Perturbed de Sitter

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The question : **Lambda or not Lambda ?**

- gravitation (& Lambda) vs cosmology (observations)

- general considerations on the « role » of Lambda

- local effects of Lambda ?

  → a model that shows it could generate anisotropies

I – Cosmological context

Context : Accelerated expansion of the universe interpreted in the

General Relativity with cosmological constant (LGR) framework

→ Concordance LambdaCDM (LCDM) model

LCDM **advantages** … :

- well known & tested physics : gravitation / general relativity
  (but with a cosmological constant ←→ vacuum as perfect fluid $P = -\varepsilon$)

- **the model works very well !** (SN1a, CMBR, …)

- refers to vacuum energy in physics (Casimir effect, ….)

… but **unsolved problems** :

- bad interface with quantum field theory : **120 (60 ?) orders between the cosmological Lambda & its QFT expected value** (vacuum energy)

- coïncidence pb, …

→ some authors prefer other options

change gravity theory	- ~~general relativity~~ → gravity = scalar-tensor, $f(R)$, ….
change matter content	- matter content includes exotic (dark) matter/energy
remove symetries	- inhomogeneities (voids, ….)
	- ….

II – General considerations on the cosmological constant effects

Discarding here this controversy, the fact the interpretation in terms of Λ results in a valuable cosmological scenario raises the question :

could Λ result in observable **effects at scales smaller than cosmological scales** ?

no Λ clustering effect ➔ cosmo amplitude → amplitude for all scales (in some sense…)

Works made along these lines (LGR) :

matter
- motions about black holes → incidences on accretion disks (?) [refs …]
- gravitational equilibrium [refs …]
- solar system : periastron shift, … [refs …]
- weak local value of the Hubble parameter (~ 60 km/s/Mpc vs ~ 70) [refs …]

light
- lensing [refs …]

Often expected local effect on local structures, like clusters : the common wisdom says

« the cosmological constant acts as a **radial** repulsive force proportional to the distance »

A general proof of this claim ??????

Supported by **Schwarzschild-de Sitter** solution …

$$ds^2 = -\left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right) dt^2 + \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

weak field

$$\vec{g}_{eff} = -m\frac{\vec{r}}{r^3} + \frac{\Lambda}{3}\vec{r}$$

… and by **RW-cosmological models** (including the Einstein static universe) ...

… but in all these models, the **spherical symmetry is present from the very start !!!**

… and … **solutions are known that do not share this property** (Lambda-Kassner)

➔ Λ **may result in non-spherical effects**

Λ vs **vacuum homogeneous** solutions vacuum in LGR : $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$

Bianchi I metrics : $ds^2 = -dt^2 + g_{ij}(t)\,dx^i dx^j$

$\Lambda \neq 0$ solutions

$\Lambda = 0$ solutions

Λ - Kassner $\xrightarrow{\Lambda \to 0}$ Kassner : $ds^2 = -dt^2 + t^{2p}dx^2 + t^{2q}dy^2 + t^{2r}dz^2$

with $p + q + r = p^2 + q^2 + r^2 = 1$

$t \to \infty$ $\quad H_x \sim H_y \sim H_z$

space isotropisation

$t \to \infty$ $\begin{cases} H_x = p/t \\ H_y = q/t \\ H_z = r/t \end{cases}$ $\quad \dfrac{H_x}{H_y} = \dfrac{p}{q} = cst \neq 1$

NO space isotropisation !!!

de Sitter

$ds^2 = -dt^2 + e^{2Kt}\left(dx^2 + dy^2 + dz^2\right)$

$\xrightarrow{3K^2 \equiv \Lambda \to 0}$

Minkowski : $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$

(= Kassner / $p = 1 , q = r = 0$)

Isotropic (& homogeneous) solutions

→ At the cosmological level, a non-zero cosmological constant drives a vacuum (asympt vac ?) expanding (Bianchi I) universe into an isotropic state

III - How to determine the general Lambda effect on structures ?

Preliminary study : expand LGR equation $R_{\alpha\beta} = 8\pi\left(T_{\alpha\beta} - \frac{1}{2}Tg_{\alpha\beta}\right) + \Lambda g_{\alpha\beta}$

- about Minkowski $g_{\alpha\beta} = m_{\alpha\beta} + h_{\alpha\beta}$ with $|h_{\alpha\beta}| << 1$
- without any prior symmetry assumption

→ OK for **local effects** (← Minkowski is NOT a LGR (vacuum) solution) …

Not necessarily isotropic …. (Chauvineau & Regimbau, 2012)

But : what if **more than** just **local** questions are into consideration ?

