Markov chain Monte-Carlo orbit computation for binary asteroids

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Introduction

We present a novel orbit computation method for resolved binary asteroids. The new technique combines the well known Thiele-Innes method with Metropolis-Hastings algorithm. The new method will be used in the Gaia data processing pipeline.

Binary asteroids and the Gaia mission

A number of binary asteroids will be observed in the course of the Gaia mission. Particulairy the resolved binaries (when the two components are well separated and can be detected as two different bodies) or astrometric binaries (that is when the system appers as a single object but the binarity can be detected by a wobble of the photocenter around the center of mass) are of high interest [1], [2] [3]. For those objects improved orbits, masses and possibly densities can be computed.

It has been shown [3] that asteroid binaries both with large ($\approx$ 100 km) and small (< 10 km) primary bodies will be detected given favourable geometric conditions will be detectable by Gaia. For example binary asteroids with 0.3 arc second seperation or larger and small magnitude difference (< 2 mag) between the two components will be visible to Gaia. Gaia will fill the gap between adaptive optics imaging and radar detection techniques, thus has a potential of discovering binary asteroids that are not accessable to other techniques.

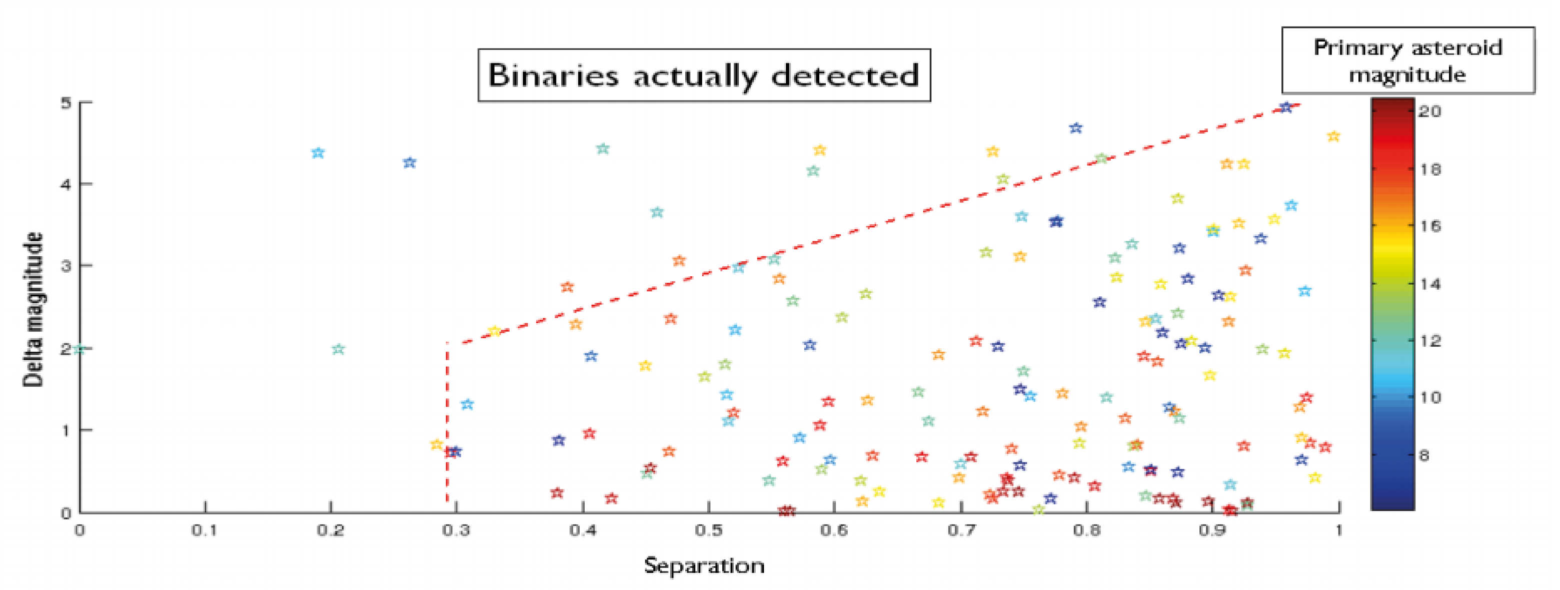

Figure 1: Binary asteroids detectable by Gaia. Angular separation vs. magnitude difference. Figure produced by [3].

