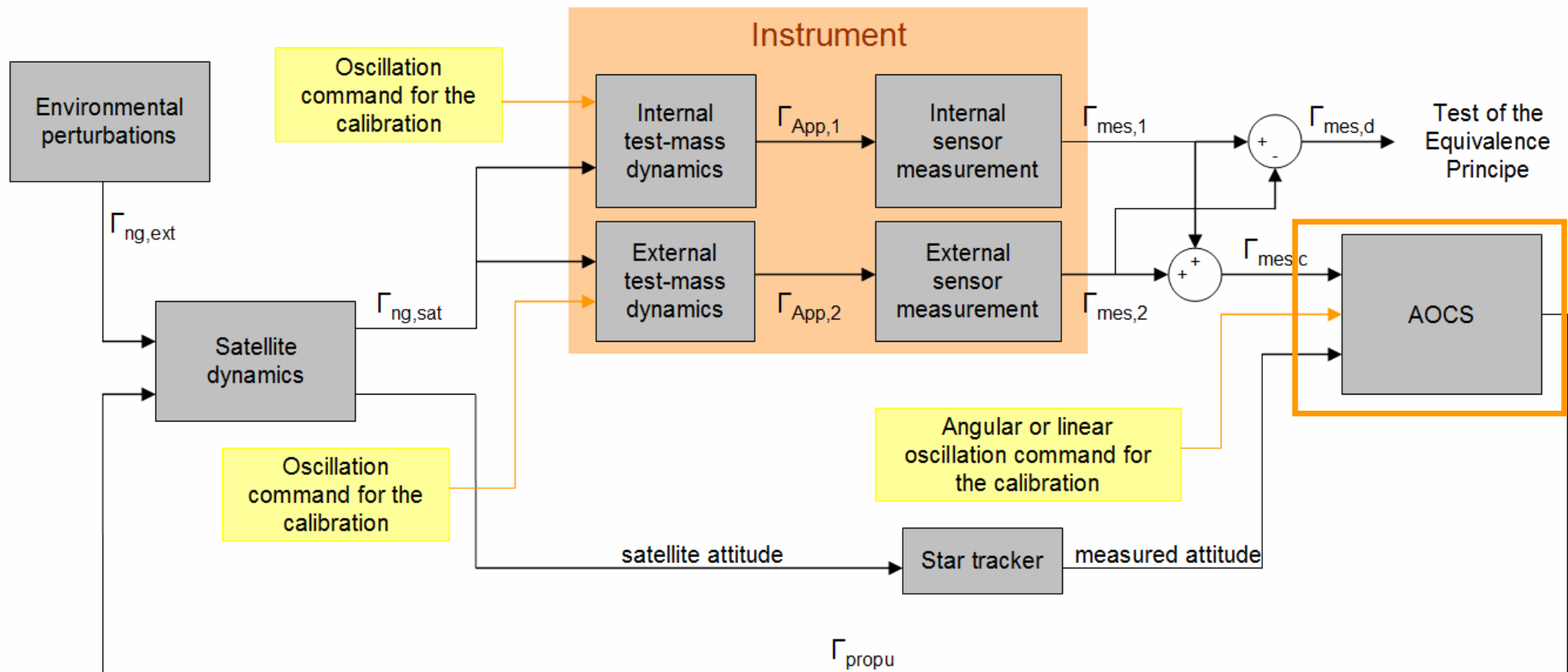


Measurement of the applied acceleration

Input	Output
Electrostatic acceleration applied to test-mass k to keep it centered $\Gamma_{App,k}$	Measured acceleration $\Gamma_{mes,k}$

- Bias
- Noise
- Sensitivity matrix : scale factor, coupling, misalignement
- Quadratic factors

Attitude and Orbit Control System (AOCS)



Drag-free system

Input

Output

Common mode acceleration measurement of the test-masses $\Gamma_{mes,c}$

Acceleration applied to the satellite by the thrusters Γ_{propu}

– Hybridation of the attitude measured with the star tracker (LF) and the angular acceleration measured with the instrument (HF)

– AOCS calculator: fonction de transfert : $\Gamma_{DF} = TF_{DF} (\Gamma_{mes,c} + C_{\text{étalonnage}})$

– Thrusters: sensitivity matrix and noise $\Gamma_{propu} = -[M_{propu}] \Gamma_{DF} + \Gamma_{n,DF}$