For instance, if one has to :

- match local (anisotropic) effects to (isotropic) cosmological expansion ?
- consider in a coherent way local effects here & there ?

If **more than** just **local** ➔ **expansion about an exact LGR** is required

➔ choice : expand about **de Sitter** :

- **simplest** exact LGR
- cosmological context
- vacuum ($T_{\alpha\beta} = 0$) → isolate **vacuum** (w.r.t. matter) **effects**

→ **impact** of perturbations **on *z* distribution** (cosmological observable)

De Sitter in Robertson-Walker coordinates

$$ds^2 = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right) \text{ with } a = e^{Kt} \quad \& \quad K = \sqrt{\frac{\Lambda}{3}}$$

$$ds^2 = -\left(1 - \frac{\Lambda \tilde{r}^2}{3}\right)d\tilde{t}^2 + \left(1 - \frac{\Lambda \tilde{r}^2}{3}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

→ perturbed metric (use gauge freedom to maintain synchronous coord.)

$$ds^2 = -dt^2 + a^2\left[\delta_{ij} + \theta_{ij}\left(t, x^k\right)\right]dx^i dx^j \quad \text{with} \quad \left|\theta_{ij}\right| << 1$$

$$\left(x^1, x^2, x^3\right) \equiv (x, y, z)$$

→ Insert in $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$ & linearize → a way to (linearized) solutions :

Choose any $V(x,y,z)$ & $\Psi_i(x,y,z) \xrightarrow{U \text{ def}} 4K^2 U(x,y,z) \equiv div\left(\vec{\Psi} - \vec{\partial}V\right)$

$$\theta_{kk} = a^{-2}U + V$$

$$\partial_k \theta_{ik} = a^{-2}\partial_i U + \Psi_i$$

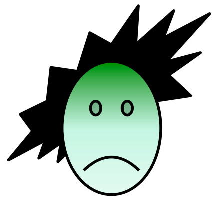
$$\partial_k \partial_k \theta_{ij} - a^{-1}\partial_t\left(a^3 \partial_t \theta_{ij}\right) = a^{-2}\partial_i \partial_j U + \partial_i \Psi_j + \partial_j \Psi_i - 2K^2 U \delta_{ij} - \partial_i \partial_j V$$

Linear system → general solution = particular sol. + homogeneous general sol.

ok (in Fourier form)

Let us consider the **homogeneous case** : $U = V = \Psi = 0$

$$\theta_{kk} = 0 \quad \& \quad \partial_k \theta_{ik} = 0 \quad \& \quad \partial_k \partial_k \theta_{ij} = a^{-1} \partial_t \left(a^3 \partial_t \theta_{ij} \right)$$



Solution presented as the sum of its spatial Fourier components :

$$\theta_{ij}(t,\vec{x}) = \int \left[\overline{C}_{ij} \left(\frac{\mu}{a} \cos\frac{\mu}{a} - \sin\frac{\mu}{a} \right) + \widetilde{C}_{ij} \left(\frac{\mu}{a} \sin\frac{\mu}{a} + \cos\frac{\mu}{a} \right) \right] \cos\left(K \vec{\mu} \vec{x}\right) d^3\vec{\mu}$$
$$+ \int \left[\overline{S}_{ij} \left(\frac{\mu}{a} \cos\frac{\mu}{a} - \sin\frac{\mu}{a} \right) + \widetilde{S}_{ij} \left(\frac{\mu}{a} \sin\frac{\mu}{a} + \cos\frac{\mu}{a} \right) \right] \sin\left(K \vec{\mu} \vec{x}\right) d^3\vec{\mu}$$

with $\overline{C}_{ij}(\vec{\mu})$ & $\widetilde{C}_{ij}(\vec{\mu})$ & $\overline{S}_{ij}(\vec{\mu})$ & $\widetilde{S}_{ij}(\vec{\mu})$ any real functions satisfying

$$\overline{C}_{ii} = \widetilde{C}_{ii} = \overline{S}_{ii} = \widetilde{S}_{ii} = 0 \quad \& \quad \overline{C}_{ij}\mu^j = \widetilde{C}_{ij}\mu^j = \overline{S}_{ij}\mu^j = \widetilde{S}_{ij}\mu^j = 0$$