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Conclusions and future plans

- We have presented a novel method for orbit computation for binary asteroids
- The new method was validated by comparing results with those obtained by other methods for well known binary systems
- The new method will be implemented into the Gaia processing pipeline and will be used in the short-term processing.

The method requires some additional tunning including the parallax computation and developing automatic convergence diagnostic tools. More extensive testing of the novel method is also required.

References

References

[1] Hestroffer, Daniel et al.: *The Gaia mission and the asteroids.*.Dynamics of Small Solar System Bodies and Exoplanets. Springer Berlin Heidelberg, 2010. 251-340.

[2] Hestroffer, Daniel: *Preparing GAIA for the Solar System.* EAS Publications Series, 2002. 359-364.

[3] Tanga, Paolo and Hestroffer, Daniel: *Gaia as a Solar System observatory: Perspectives for binary asteroids.*. Proceedings of the workshop. Vol. 1. 2012.

[4] Aitken, R., 1918. *The binary stars.*. Semicentennial publications of the University of California, 1868-1918. D.C. McMurtrie.

[5] Argyle, B., 2004. Observing and Measuring Visual Double Stars. No. vol. 1 in Patrick Moore's Practical Astronomy Series. Springer.

[6] Chib, Siddhartha, and Edward Greenberg. *Understanding the metropolis-hastings algorithm.*. The American Statistician 49.4 (1995): 327-335.

[7] Virtanen, J., Muinonen, K., Bowell, E., 2001. Statistical ranging of asteroid orbits. Icarus 154 (2), 412–431.

[8] Muinonen, K., Bowell, E., 1993. Asteroid orbit determination using bayesian probabilities. Icarus 104 (2), 255–279.

Markov-chain Monte Carlo with Thiele-Innes

The inverse problem of computing orbits of binary asteroids simialrly to individual objects [8, 7] can be taken to to probablistic in nature and therefore treated using the Bayesian approach and appropriate statistical methods. In particular we have developed a novel method combining the well known Thiele -Innes method [4, 5] and numerical methods. Principally we use the Metropolis-Hastings [6] algorithm to sample the parameters of the Thiele-Inners method. From the whole set of N observations $\psi_i = (x_i, y_i)$ made at observation times t_i (where $i = 1.., N$) we randomly select three observations from the same tangent plane and a staring orbital period P. We refer to those seven parameters as the sampling parameters and denote by $\mathbf{S} = (x_1, x_2, x_3, y_1, y_2, y_3, P)$. From the three selected observations and the period we compute a starting orbital elements $\mathbf{P} = (a, e, i, \Omega, \omega, P, \mathcal{M} = (m_1 + m_2))$ using the Thiele-Innes method. Once a starting orbit have been computed we start Markov chain Monte-Carlo (MCMC) sampling of the parameters $\mathbf{S}$ by adding random deviates to the selected three observations and the orbital period. In practice at each iteration t in a chain a new candidate sampling parameters are proposed using so-called proposal densities. In particular we make use of Gaussian proposal densities for all the seven sampling parameters. For the cartesian coordinates we use proposal densities that are centered around the last accepted sampling parameters in the chain and the size of the proposal density is proportional to the observational noise: $x_i^{(c)} \propto N(x_i^{(t-1)}, \sigma_{x_i})$, $y_i^{(c)} \propto N(y_i^{(t-1)}, \sigma_{y_i})$ (where $i = 1, 2, 3$) for x and y coordinates respectively. For the orbital period we use a normal distribution that is centered around the last accepted period $P^{(c)} \propto N(P^{(t-1)}, \sigma_P)$. The size of the proposal density for the orbital period σ_P is the only parameter to be tuned in the method, but in general in most of the cases, an educated guess of the size of that parameter can be made. Once a new candidate sampling parameters have been generated $\mathbf{S}^{(c)} = (x_1^{(c)}, x_2^{(c)}, x_3^{(c)}, y_1^{(c)}, y_2^{(c)}, y_3^{(c)}, P^{(c)})$ the M-H acceptance coefficient a_r is uded to accept or reject the sample parameters. In practice if the new orbit produces a better fit to the full observational data set, it is always accepted. If it produces a worse fit, it is accepted with the probability equal to a_r. The sampling is repeated until a large enough number of orbits have been obtained. After the sampling is complited convergence diagnostics has to be performed to insure that the stationary distribution was reached and to test for the lenght of burn-in period (the time required for the chain to reach the stationary). The obtained distributions of the orbital parameters reflect the properties of orbital element uncertainties.