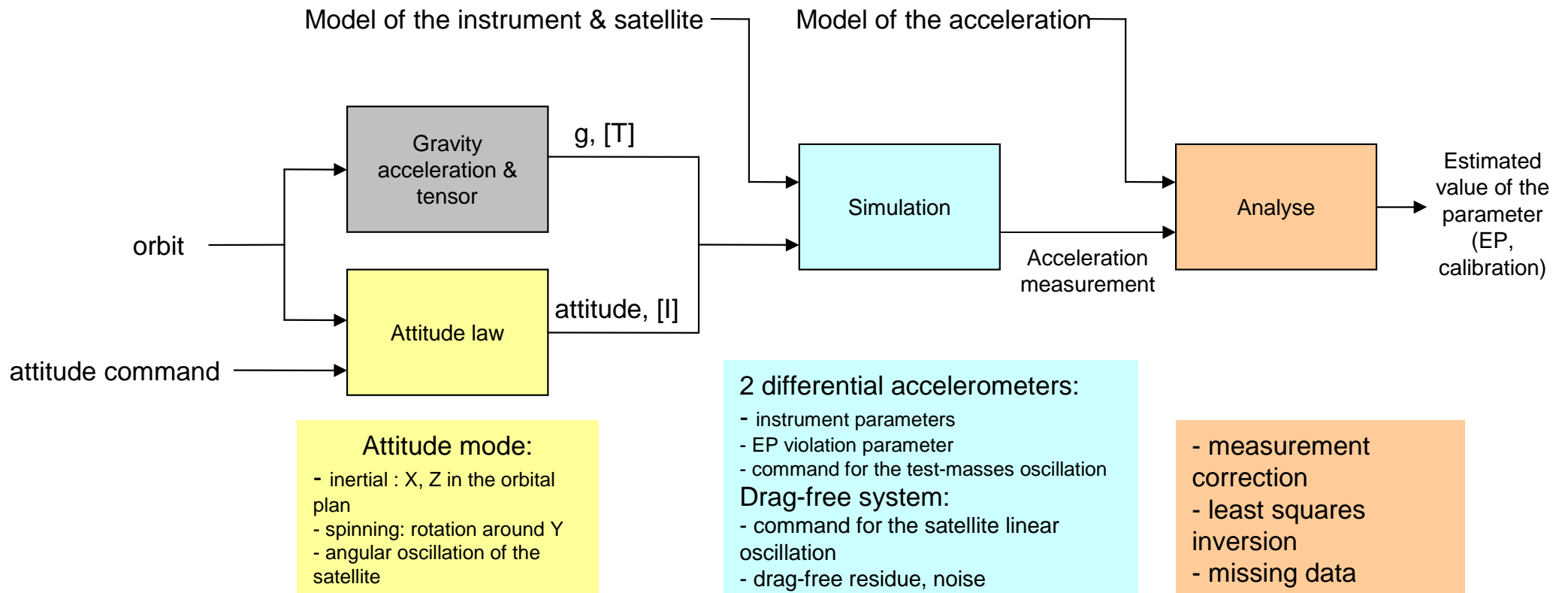
Results of the simulation

Parameter to be calibrated	Max value	Estimation accuracy: specification	Estimation accuracy: worst result	Estimation accuracy: mean result	Estimation accuracy: standard deviation
$K_{1dx} \Delta_x$	20 μm	0.1 μm	0.04 μm	0.01 μm	7.3 nm
$K_{1dx} \Delta_z$	20 μm	0.1 μm	0.05 μm	0.03 μm	6.5 nm
$K_{1dx} \Delta_y$	20 μm	2 μm	0.2 μm	0.05 μm	0.04 μm
$\eta_{cy} - \theta_{cy}$	$2.6 \cdot 10^{-3}$ rad	$9.0 \cdot 10^{-4}$ rad	$1.0 \cdot 10^{-3}$ rad	$3.1 \cdot 10^{-4}$ rad	$2.3 \cdot 10^{-4}$ rad
$\eta_{cz} + \theta_{cz}$	$2.6 \cdot 10^{-3}$ rad	$9.0 \cdot 10^{-4}$ rad	$1.1 \cdot 10^{-3}$ rad	$2.6 \cdot 10^{-4}$ rad	$2.6 \cdot 10^{-4}$ rad
$(K_{1dx}/K_{1cx})'$	10^{-2}	$1.5 \cdot 10^{-4}$	$5 \cdot 10^{-6}$	$1.6 \cdot 10^{-6}$	$1.2 \cdot 10^{-6}$
Θ_{dy}	$1.6 \cdot 10^{-4}$ rad	$5 \cdot 10^{-5}$ rad	$2 \cdot 10^{-5}$ rad	$1.0 \cdot 10^{-6}$ rad	$8.3 \cdot 10^{-7}$ rad
Θ_{dz}	$1.6 \cdot 10^{-4}$ rad	$5 \cdot 10^{-5}$ rad	$4 \cdot 10^{-6}$ rad	$1.2 \cdot 10^{-6}$ rad	$1.7 \cdot 10^{-6}$ rad
K_{2dxx}/K_{1cx}^2	14000 s ² /m	250 s ² /m	124 s ² /m	25 s ² /m	23 s ² /m
K_{2cxx}/K_{1cx}^2	14000 s ² /m	1000 s ² /m	274 s ² /m	62 s ² /m	54 s ² /m

Estimation accuracy after calibration : statistical analysis with 100 simulations

Session duration: 10 orbits, except for $\eta_{cz} + \theta_{cz}$: 40 orbits

Mission simulator



Calibration sessions + session for the EP test → Correction of the effects of the instrumental parameters → Estimation of the EP parameter with 10^{-15} accuracy

Calibration process definition

- The budget of the measurement equation before calibration does not comply with the objective of the EP test accuracy
- Several in flight calibrations are necessary during the space experiment
- Parameters to be calibrated have been identified and appropriate methods of calibration have been proposed.

Calibration process validation

- Development of a simulation software including models of the instrument and the satellite drag-free system and implementation of the calibration methods
- Results compatible with the specifications

Data processing validation

- Association with a dedicated software for the EP test sessions in order to test the entire mission scenario
- Correction of the measurement with the parameters estimated during the calibration process: estimation of the EP parameter to validate the mission performance
- Influence of the numerical effects (missing data)