→ each family of 6 functions B_{ij} (B for $\overline{C}_{ij}$, $\widetilde{C}_{ij}$, $\overline{S}_{ij}$ or $\widetilde{S}_{ij}$) satisfies 4 constraints

$B_{ii} = 0$
$B_{ij}\mu^j = 0$

Choose any $B(\vec{\mu})$ & $\Phi(\vec{\mu})$

$(\overline{C}, \overline{\Phi}_C)$
....
$(\widetilde{S}, \widetilde{\Phi}_S)$

$$\begin{pmatrix} B_{11} \\ B_{22} \\ B_{33} \\ B_{12} \\ B_{23} \\ B_{13} \end{pmatrix} = B \cdot \begin{pmatrix} 2\mu_1\mu_2\mu_3 \cos\Phi \\ 2\mu_1\mu_2\mu_3 \cos(\Phi + 2\pi/3) \\ 2\mu_1\mu_2\mu_3 \cos(\Phi + 4\pi/3) \\ \mu_3\mu_3\mu_3 \cos(\Phi + 4\pi/3) - \mu_1\mu_1\mu_3 \cos\Phi - \mu_2\mu_2\mu_3 \cos(\Phi + 2\pi/3) \\ \mu_1\mu_1\mu_1 \cos\Phi - \mu_1\mu_2\mu_2 \cos(\Phi + 2\pi/3) - \mu_1\mu_3\mu_3 \cos(\Phi + 4\pi/3) \\ \mu_2\mu_2\mu_2 \cos(\Phi + 2\pi/3) - \mu_2\mu_3\mu_3 \cos(\Phi + 4\pi/3) - \mu_1\mu_1\mu_2 \cos\Phi \end{pmatrix}$$

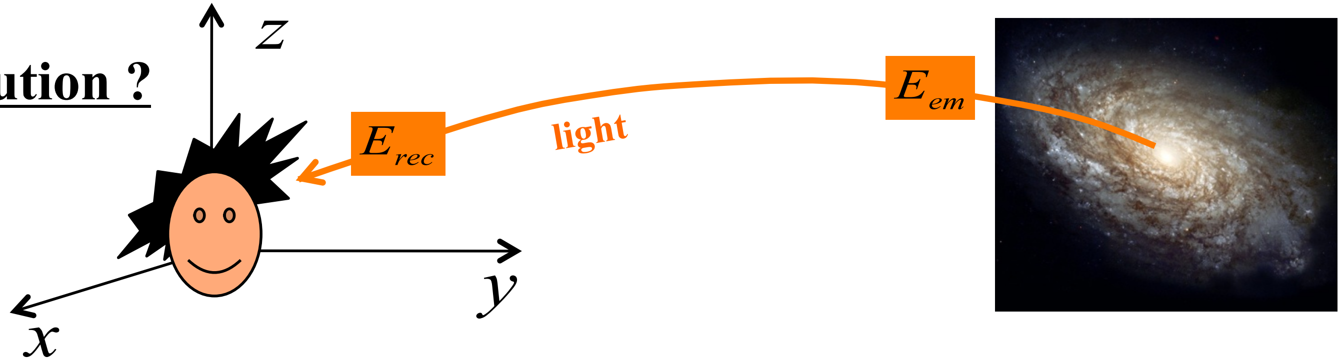
$$ds^2 = -dt^2 + a^2\left(\delta_{ij} + \theta_{ij}\right)dx^i dx^j \quad \text{with} \quad \left|\theta_{ij}\right| << 1$$

isotropic part (de Sitter)

anisotropic part → Expected anisotropic incidence on redshift distribution

Induced redshift distribution ?

$$1 + z = \frac{E_{em}}{E_{rec}}$$



$$\text{with} \quad E = -g_{\alpha\beta}\left(\frac{dx^\alpha}{dp}\right)_{\text{ph}}\left(\frac{dx^\beta}{d\tau}\right)_{\text{obs}} \quad \xrightarrow[\text{source \& obs}]{\text{comobile}} \quad E = \left(\frac{dt}{dp}\right)_{\text{ph}}$$

geodesics + isotropy (photon) $\xrightarrow{T = 1/a}$

$$\frac{d\left(aE\right)}{dp} = \frac{1}{2}Ka^2\left(\frac{dx^i}{dp}\frac{dx^j}{dp}\right)_{\text{ph}}\frac{\partial\theta_{ij}}{\partial T}$$