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Preliminary results

To validate the new method, we compute orbital distributions for well-constrained systems. In particular we selected the binary asteroids 1998 WW_{31} 2000 QL_{251} as test cases. The two figures below present preliminary results for those two objects. The orbital elements agree with results obtained by other methods.

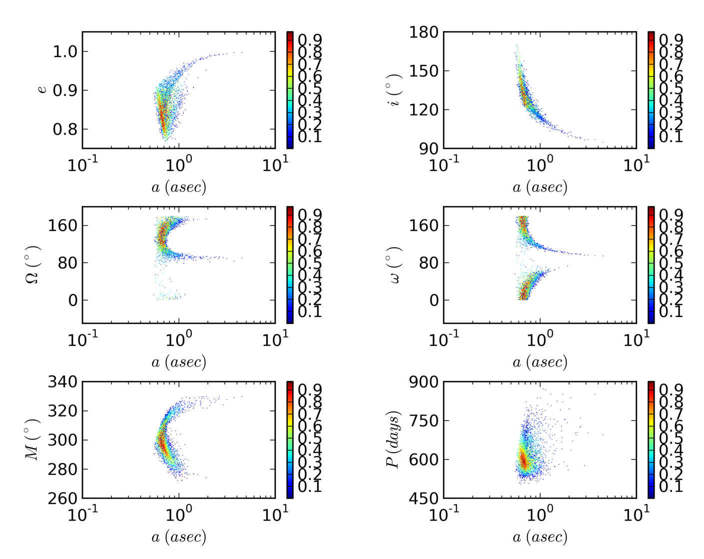

Figure 2: Distribution of orbital elements for asteroid 1998 WW_{31}. Color corresponds to normalized probability density value.

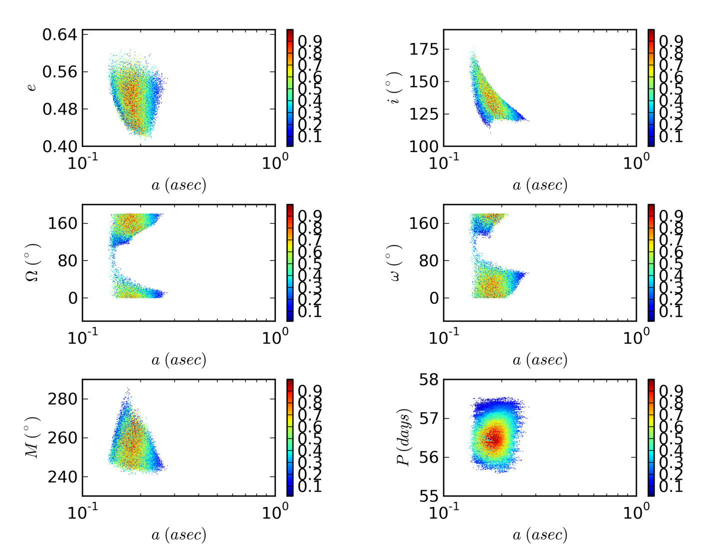

Figure 3: Distribution of orbital elements for asteroid 2000 QL_{251}. Color corresponds to normalized probability density value.

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