Simplest « mono-mode » case

Let us consider the case where the free Fourier amplitudes are chosen as

$$\tilde{C}(\vec{u}) = \tilde{c}\,\delta(\vec{u}-\vec{\sigma}) \quad \& \quad \tilde{\Phi} = \overline{C} = \tilde{S} = \overline{S} = 0 \quad \text{with} \quad \vec{\sigma} = \begin{pmatrix} 0 \\ 0 \\ \sigma > 0 \end{pmatrix} \quad \& \quad \tilde{c} = \text{cst}$$

$$1 + z_{\vec{N}} = \frac{a_{\text{obs}}}{a}\left(1 + \frac{\tilde{c}\,\sigma^3}{2} N^x N^y \left(\sigma T_{\text{obs}}\right)^2 \Gamma_{\vec{N}}\right) \qquad \left(\text{with} \quad T_{\text{obs}} = \frac{1}{a_{\text{obs}}}\right)$$

direction of observation

global anisotropic correction

$$\text{where} \quad \Gamma_{\vec{N}} = z\cos(\sigma T_{\text{obs}}) + \frac{z^2}{2}\left[\cos(\sigma T_{\text{obs}}) - \sigma T_{\text{obs}}\sin(\sigma T_{\text{obs}})\right]$$
$$- \frac{z^3}{3}\sigma T_{\text{obs}}\left[\sin(\sigma T_{\text{obs}}) + \frac{1+(N^z)^2}{2}\sigma T_{\text{obs}}\cos(\sigma T_{\text{obs}})\right] + O(z^4)$$

$\frac{a_{\text{obs}}}{\text{a}}$

further correction effective at ~ high z

General case

$$1 + z_{\vec{N}} = \frac{a_{\text{obs}}}{a}\left(1 - \frac{1}{2}\int d^3\vec{\mu}\left[I\left(\eta,\vec{\mu},\vec{N}\right) - I\left(\eta_{\text{obs}},\vec{\mu},\vec{N}\right)\right]\right)$$

where (with $Q_{\mu N} = \cos\left(\vec{u},\vec{N}\right)$ & $\widetilde{C}_N = \widetilde{C}_{ij}N^iN^j$, ...)

$$I\left(\eta \equiv \mu T,\vec{\mu},\vec{N}\right) = \left(\widetilde{C}_N + \overline{S}_N\right)\frac{\cos\left[\left(1+Q_{\mu N}\right)\eta - Q_{\mu N}\eta_{\text{obs}}\right]}{2\left(1+Q_{\mu N}\right)^2} + \left(\widetilde{C}_N - \overline{S}_N\right)\frac{\cdots\cdots}{\cdots\cdots}$$

$$+\eta\left[\left(\widetilde{C}_N + \overline{S}_N\right)\frac{\cdots\cdots}{\cdots\cdots} + \left(\widetilde{C}_N - \overline{S}_N\right)\frac{\cdots\cdots}{\cdots\cdots}\right]$$

$$+\left(\widetilde{S}_N - \overline{C}_N\right)\frac{\cdots\cdots}{\cdots\cdots} - \left(\widetilde{S}_N + \overline{C}_N\right)\frac{\cdots\cdots}{\cdots\cdots}$$

$$+\eta\left[-\left(\widetilde{S}_N - \overline{C}_N\right)\frac{\cdots\cdots}{\cdots\cdots} + \left(\widetilde{S}_N + \overline{C}_N\right)\frac{\cdots\cdots}{\cdots\cdots}\right]$$

Conclusions (?)

The present study shows that :

- cosmological constant acts anisotropically in the general case
- global $N^x N^y \sim \sin(2\alpha)$ effect
- more complex effects at « high » z
- relative anis. effect locally proportional to z

(last three items) ← studied mono-mode case

Going further (?)

- mono-mode results → general case ? (some results ….)
- comobility hypothesis → local impact of cosmo. cst. on local motions ?
- what happens/changes if matter is present ?
- link with observed dynamics in clusters ? (In our local group ?)
- ……


….
cosmological constant, de Sitter,
perturbations, anisotropies ….
….
… and thank you for your attention !!!
zzzz….